Table of content:

Mechanics of a Lamina

Book Geoff Eckold, Chapter 3, pp 49-65

A **Laminate** is consisting of several **Laminas or Plies or Layers.**

A **Lamina** is consisting of Fibers and Matrix.

Micromechanics (in μm-mm range) is dealing for example with the determination of **Lamina** constitutive properties from those of Fiber and Matrix, Fiber-Matrix interface stresses, etc.

Assumptions:

- Linear Elasticity: Matrix and Fiber behave as linear elastic material (viscoelasticity of Matrix: see previous chapter)

- Perfect bond, no strain discontinuity across interface

- Fibers are arranged in a regular or repeating array

Functional requirements for Fibers:

- High E-Modulus

- High ultimate strength

- Low variation between individual fibers

- Retain the strength during handling and fabrication

- Uniform diameter and surface

Functional requirements for Matrix:

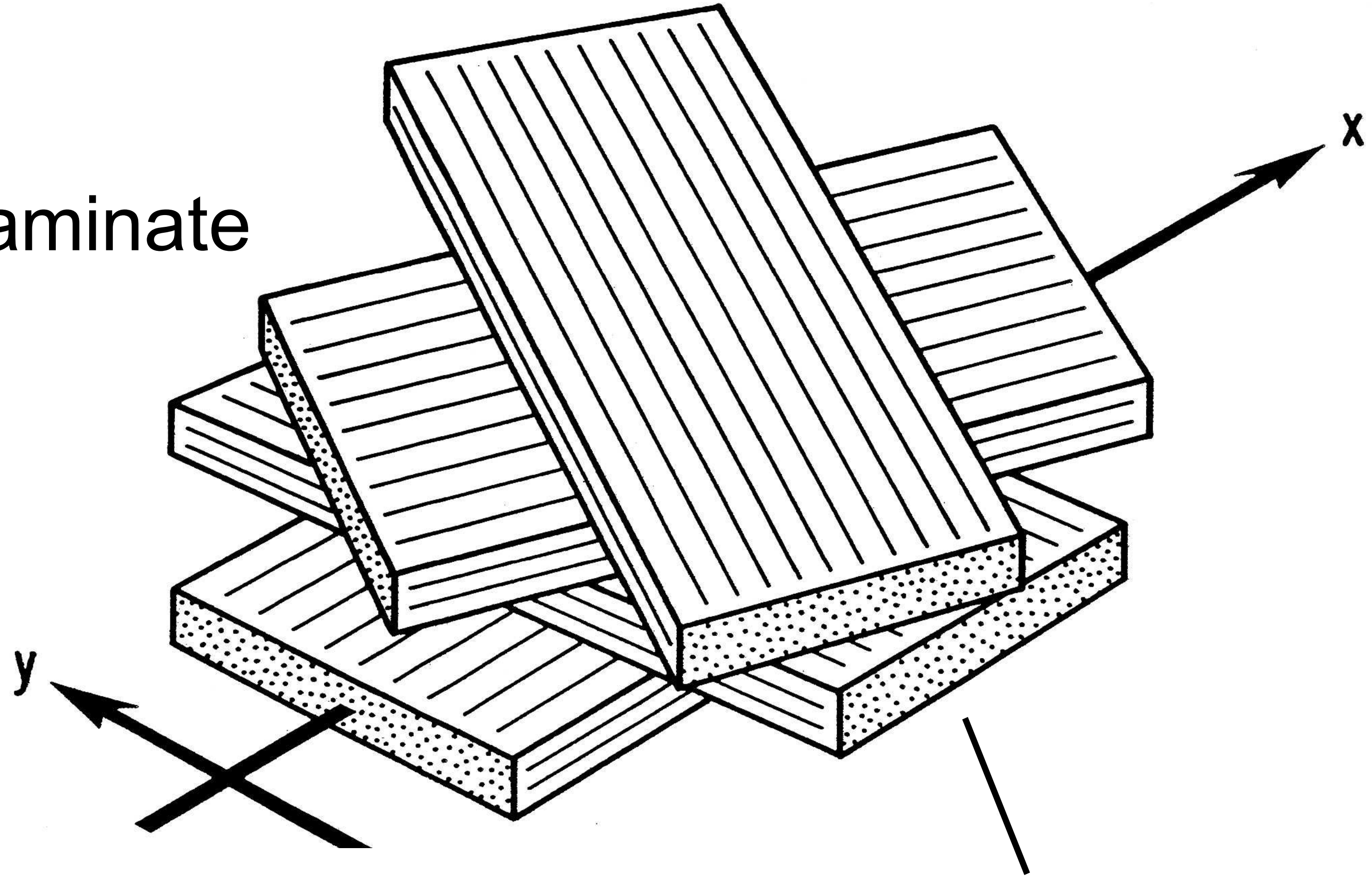
- Bind together the fibers and protect their surfaces

- Transfer stresses to the fibers efficiently

- Chemically compatible with fibers over a long period

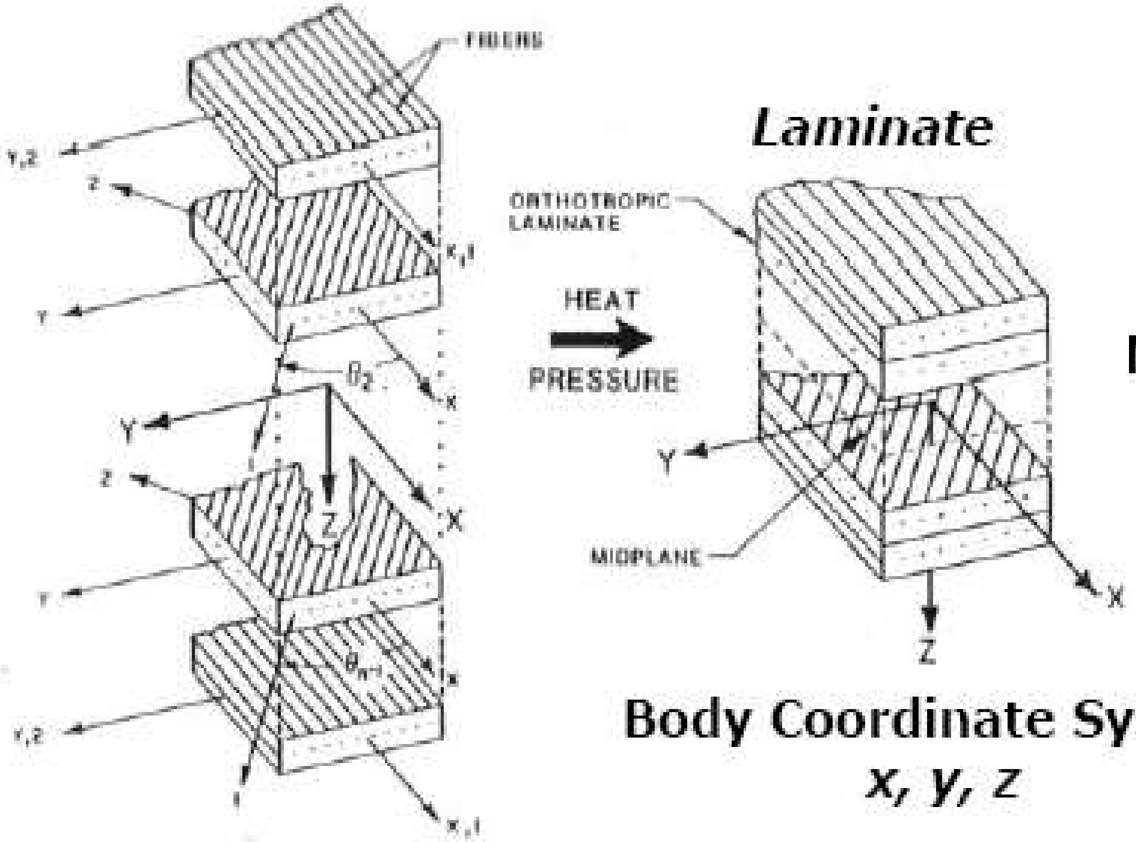
- Thermally compatible with fibers

Modell of a Laminate

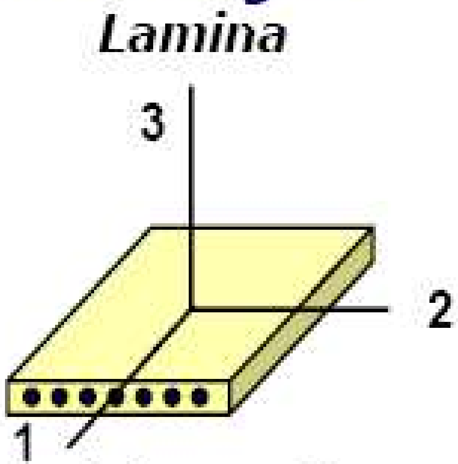


A Unidirectionally Reinforced Lamina (UD)

Lamina – Laminate Coord. Sys.

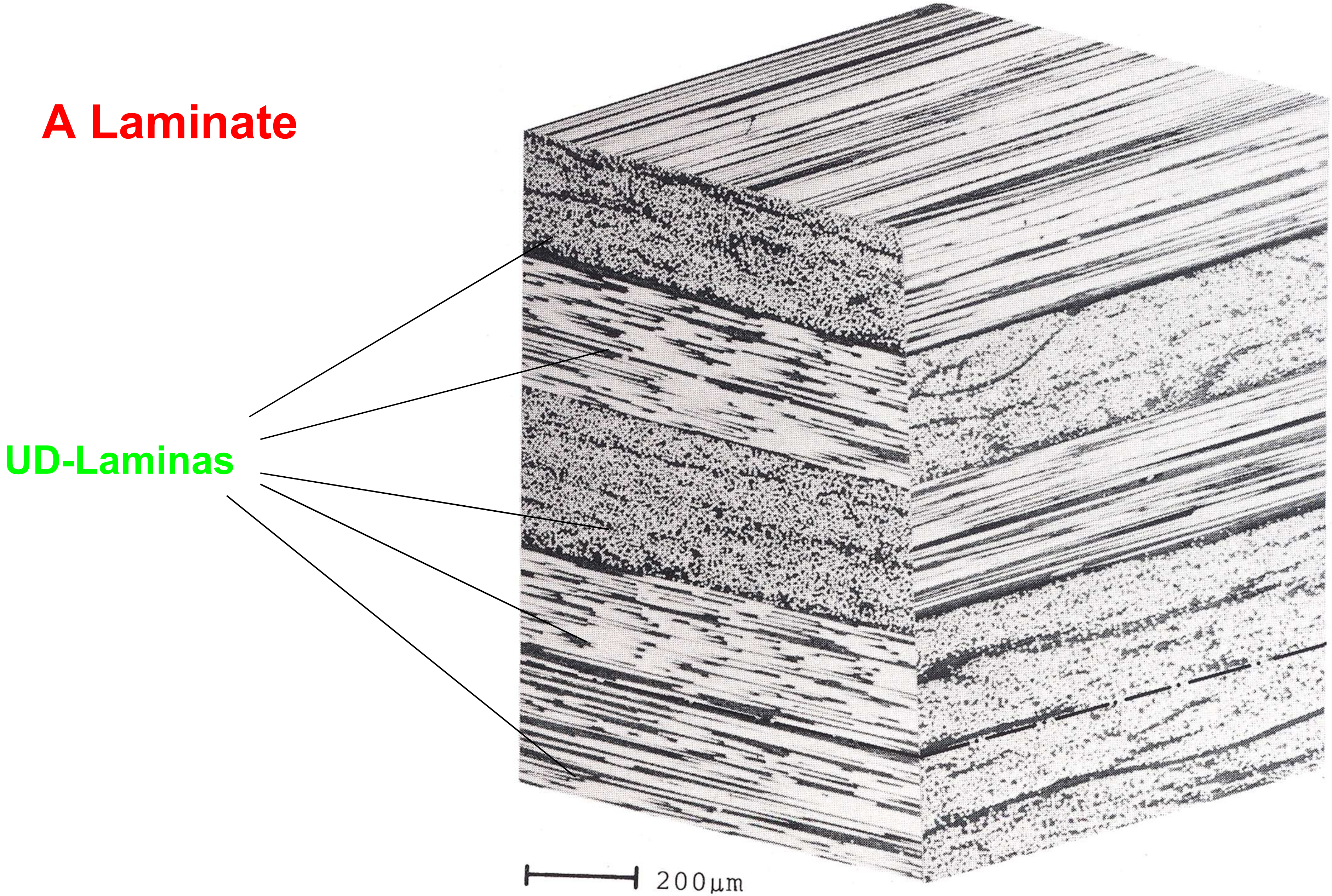


Body Coordinate System
x, y, z

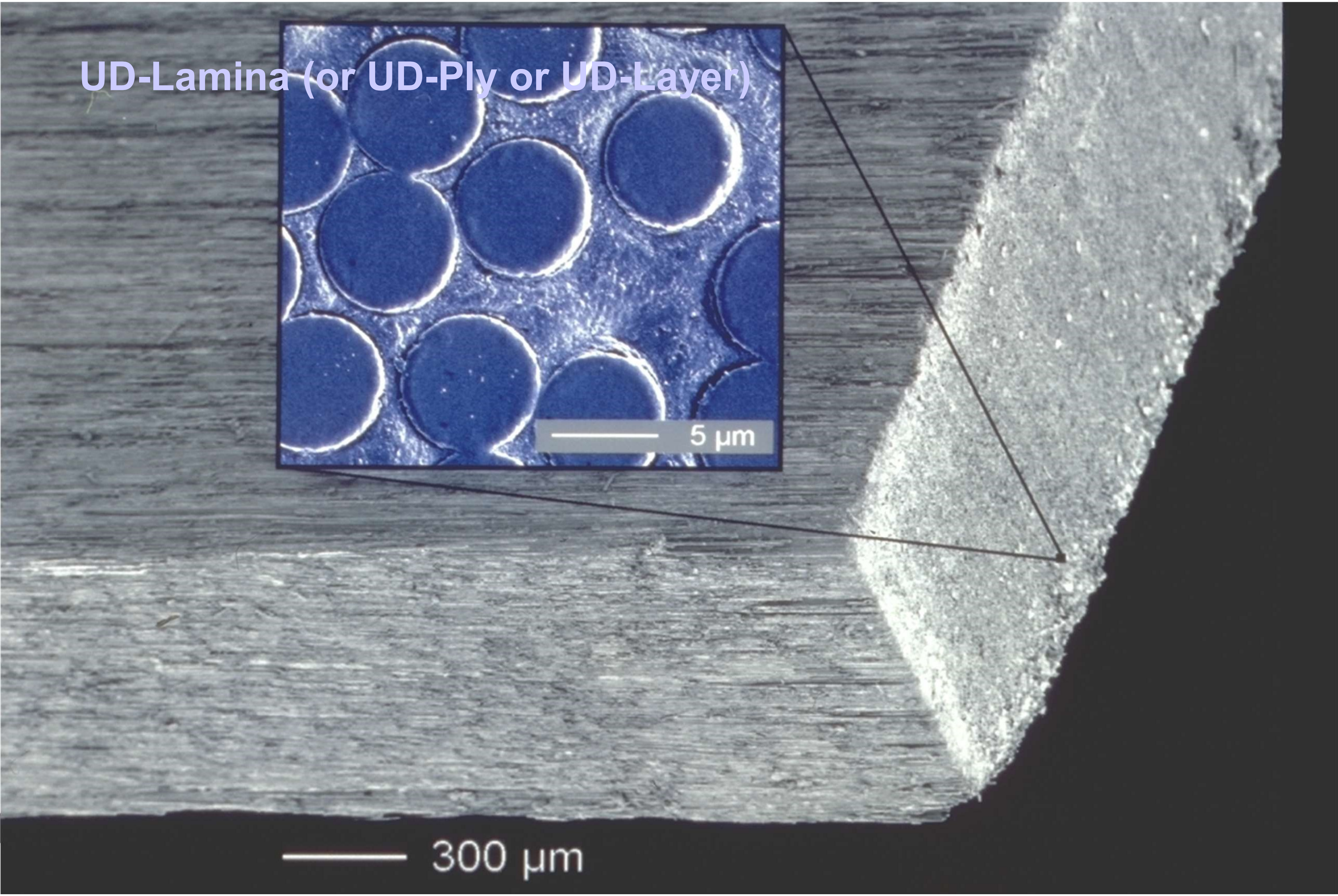


Material Coordinate System
1, 2, 3

A Laminate



A Laminate


UD-Lamina (or UD-Ply or UD-Layer)
5 µm
300 µm

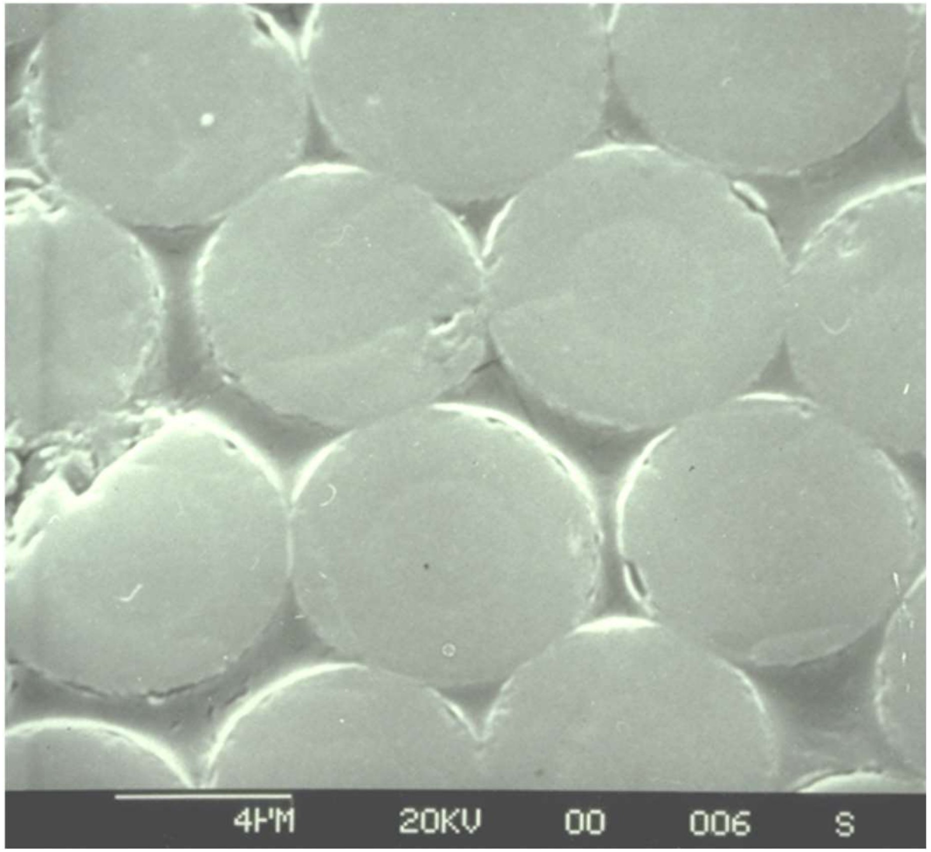
Definitions:

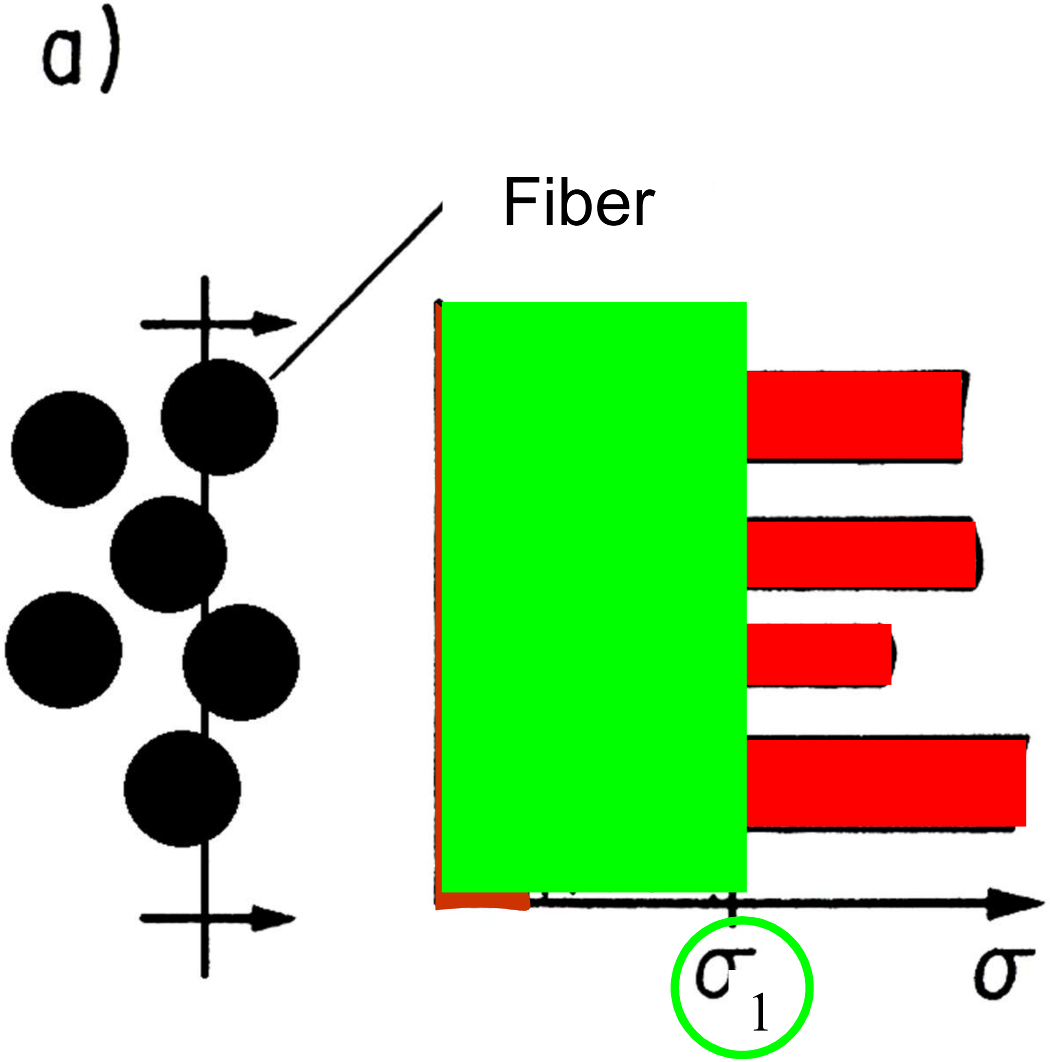
- **Homogeneous**: Properties are not function of the position of the material points

- **Isotropy**: Properties are not function of the orientation.
  2 independent material constants: E and ν

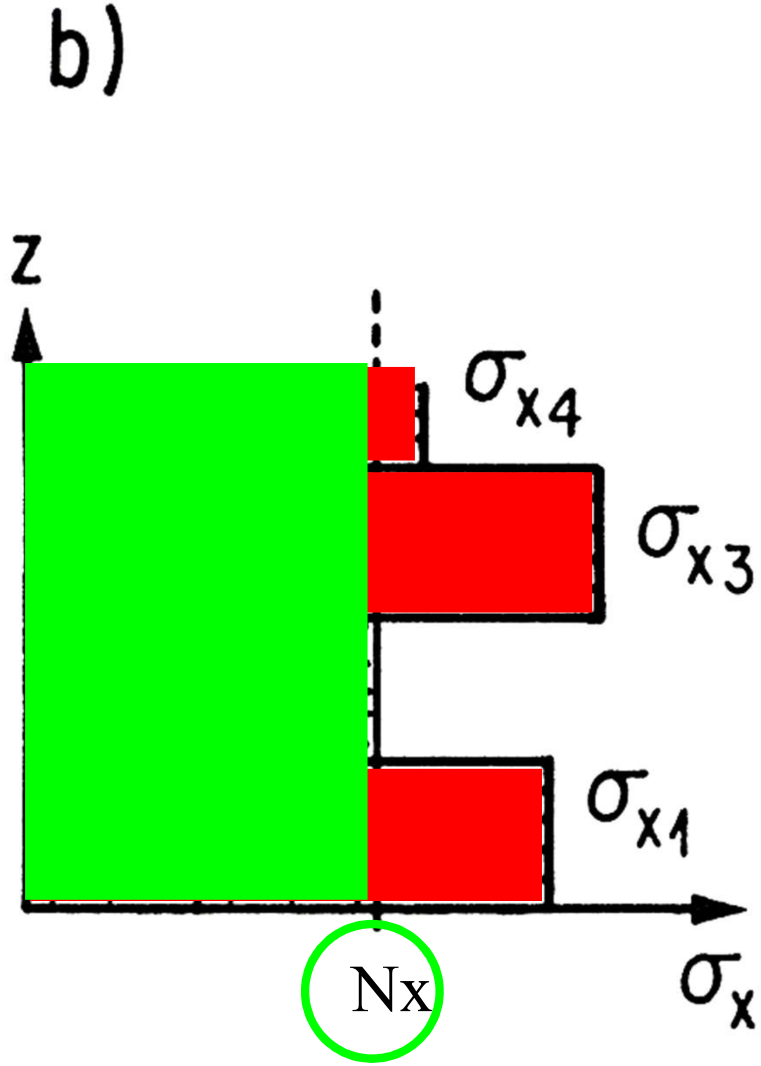
- **Anisotropy**: Properties are function of the orientation with no planes of symmetry.
  21 independent material constants

Micromechanics





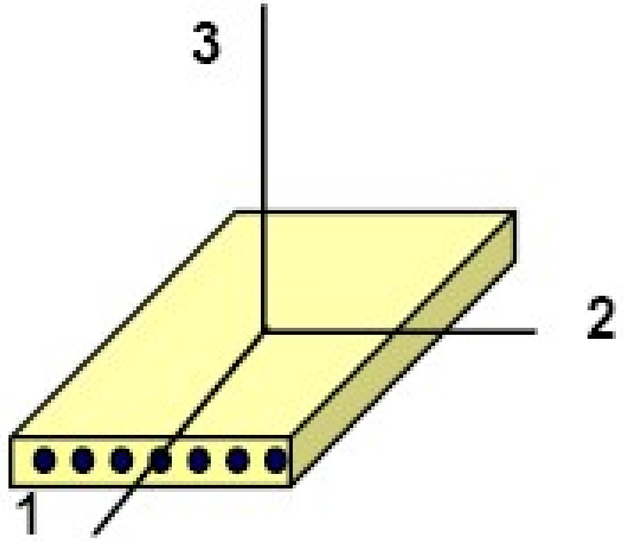
Macromechanics

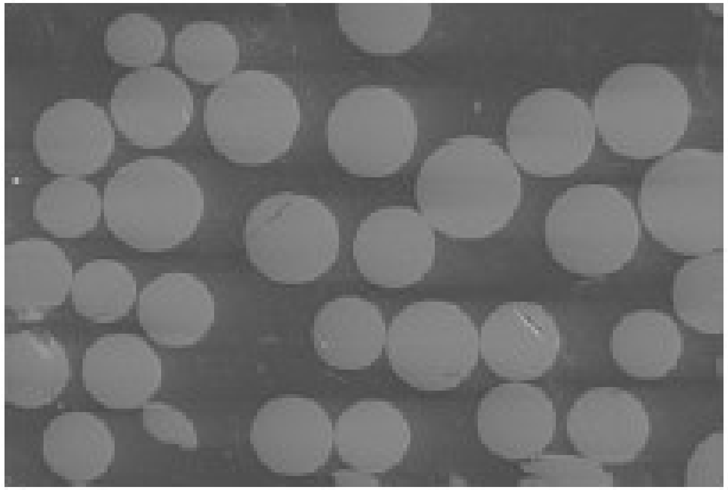


Equivalent Homogeneous Material

Important Assumption:

- We assume that the fiber and matrix properties can be smeared and represented as an equivalent homogenous material with orthotropic material properties
- This allows us to develop the stress strain behavior of the material making the structural level response tractable
- Otherwise we would have to deal with micromechanics





Unidirectional Lamina
(UD-Lamina)

Stiffness of a UD-Lamina

Orthotropic Lamina – Symmetric Compliance Matrix

Due to the 3 reciprocal relations, only 9 independent elastic constants are needed

$$S_{11} = \frac{1}{E_{11}} \quad S_{12} = \frac{-\nu_{12}}{E_{11}} \quad S_{13} = \frac{-\nu_{13}}{E_{11}}$$

$$S_{22} = \frac{1}{E_{22}} \quad S_{23} = \frac{-\nu_{23}}{E_{22}} \quad S_{33} = \frac{1}{E_{33}}$$

$$S_{44} = \frac{1}{G_{23}} \quad S_{55} = \frac{1}{G_{13}} \quad S_{66} = \frac{1}{G_{12}}$$

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix}$$

Zero entries in the upper right and lower left portions of the compliance mat rix characterize orthotropic behavior

Orthotropic Lamina – Symmetric Stiffness Matrix

The inverse relations are…

$$C_{11} = \frac{S_{22}S_{33} - S_{23}S_{23}}{S} \quad C_{12} = \frac{S_{13}S_{23} - S_{12}S_{33}}{S}$$

$$C_{22} = \frac{S_{33}S_{11} - S_{13}S_{13}}{S} \quad C_{13} = \frac{S_{12}S_{23} - S_{13}S_{22}}{S}$$

$$C_{33} = \frac{S_{11}S_{22} - S_{12}S_{12}}{S} \quad C_{23} = \frac{S_{12}S_{13} - S_{23}S_{11}}{S}$$

$$C_{44} = \frac{1}{S_{44}} \quad C_{55} = \frac{1}{S_{55}} \quad C_{66} = \frac{1}{S_{66}}$$

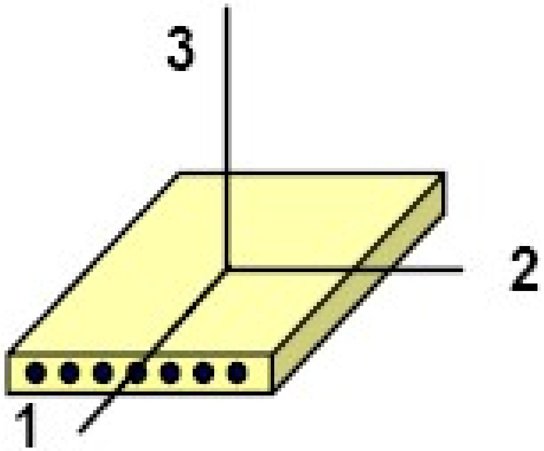
$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix}$$

where

$$S = S_{11}S_{22}S_{33} - S_{11}S_{23}S_{23} - S_{22}S_{13}S_{13} - S_{33}S_{12}S_{12} + 2S_{12}S_{23}S_{13}$$

Transversely Isotropic

Between **orthotropic** material behavior and **isotropic** material behavior is **transversely isotropic** behavior. We assume properties in the 2 and 3 directions are similar. We assume that…



$$E_{22} = E_{33} \qquad \nu_{12} = \nu_{13} \qquad G_{12} = G_{13}$$

$$\text{and} \quad G_{23} = \frac{E_{22}}{2(1+\nu_{23})}$$

Transversely Isotropic

If we assume that properties in the 2 and 3 directions are similar, we arrive at the compliance matrix…

$$S_{11} = \frac{1}{E_{11}} \quad S_{12} = \frac{-\nu_{12}}{E_{11}} \quad S_{22} = \frac{1}{E_{22}} \quad S_{23} = \frac{-\nu_{23}}{E_{22}}$$

$$S_{33} = \frac{1}{E_{33}} \quad S_{44} = \frac{1}{G_{23}} = \frac{2(1+\nu_{23})}{E_{22}} \quad S_{55} = \frac{1}{G_{12}}$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{12} & S_{23} & S_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{55} \end{bmatrix}$$

We now have 5 independent material properties

E_{11}, E_{22}, ν_{12}, ν_{23}, G_{12}

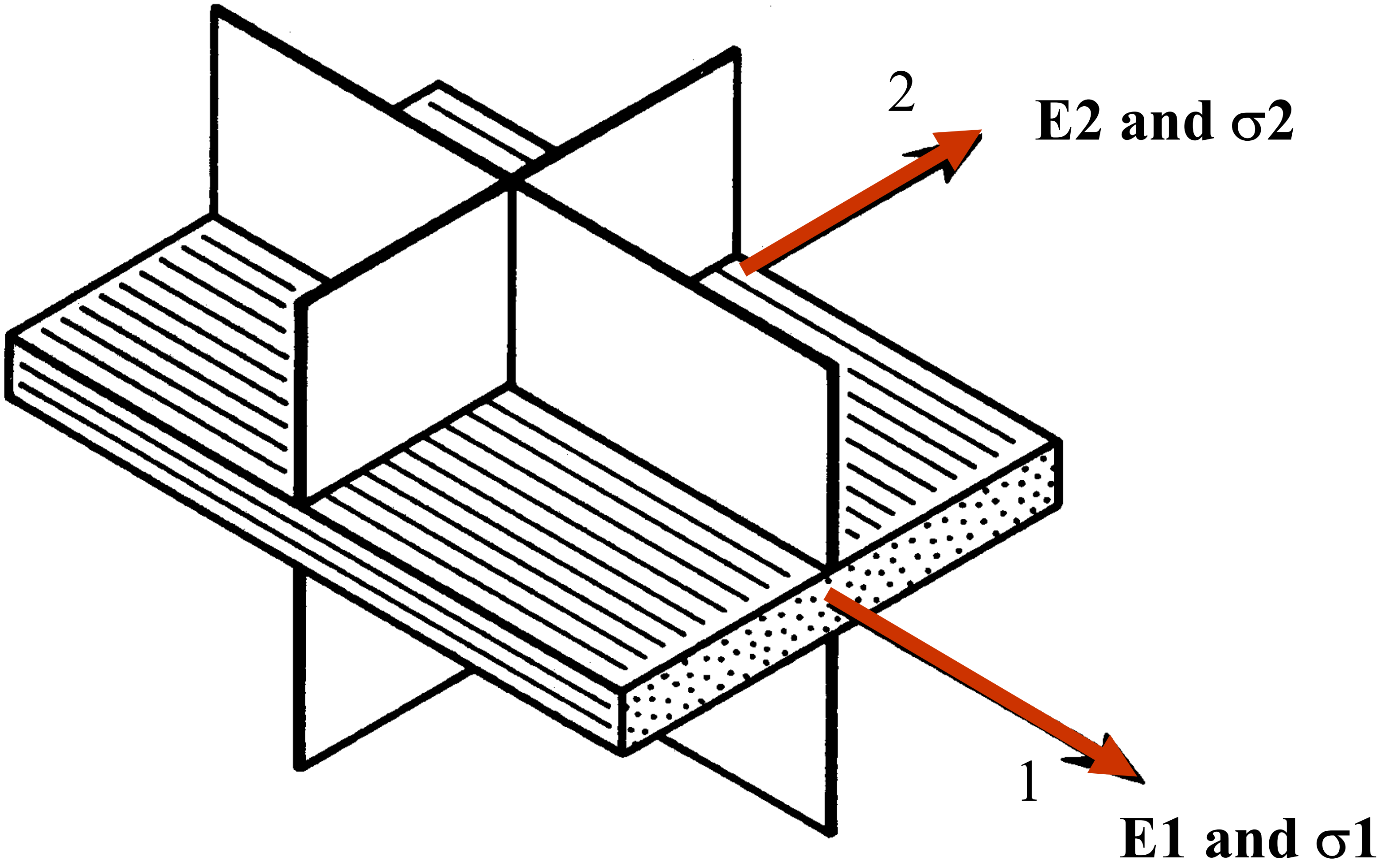
Transversely Isotropic

If we assume that properties in the 2 and 3 directions are similar, we arrive at the stiffness matrix…

$$\begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{12} & C_{23} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{55} \end{bmatrix}$$

Transversely Isotropic: a material which contains a plane in which the mechanical properties are equal in all directions

Symmetrical planes of a transverse isotrop UD-Lamina



Plane Stress: Out-of-Plane Strains

$$\sigma_{33} = \tau_{13} = \tau_{23} = 0$$

As a result the out-of-plane strains are…

$$\gamma_{13} = \gamma_{23} = 0$$

$$\varepsilon_{33} = S_{13}\sigma_{11} + S_{23}\sigma_{22}$$

Note that the out-of-plane normal strain ε_{33} **is not zero**, as a result of the Poisson's ratios ν_{13}, ν_{23},acting through the S_{13} & S_{23}!

Reduced Compliance Matrix

The Reduced Compliance Matrix is a result of our assumption of transverse isotropy and plane stress…

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix}$$

Transversely Isotropic

$$S_{11} = \frac{1}{E_{11}} \quad S_{12} = \frac{-\nu_{12}}{E_{11}} = \frac{-\nu_{21}}{E_2}$$

Why ??

$$S_{22} = \frac{1}{E_{22}} \quad S_{66} = \frac{1}{G_{12}}$$

4 independent material properties

E_{11}, E_{22}, ν_{12}, G_{12}

Isotropic

$$S_{11} = S_{22} = \frac{1}{E} \quad S_{12} = \frac{-\nu}{E}$$

$$S_{66} = \frac{1}{G} = \frac{2(1+\nu)}{E}$$

2 independent material properties

E,ν *or* G, ν *or* E, G

Strain-Stress relation of a UD-Lamina, Plane stress

$$\varepsilon_1 = \frac{1}{E_1}\sigma_1 - \frac{\nu_{21}}{E_2}\sigma_2$$

$$\varepsilon_2 = -\frac{\nu_{12}}{E_1}\sigma_1 + \frac{1}{E_2}\sigma_2$$

$$\gamma_{12} = \frac{1}{G_{12}}\tau_{12}$$

Inversion of the Compliance Matrix

$$\sigma_1 = \frac{E_1}{1-\nu_{21}\nu_{12}}\varepsilon_1 + \frac{\nu_{12}E_2}{1-\nu_{21}\nu_{12}}\varepsilon_2$$

$$\sigma_2 = \frac{\nu_{21}E_1}{1-\nu_{21}\nu_{12}}\varepsilon_1 + \frac{E_2}{1-\nu_{21}\nu_{12}}\varepsilon_2$$

$$\tau_{12} = G_{12}\gamma_{12}$$

Stiffness Matrix Q of a UD-Lamina

	ε_1	ε_2	γ_{12}
σ_1	Q_{11}	Q_{12}	
σ_2	Q_{21}	Q_{22}	
τ_{12}			Q_{66}

Stiffnesses Q_{ij} of a UD-Lamina

$$Q_{11} = \frac{E_1}{1 - \nu_{21}\nu_{12}}; Q_{22} = \frac{E_2}{1 - \nu_{21}\nu_{12}}; Q_{66} = G_{12}$$

$$Q_{21} = \frac{\nu_{21}E_1}{1 - \nu_{21}\nu_{12}}; Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{21}\nu_{12}}$$

Symmetry ?

Stiffness matrix Q:

$$Q = \begin{bmatrix} Q11 & Q12 & 0 \\ Q21 & Q22 & 0 \\ 0 & 0 & Q66 \end{bmatrix} \left(\frac{N}{mm^2} \right)$$

Compliance matrix S:

Symmetry ?

$$S = \begin{bmatrix} S11 & S12 & 0 \\ S21 & S22 & 0 \\ 0 & 0 & S66 \end{bmatrix} \left(\frac{mm^2}{N} \right)$$

Symmetry ??

$$Q_{12} = Q_{21}$$

and

$$S_{12} = S_{21}$$

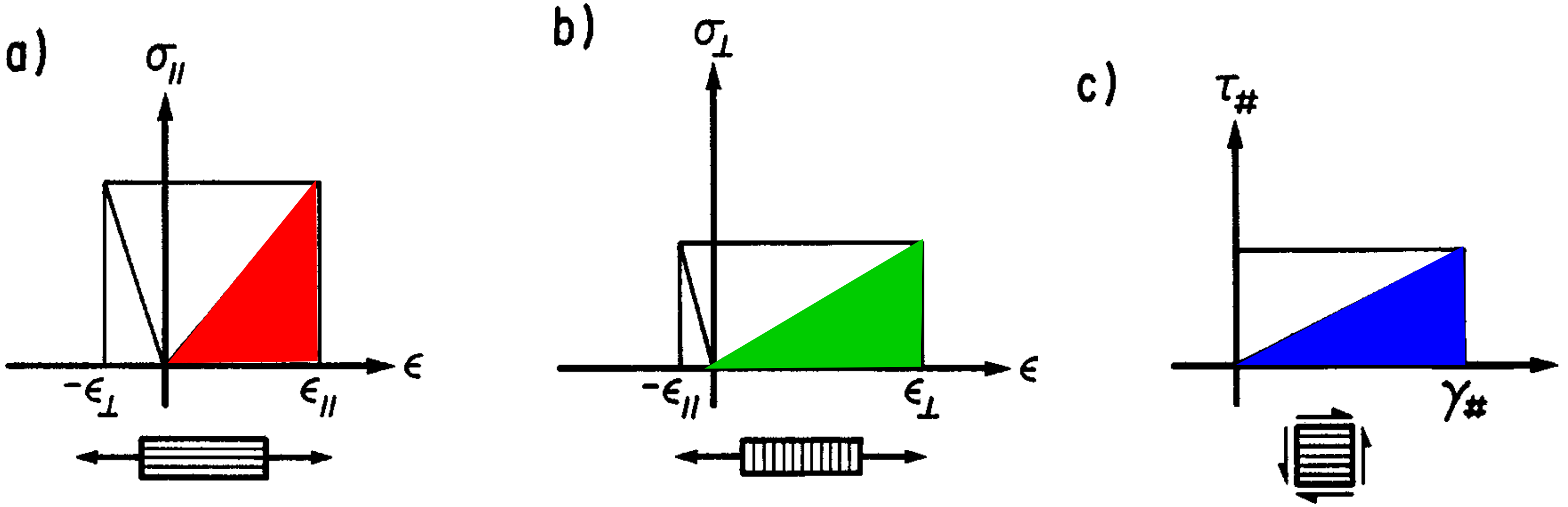
???

Coupling Terms

Elastic Energy

The stored elastic energy in the UD-Lamina is:

$$W = \frac{1}{2}\left[\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 + \tau_{12}\gamma_{12}\right]$$



Replace the strains with stresses using the compliance matrix as follow:

	σ_1	σ_2	τ_{12}
ε_1	S_{11}	S_{12}	
ε_2	S_{21}	S_{22}	
γ_{12}			S_{66}

We obtain :

$$W = \frac{1}{2}\left[S_{11}\sigma_1{}^2 + \left(S_{12} + S_{21}\right)\sigma_1\sigma_2 + S_{22}\sigma_2{}^2 + S_{12}\tau_{12}{}^2\right]$$

Partial differentiation provides the following strain-stress relation:

$$\frac{\partial W}{\partial \sigma_1} = \left[S_{11}\sigma_1 + \frac{1}{2}\left(S_{12} + S_{21}\right)\sigma_2 \right] = \varepsilon_1$$

$$\frac{\partial W}{\partial \sigma_2} = \left[\frac{1}{2}\left(S_{12} + S_{21}\right)\sigma_1 + S_{22}\sigma_2 \right] = \varepsilon_2$$

Compare the equations with the strain-stress relation through compliance matrix

	σ_1	σ_2	τ_{12}
ε_1	S_{11}	S_{12}	
ε_2	S_{21}	S_{22}	
γ_{12}			S_{66}

we obtain:

$$S_{12} = S_{21}$$

If we use the stiffness matrix

	ε_1	ε_2	γ_{12}
σ_1	Q_{11}	Q_{12}	
σ_2	Q_{21}	Q_{22}	
τ_{12}			Q_{66}

and replace stresses with strains, we obtain in a similar way the following equation:

$$Q_{12} = Q_{21}$$

or following equation is obtained for engineering constants:

$$\nu_{21} E_1 = \nu_{12} E_2$$

or:

$$\frac{\nu_{21}}{\nu_{12}} = \frac{E_2}{E_1}$$

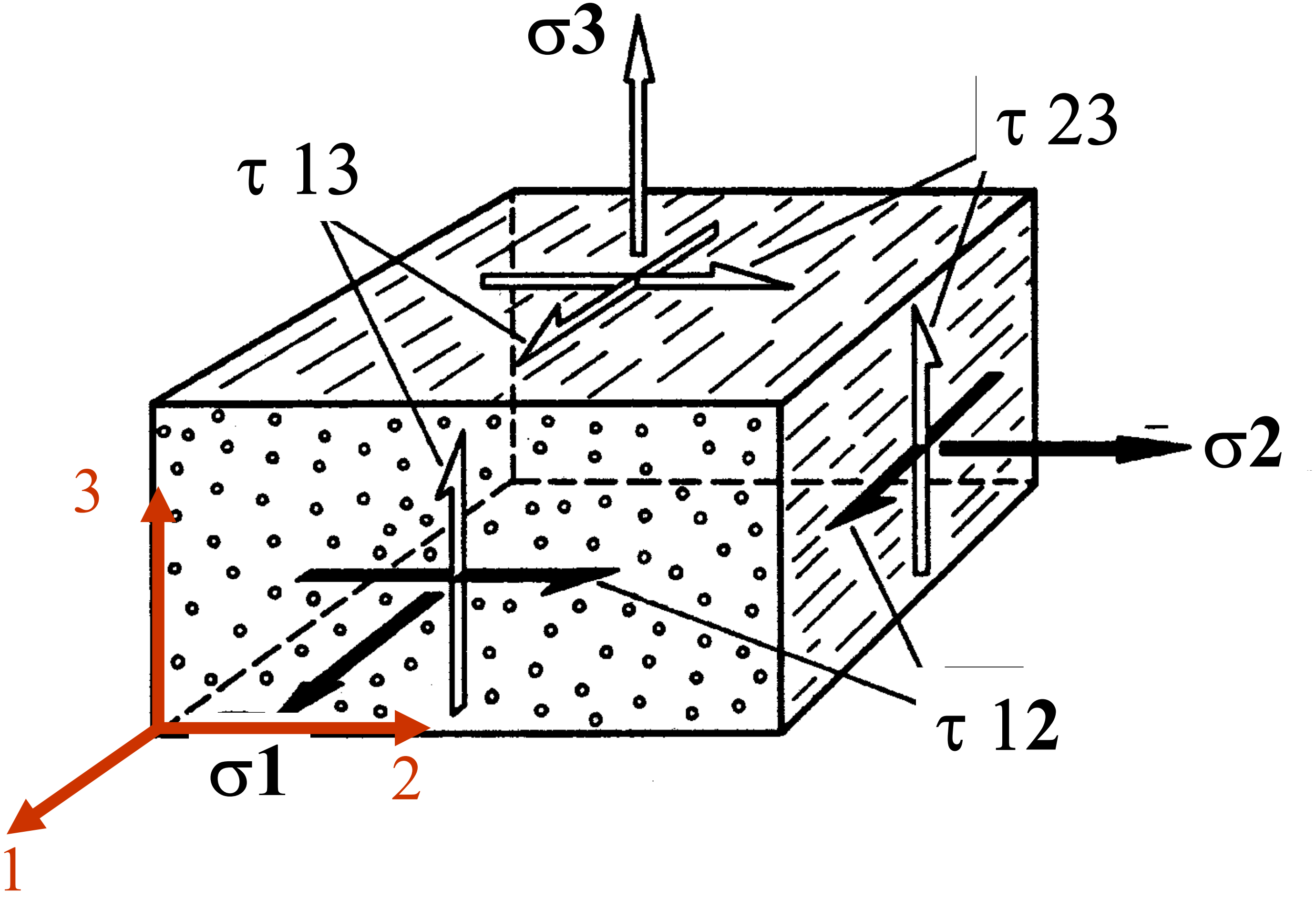
Definitions:

- **Orthotropy**: Anisotropy with 3 orthogonal planes of symmetry.
  9 independent constants

- **Transverse Isotropy**: Orthotropy with a plane at which there is Isotropy
  5 independent constants

- **Transverse Isotropy and Plane Stress**:
  4 independent constants

Stress State of a Unidirectional (UD) Lamina: Transverse Isotropy, Homogeneous



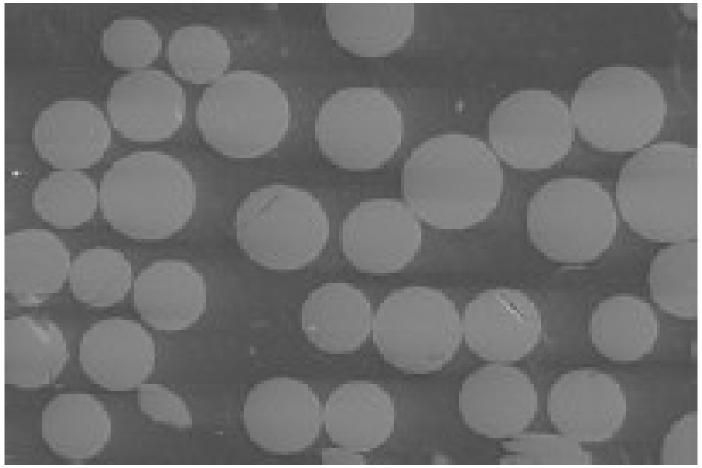
Plane Stress: $\sigma 1 \quad \sigma 2 \quad \tau\ 12$

Independent Elasticity Constants

If there are more symmetry conditions, there will be further reductions in the number of constants. For a cross-ply laminate with E1=E2, there are 3 and for an isotropic material (for example mat-laminate with randomly distributed fibers) 2 independent constants.

Estimating Elastic Composite Properties

- Assumes…
  - The fiber and matrix are bonded
  - Pure modes (extension/shear)
  - Fiber packing – representative volume
- Approaches
  - FEA
  - Mechanics of materials
  - Semi empirical equations



Elasticity constants of a UD-Lamina are dependent on the following

E_F = Fiber E-Modulus

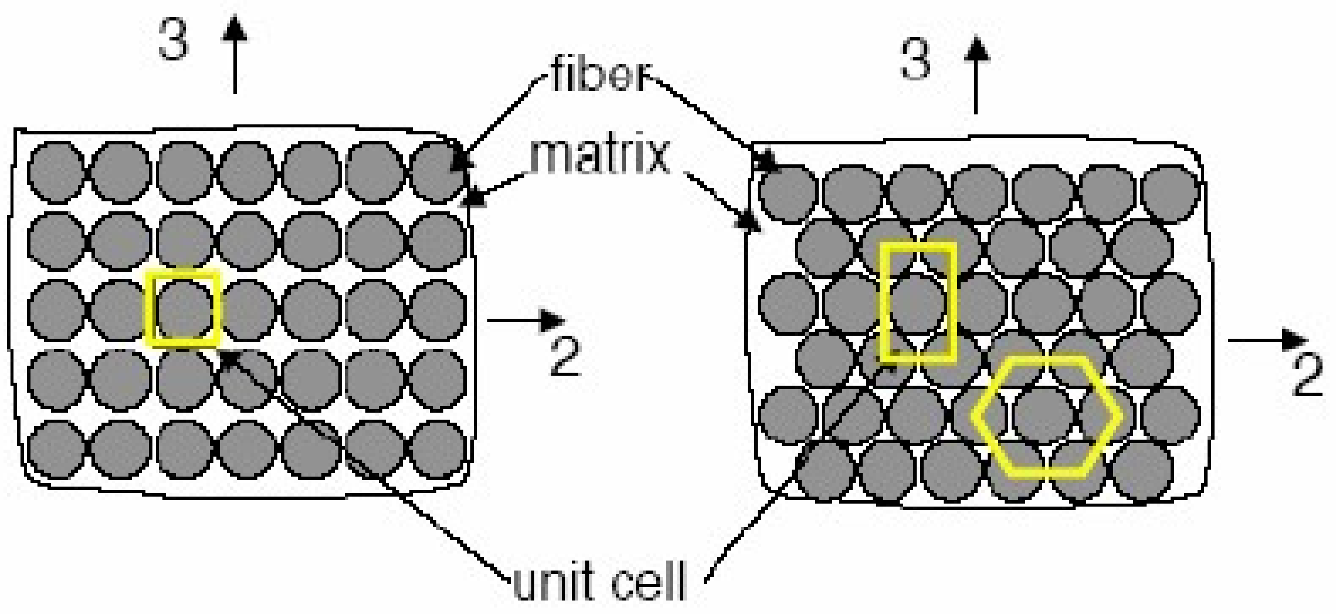
ν_F = Fiber Poisson‘s Ratio

E_M = Matrix E-Modulus

ν_M = Matrix Poisson‘s Ratio

ϕ = Fiber Volume Fraction

FEA Examination



- Idealized packing
- Representative volume
- Boundary conditions applied
- Stresses/Strains integrated to derive properties
- Local stress states/damage

Mechanics of Materials Approaches to Elastic Property Estimates

Rule of Mixtures

Equal strain

$$\varepsilon = \varepsilon_f = \varepsilon_m$$

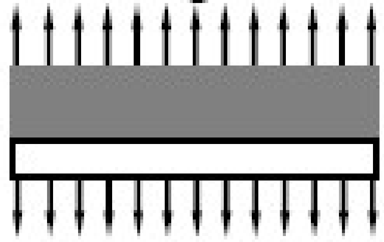
$$\sigma = \sigma_f V_f + \sigma_m (1 - V_f)$$
$$= E_f \varepsilon V_f + E_m \varepsilon (1 - V_f)$$

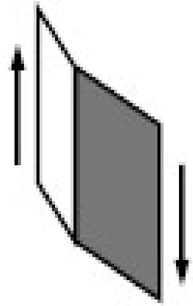
$$E_{11} = E_1^f V^f + E_m \left(1 - V^f\right)$$

Similar for ν_{12}

$$\nu_{12} = \nu_{12}^{\,f} V^f + \nu_m \left(1 - V^f\right)$$

Equal stress





$$\sigma = \sigma_f = \sigma_m$$

$$\varepsilon = \varepsilon_f V_f + \varepsilon_m (1 - V_f)$$

$$\varepsilon = \frac{\sigma_f}{E_f} V_f + \frac{\sigma_m}{E_m} (1 - V_f)$$

$$\frac{1}{E_{22}} = \frac{V^f}{E_2^f} + \frac{\left(1 - V^f\right)}{E^m}$$

$$\frac{1}{G_{12}} = \frac{V^f}{G_{12}^f} + \frac{\left(1 - V^f\right)}{G^m}$$

Not so good for E_2 & G_{12}

The longitudinal E-Modulus (parallel to the fiber direction) can be derived from the following so called rule of mixture:

$$E_1 = \Phi_F E_F + (1 - \Phi_F) E_M$$

where ϕ_F = fiber volume content of the UD-Lamina

The major poisson's ratio ν_{12} caused by longitudinal stresses following the rule of mixture is:

$$\nu_{12} = \Phi_F \nu_F + (1 - \Phi_F)\nu_M$$

And the minor poisson's ratio is:

$$\nu_{21} = \nu_{12} \frac{E_2}{E_1}$$

Semi Empirical Equations Based on Experiments

The transverse E-modulus for isotropic fibers according to ‚Puck‘ can be obtained:

$$E_2 = E_M{}^o \frac{1 + 0.85\Phi_F{}^2}{\Phi_F E_M{}^o / E_F + (1 - \Phi_F)^{1.25}}$$

$$E_M{}^o = \frac{E_M}{1 - \nu_M{}^2}$$

According to ‚Puck‘ for isotropic fibers:

$$G_{12} = G_M \frac{\left(1 + 0.60 \Phi_F{}^{0.5}\right)}{\Phi_F G_M / G_F + \left(1 - \Phi_F\right)^{1.25}}$$

According to ‚Förster‘ and ‚Schneider‘ for isotropic fibers:

$$E_2 = E_M{}^o \frac{1}{\Phi_F E_M{}^o / E_F + \left(1 - \Phi_F\right)^{1.45}}$$

$$E_M{}^o = \frac{E_M}{1 - \nu_M{}^2}$$

According to ‚Förster‘ and ‚Schneider‘ for isotropic fibers:

$$G_{12} = G_M \frac{\left(1 + 0.4\Phi_F{}^{0.5}\right)}{\Phi_F G_M / G_F + \left(1 - \Phi_F\right)^{1.45}}$$

Following ‚Tsai‘ for isotropic fibers:

$$E_2 = \frac{E_M \left(1 + \xi \quad \eta \quad \Phi_F\right)}{1 - \eta \quad \Phi_F} \quad where$$

$$\eta = \frac{E_F / E_M - 1}{E_F / E_M + \xi} \quad and \quad \xi = 2$$

Following ‚Tsai‘ for isotropic fibers:

$$G_{12} = \frac{G_M\left(1 + \xi \quad \eta \quad \Phi_F\right)}{1 - \eta \quad \Phi_F} \quad where$$

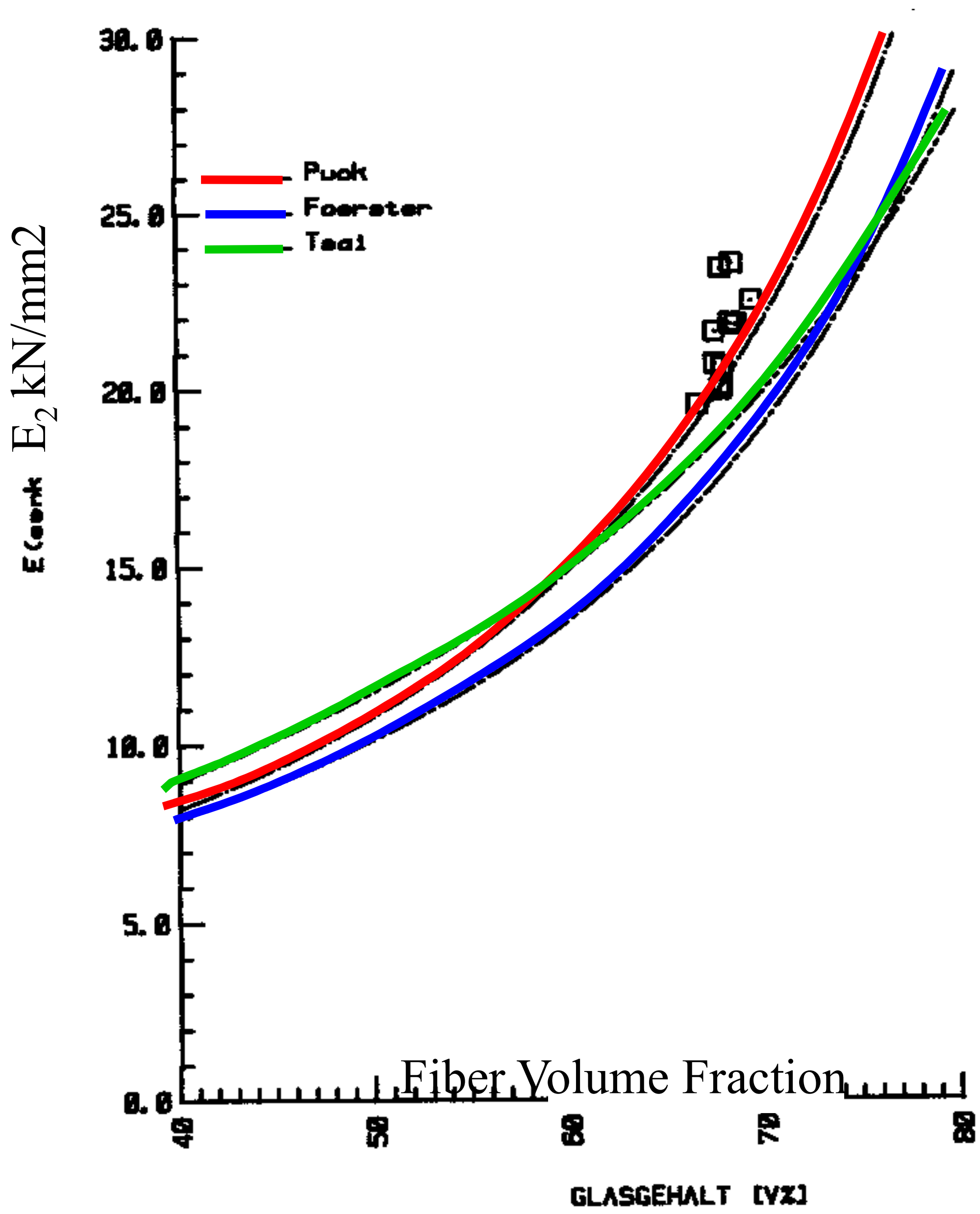
$$\eta = \frac{G_F / G_M - 1}{G_F / G_M + \xi} \quad and \quad \xi = 1$$

Comparisons for E_2

$$E_2 = E_M{}^o \frac{1 + 0.85\Phi_F{}^2}{\Phi_F E_M{}^o / E_F + (1 - \Phi_F)^{1.25}}$$

$$E_2 = E_M{}^o \frac{1}{\Phi_F E_M{}^o / E_F + (1 - \Phi_F)^{1.45}}$$

$$E_2 = \frac{E_M (1 + \xi \quad \eta \quad \Phi_F)}{1 - \eta \quad \Phi_F}$$

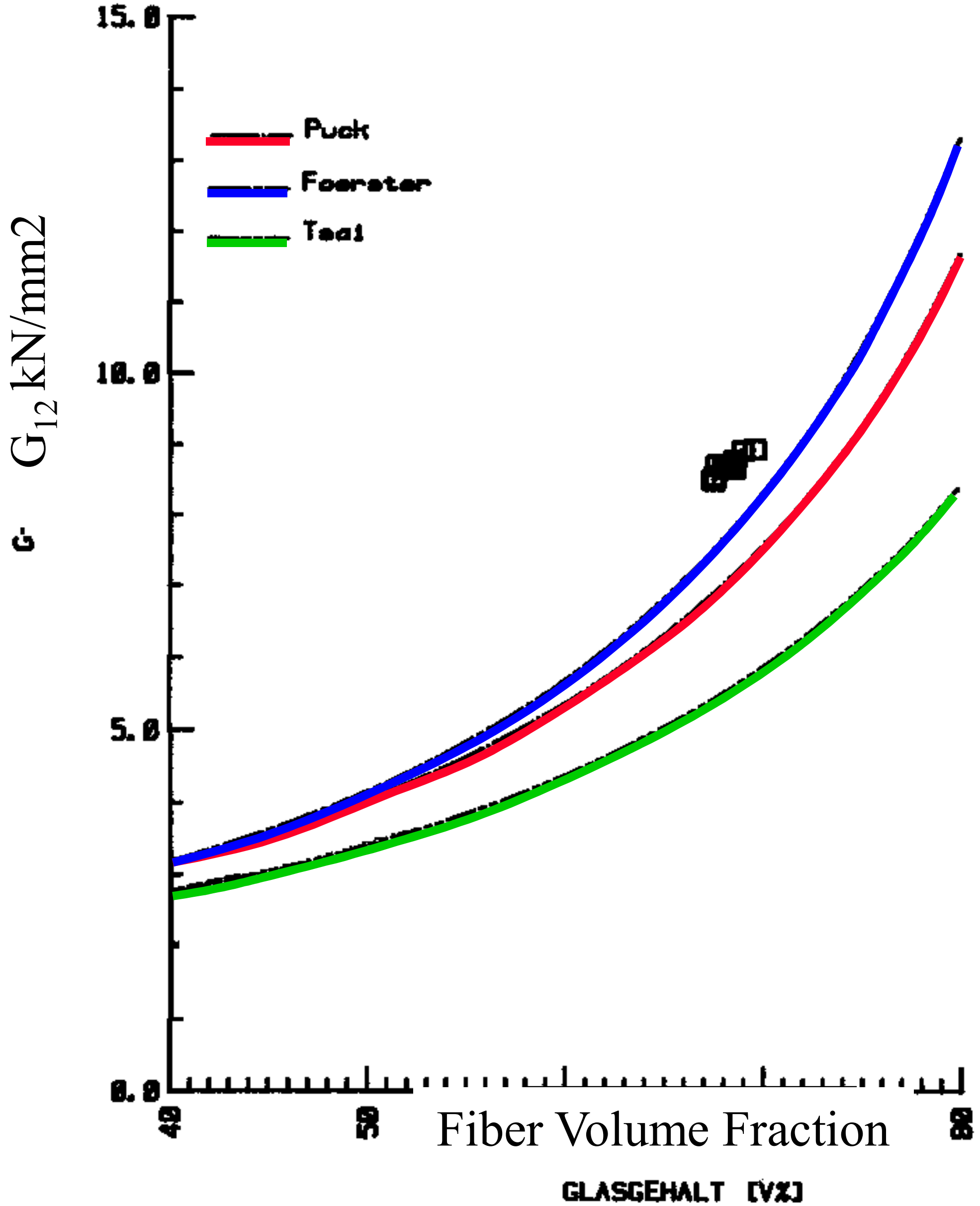


Comparisons for G_{12}

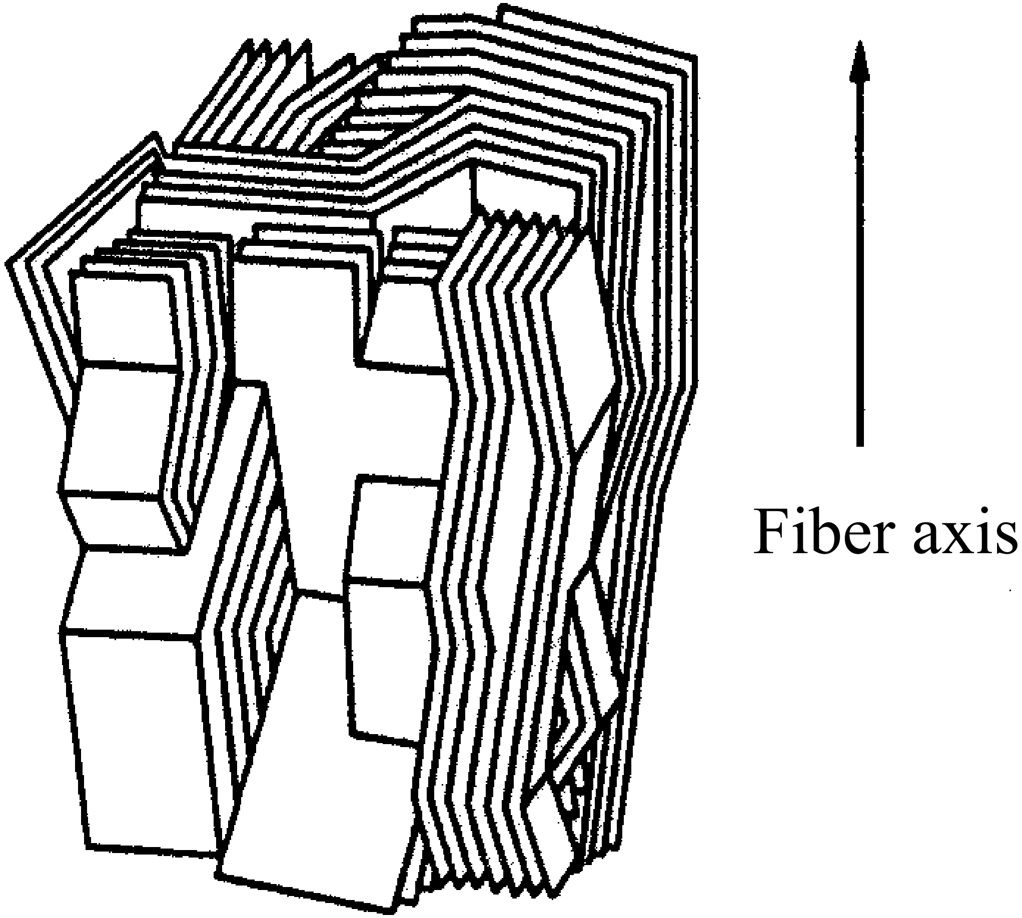
$$G_{12} = G_M \frac{\left(1+0.60\Phi_F^{0.5}\right)}{\Phi_F G_M/G_F + \left(1-\Phi_F\right)^{1.25}}$$

$$G_{12} = G_M \frac{\left(1+0.4\Phi_F^{0.5}\right)}{\Phi_F G_M/G_F + \left(1-\Phi_F\right)^{1.45}}$$

$$G_{12} = \frac{G_M\left(1+\xi \quad \eta \quad \Phi_F\right)}{1-\eta \quad \Phi_F}$$



Anisotropic fibers



For anisotropic fibers like C-fibers following equations can be applied:

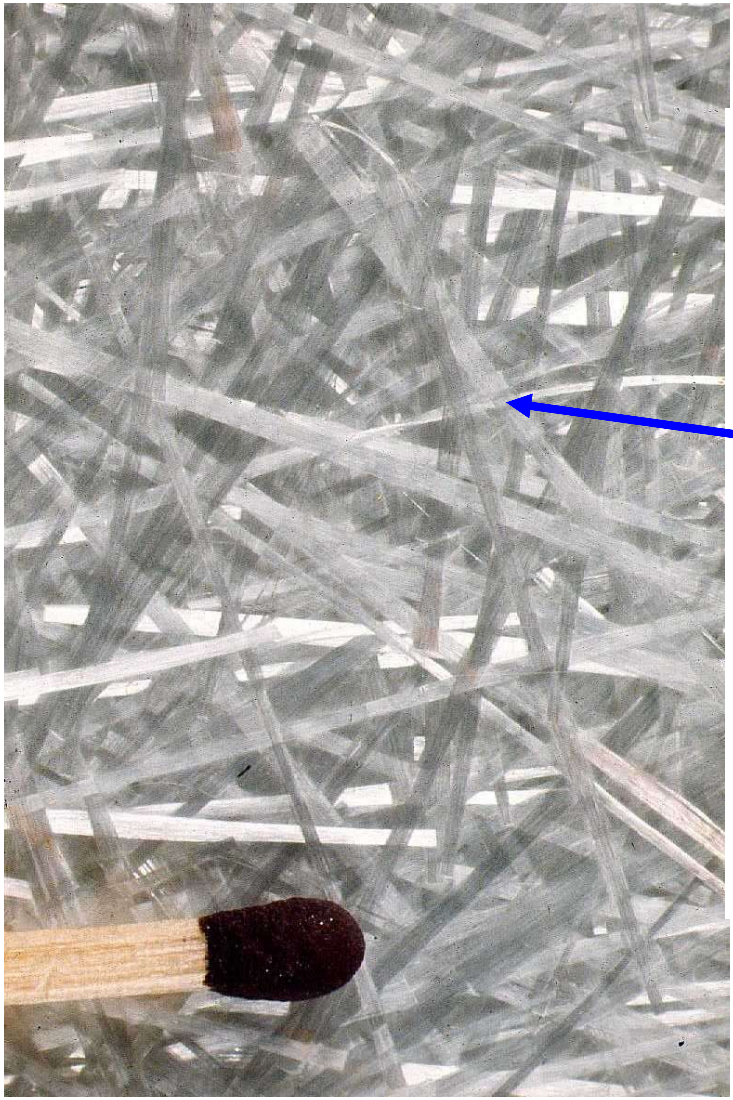
$$E_2 = \frac{E_M{}^o \left(1 + \Phi_F{}^3\right)}{\left(1 - \Phi_F\right)^{0.75} + 6 \quad \Phi_F \quad E_M{}^o / E_F}$$

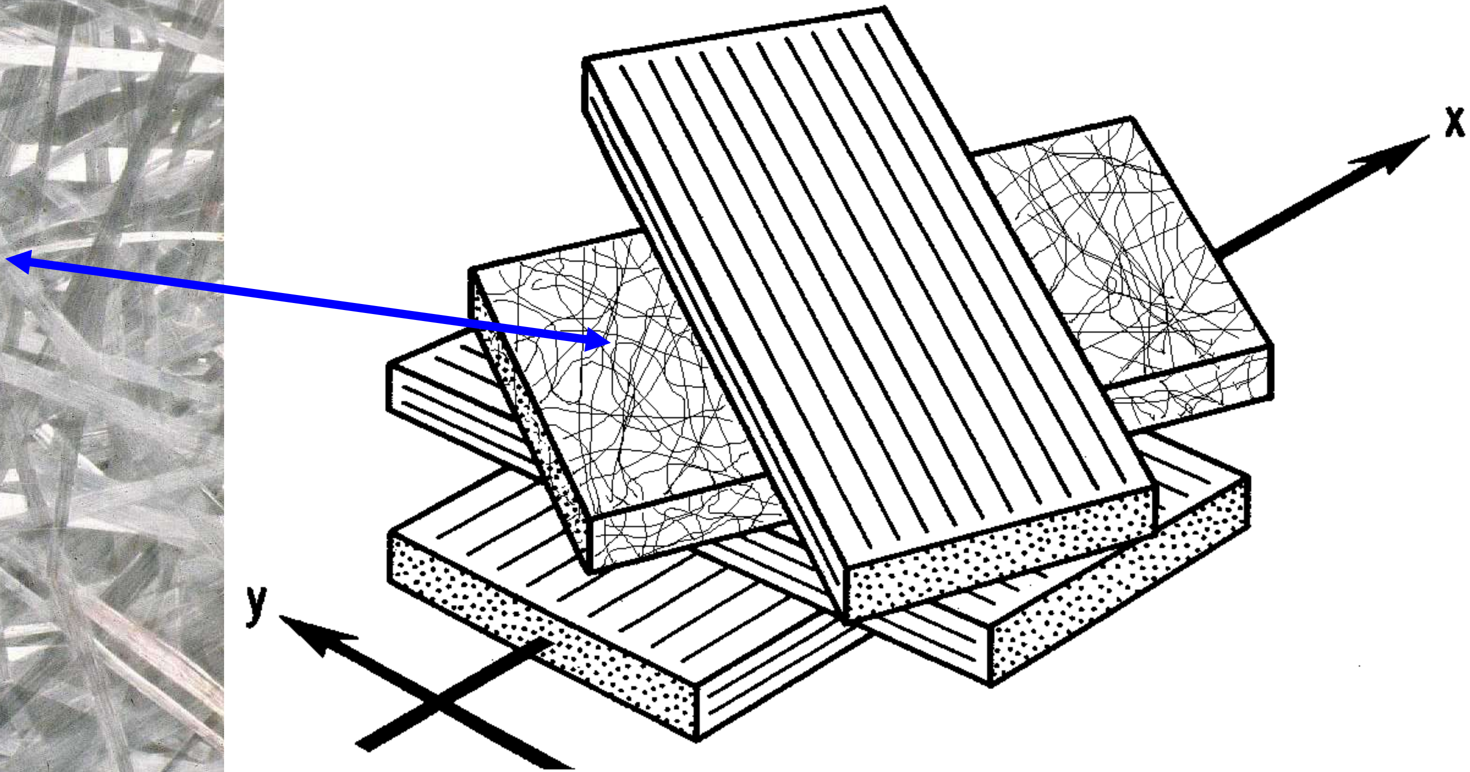
$$E_M{}^o = \frac{E_M}{1 - \nu_M{}^2}$$

and

$$G_{12} = \frac{G_M \left(1 + 0.25 \quad \Phi_F{}^{0.5}\right)}{\left(1 - \Phi_F\right)^{1.25} + 1.25 \quad \Phi_F \quad G_M / G_F}$$

Glass-Mat Lamina





Following equations can be applied to E-Glass Mat Lamina according to Puck :

$$E \approx 29'630 \quad \Phi_F - 4'710 \quad \Phi_F{}^2 + 3'920$$

$$\nu \approx 0.34 - 0.075 \quad \Phi_F$$

$$G \approx 10'970 \quad \Phi_F + 1'370$$

Elasticity constants of some UD-Laminas

Lamina type	T 300/5208	B (4)/5505	AS/3501	Scotchply 1002	Kevlar 49 / Epoxy
Fiber	C-Fibers from Toray	Boron	C –Fibers from Hercules	E-Glass	Aramid from E.I. Dupont de Nemours
Matrix	EP from Narmco	EP-Prepreg from Avco	EP-Prepreg from Hercules	EP-Prepreg from 3M	EP
Fiber volume fraction Φ (%)	70	50	66	45	60
Density (g/cm^3)	1.6	2.0	1.6	1.8	1.46
E_1 (N/mm^2)	181'000	204'000	138'000	38'600	76'000
E_2 (N/mm^2)	10'300	18'500	8'960	8'270	5'500
ν_{12}	0.28	0.23	0.30	0.26	0.34
G_{12} (N/mm^2)	7'170	5'590	7'100	4'140	2'300

…Elasticity constants of some UD-Laminas

Lamina type	T 300/5208	B (4)/5505	AS/3501	Scotchply 1002	Kevlar 49 / Epoxy
E_1 (N/mm^2)	181'000	204'000	138'000	38'600	76'000
E_2 (N/mm^2)	10'300	18'500	8'960	8'270	5'500
ν_{12}	0.28	0.23	0.30	0.26	0.34
G_{12} (N/mm^2)	7'170	5'590	7'100	4'140	2'300
S_{11} (mm^2/N)	$5.525 \cdot 10^{-6}$	$4.902 \cdot 10^{-6}$	$7.246 \cdot 10^{-6}$	$25.91 \cdot 10^{-6}$	$13.16 \cdot 10^{-6}$
S_{22} (mm^2/N)	$97.09 \cdot 10^{-6}$	$54.05 \cdot 10^{-6}$	$111.6 \cdot 10^{-6}$	$120.9 \cdot 10^{-6}$	$181.8 \cdot 10^{-6}$
S_{12} (mm^2/N)	$-1.547 \cdot 10^{-6}$	$-1.128 \cdot 10^{-6}$	$-2.174 \cdot 10^{-6}$	$-6.744 \cdot 10^{-6}$	$-4.474 \cdot 10^{-6}$
S_{33} (mm^2/N)	$139.5 \cdot 10^{-6}$	$172.7 \cdot 10^{-6}$	$140.8 \cdot 10^{-6}$	$241.5 \cdot 10^{-6}$	$434.8 \cdot 10^{-6}$
Q_{11} (N/mm^2)	181'800	205'000	138'000	39'160	76'640
Q_{22} (N/mm^2)	10'340	18'580	9'013	8'392	5'546
Q_{12} (N/mm^2)	2'897	4'275	2'704	2'182	1'886
Q_{33} (N/mm^2)	7'170	5'790	7'100	4'140	2'300

Thermal properties of a UD-Lamina: Expansion cofficients (Book Geoff Eckold p59)

$$\alpha_1 = \alpha_F + \frac{\alpha_M - \alpha_F}{\dfrac{\Phi E_F}{(1-\Phi)E_M} + 1}$$

And (Book Geoff Eckold p59)

$$\alpha_2 = \alpha_F \Phi + \alpha_M (1-\Phi) + \nu_F \alpha_F \Phi + \nu_M \alpha_M (1-\Phi) - [\nu_F \Phi + \nu_M (1-\Phi)]\alpha_1$$

where:

α_{F1} = Thermal expansion coefficient of fibers in fiber longitudinal direction

α_{F_2} = Thermal expansion coefficient of fibers perpendicular to fiber longitudinal direction

α_M = Thermal expansion coefficient of matrix

ν_M = Poisson's ratio of matrix

E_{F1} = Elasticity modulus of fibers in fiber longitudinal direction

E_{F_2} = Elasticity modulus of fibers perpendicular to fiber longitudinal direction

E_M = Elasticity modulus of matrix

Φ_F = Fiber volume content

Failure Theories for a UD-Lamina

Failure Theories for a UD-Lamina

Following simple criteria can be applied to examine the **fiber failure:**

$$\left(\frac{\sigma_1}{\sigma_{1\max}}\right) = 1$$

σ_{1max} = Failure stress of a UD-Lamina in fiber direction

$$\sigma_{1\max} = \sigma_{F\max}\Phi + (1-\Phi)\sigma_{M\max}$$

Matrix failure:

$$\left(\frac{\sigma_1}{\sigma_{1M\max}}\right)^2 + \frac{\sigma_2^{\,2}}{\sigma_{2C\max}\sigma_{2T\max}} + \frac{\sigma_{2C\max} - \sigma_{2T\max}}{\sigma_{2C\max}\sigma_{2T\max}}\sigma_2 + \left(\frac{\tau_{12}}{\tau_{12\max}}\right)^2 = 1$$

where

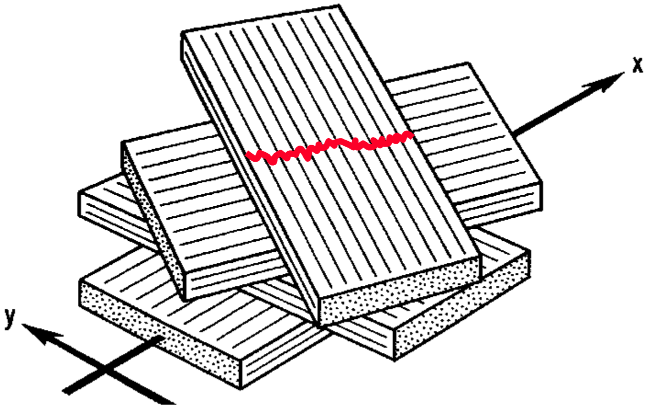
$$\sigma_{1M\max} = E_{1F}\varepsilon_{M\max}$$

$\sigma_{2C\max} =$ Compression strength perpendicular to fiber direction

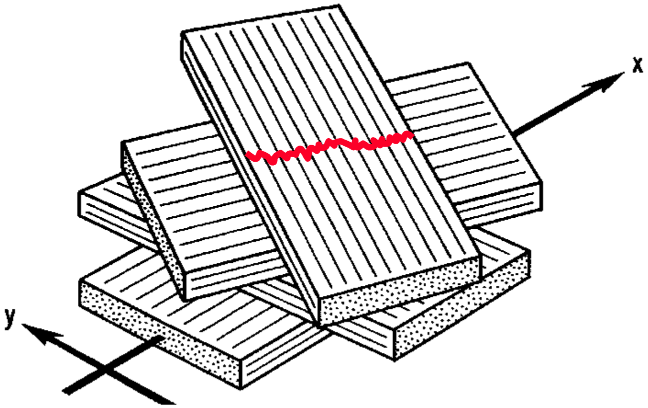
$\sigma_{2T\max} =$ Tensile strength perpendicular to fiber direction

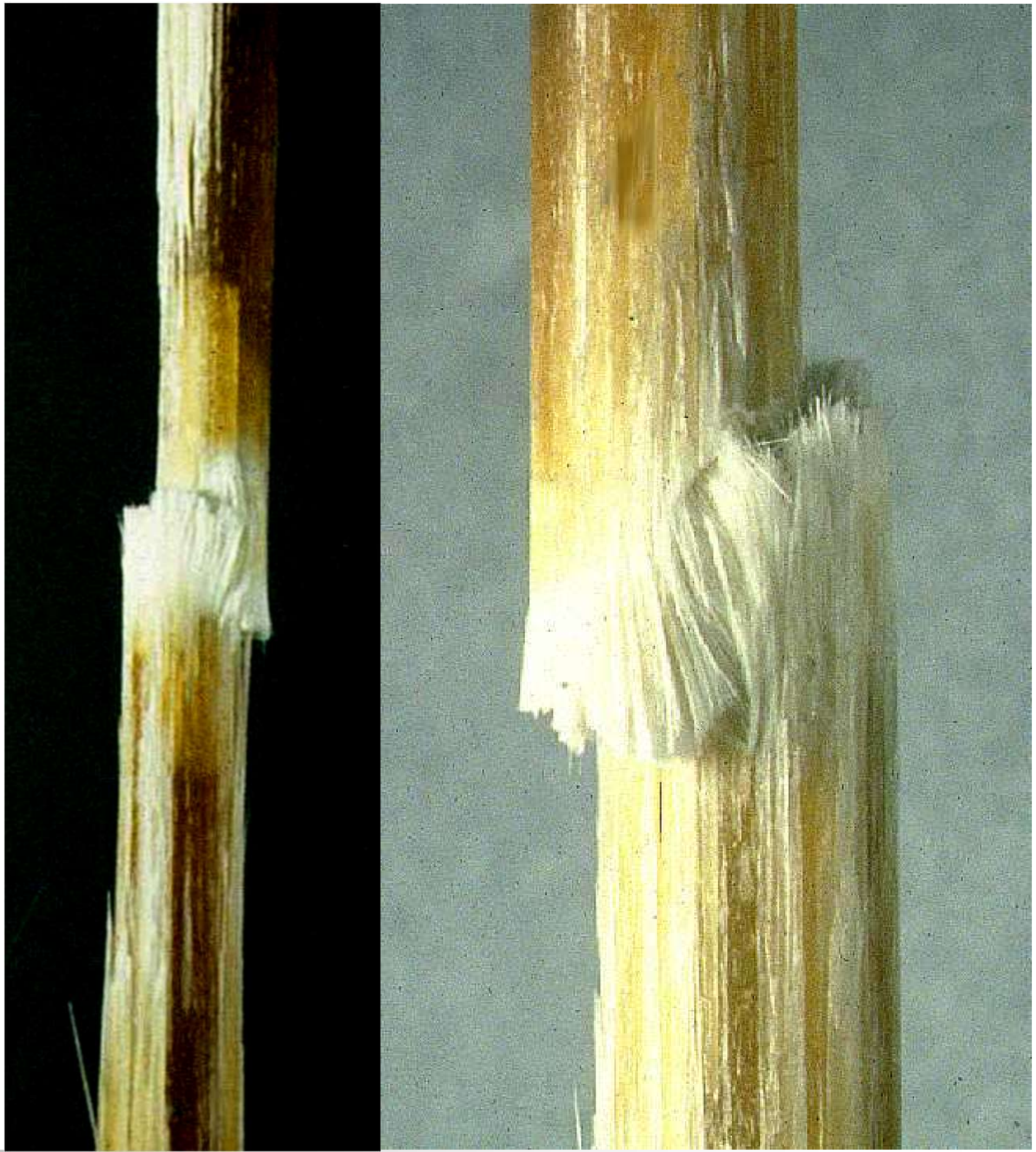
$\tau_{12\max} =$ Shear strength

Fiber failure due to tensile stress

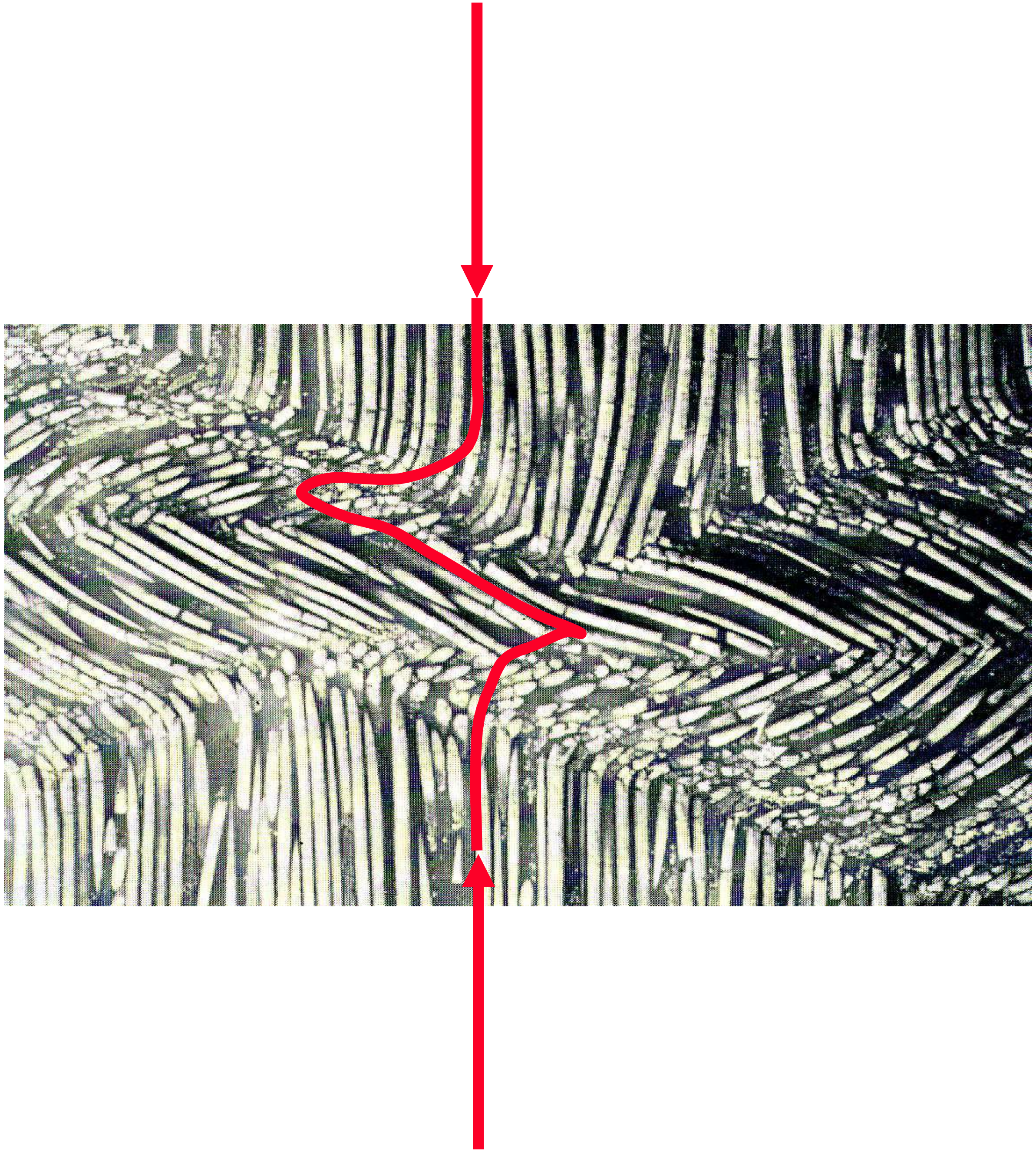


Fiber failure due to compression-stresses

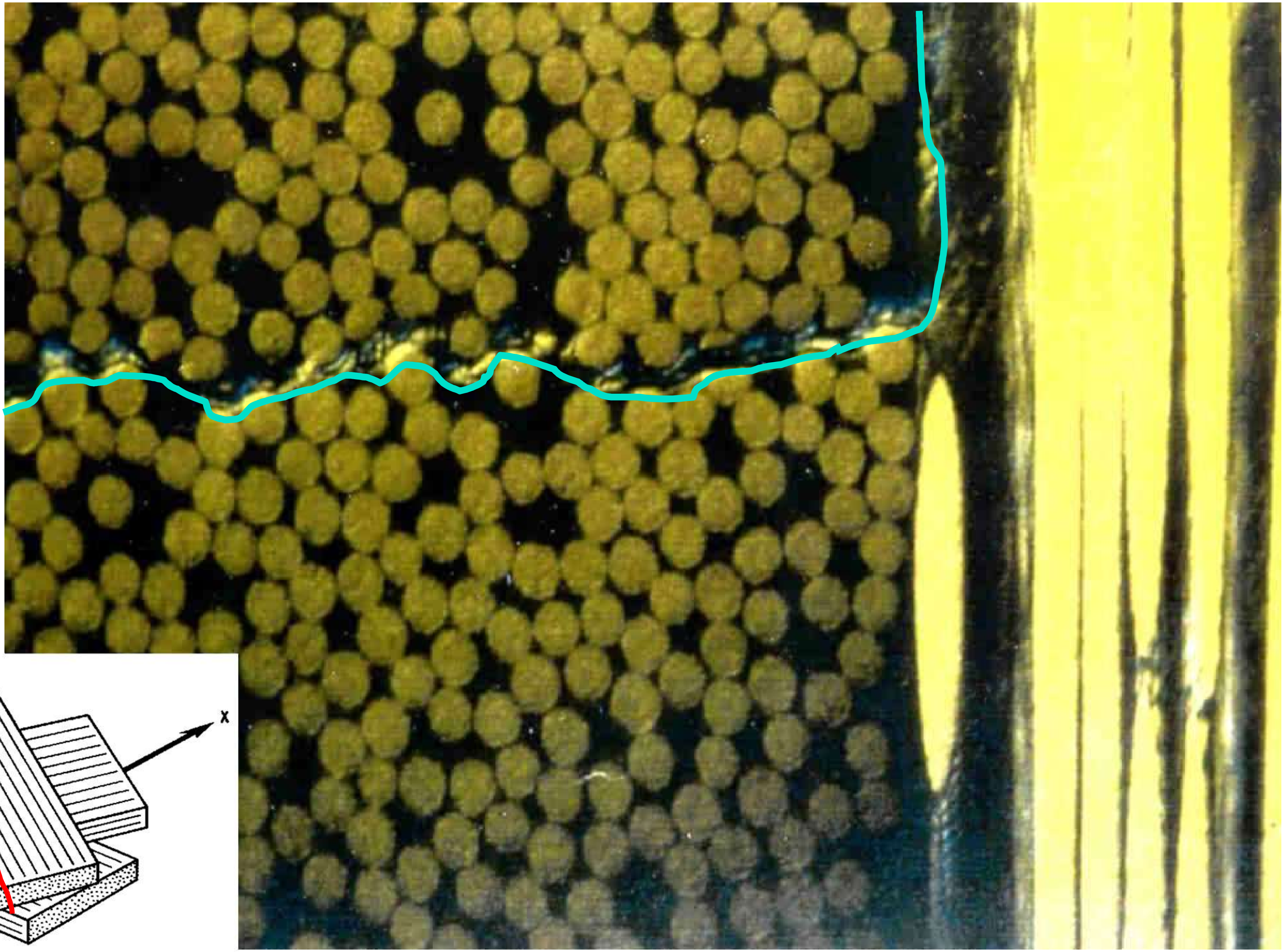


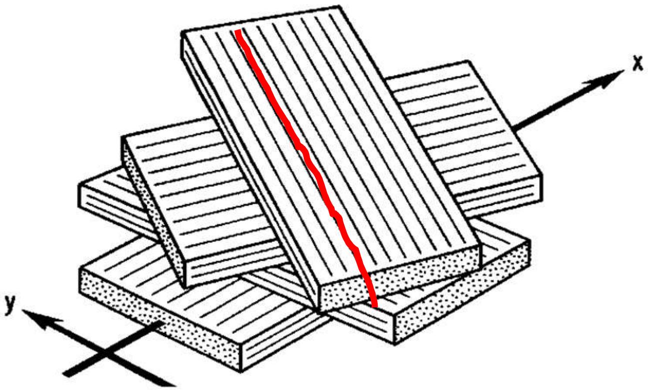


Fiber failure
due to
compression-
stresses

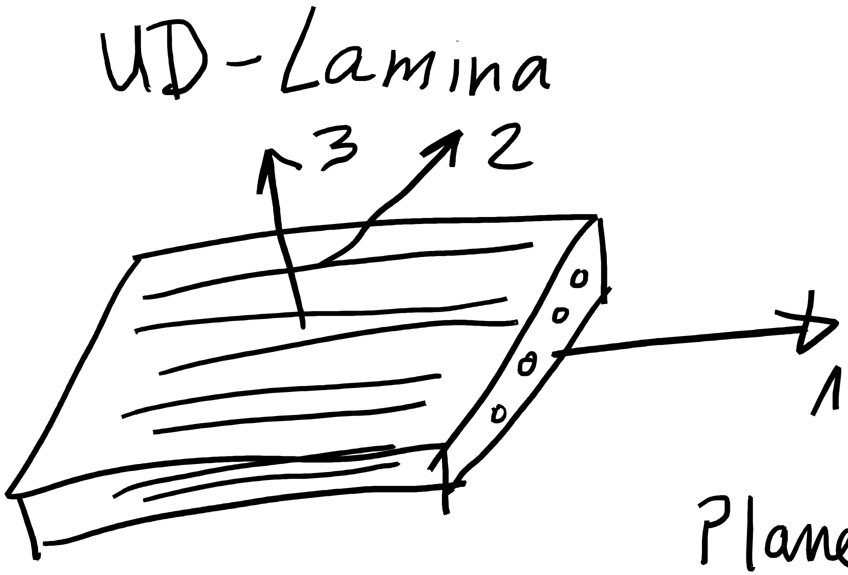


Matrix failure





UD-Lamina



orthotropic

9 independent const

E_{11}, E_{22}, E_{33}

G_{12}, G_{13}, G_{23}

$\nu_{12}, \nu_{13}, \nu_{23}$

transversal isotropic

5 ind. const.

$E_{11}, E_{22} = E_{33},$

G_{12}, ν_{12}, G_{23}

Plane stress

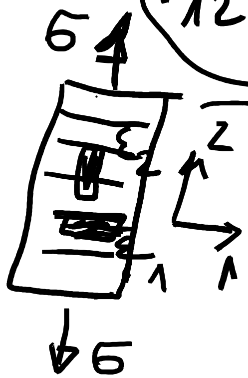
4 ind. const.

E_{11}, E_{22}

G_{12}, ν_{12}

Strain gages

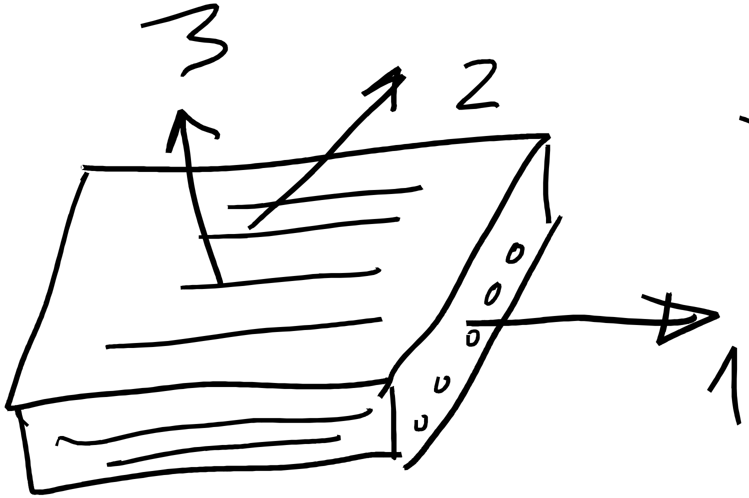
$$\nu_{12} = \frac{\varepsilon_2}{\varepsilon_1}$$



$$\nu_{21} = \frac{\varepsilon_1}{\varepsilon_2}$$

$\nu_{12} \neq \nu_{21}$!

ν_{12} : major Poisson ration / ν_{21} : Minor "



UD-Lamina stiffness matrix

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{pmatrix} = \begin{pmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{pmatrix}$$

$$Q_{12} = Q_{21} \;!$$

$$\left.\begin{array}{l} E_f\,,\ \nu_f \\ E_m\,,\ \nu_m \\ \quad\ \varphi_f \end{array}\right\} \longrightarrow \begin{array}{l} E_{11}\,,\ E_{22} \\ G_{12}\,,\ \nu_{12} \end{array} \longrightarrow \begin{pmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{pmatrix}$$

Theoretical:

$$E_{11} = E_f\,\varphi_f + E_m\,(1-\varphi_f)$$

$$\nu_{12} = \nu_f\,\varphi_f + \nu_m\,(1-\varphi_f)$$

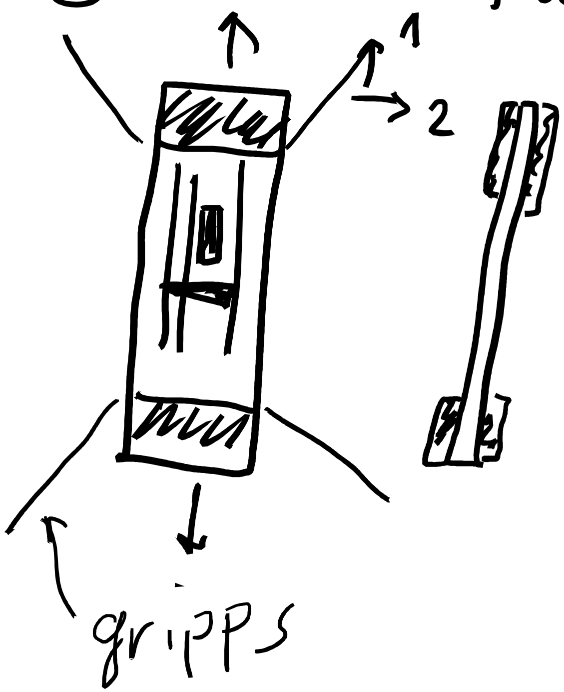
$$\begin{array}{l} E_{22} \\ G_{12} \end{array} \longrightarrow \text{See Semi empirical equations}$$

Experimental determination of UD-Lamina Properties

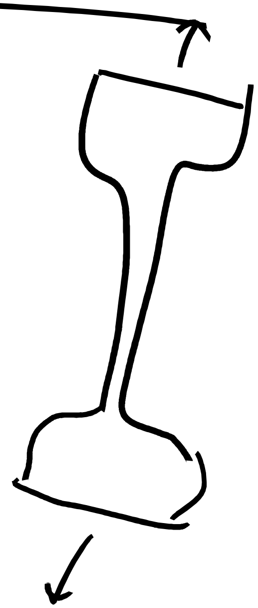
E_{11}, E_{22}, ν_{12}, G_{12}

---

(1) uniaxial tensile test: E_{11}, ν_{12}, σ_{11} failure

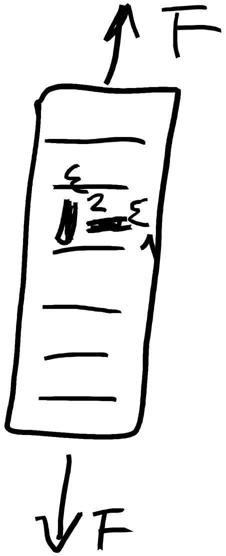


$$\begin{pmatrix} \sigma_{11} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 0 \end{pmatrix}$$ or



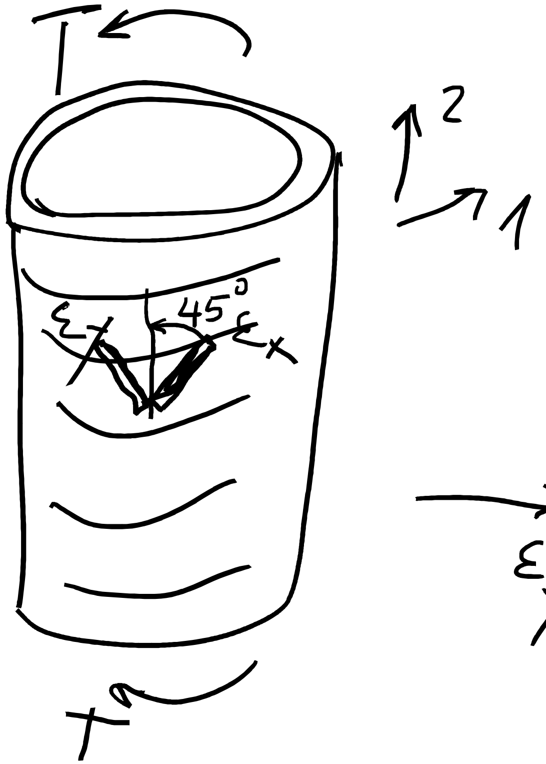
measurment uncertainities:

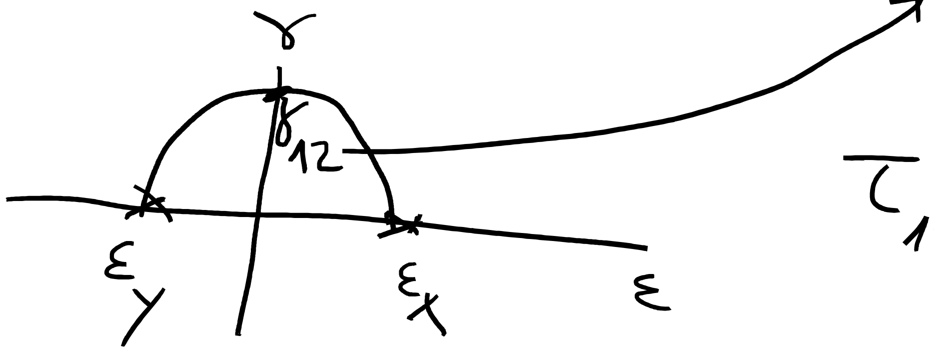
② uniaxial tensile test : E_{22}, ν_{21}, σ_{22} failure



better

③ Torsion test : G_{12} , τ_{12} failure

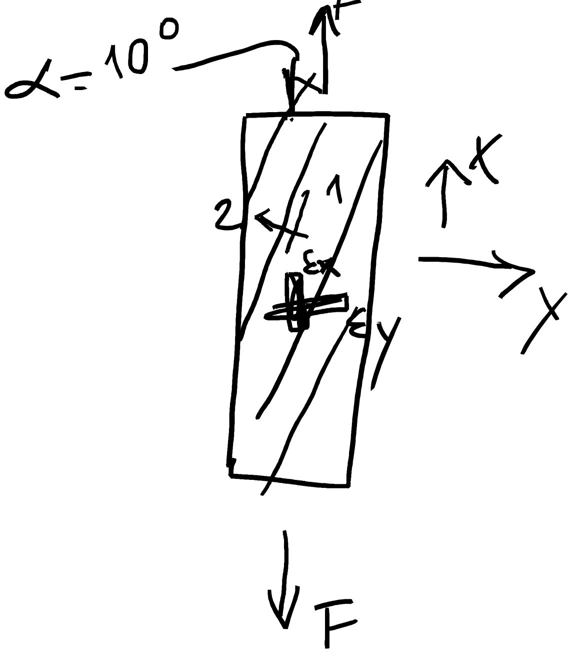




$$\tau_{12} = G_{12} \cdot \gamma_{12}$$

$$\tau_{12} = f(T, R, t)$$

④ off-axis tensile test : G_{12}

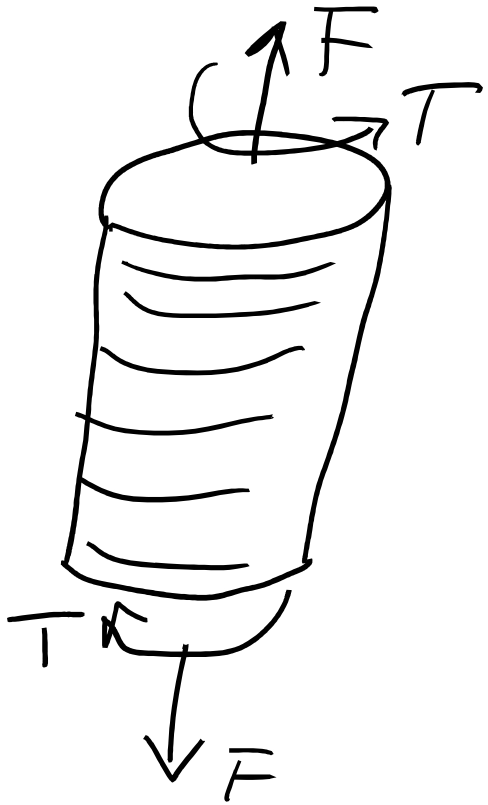


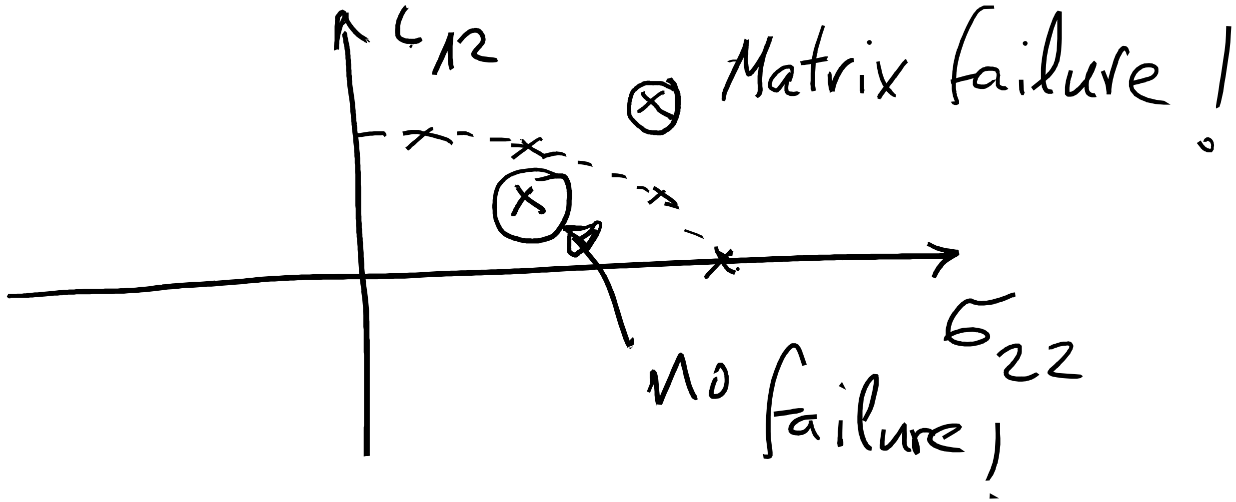
$$\begin{pmatrix}\sigma_x\\0\\0\end{pmatrix}\xrightarrow{\text{rotate}}\begin{pmatrix}\sigma_{11}\\\sigma_{22}\\\tau_{12}\end{pmatrix}$$

$$\begin{pmatrix}\varepsilon_x\\\varepsilon_y\\0\end{pmatrix}\xrightarrow{\text{rotate}}\begin{pmatrix}\varepsilon_{11}\\\varepsilon_{22}\\\gamma_{12}\end{pmatrix}$$

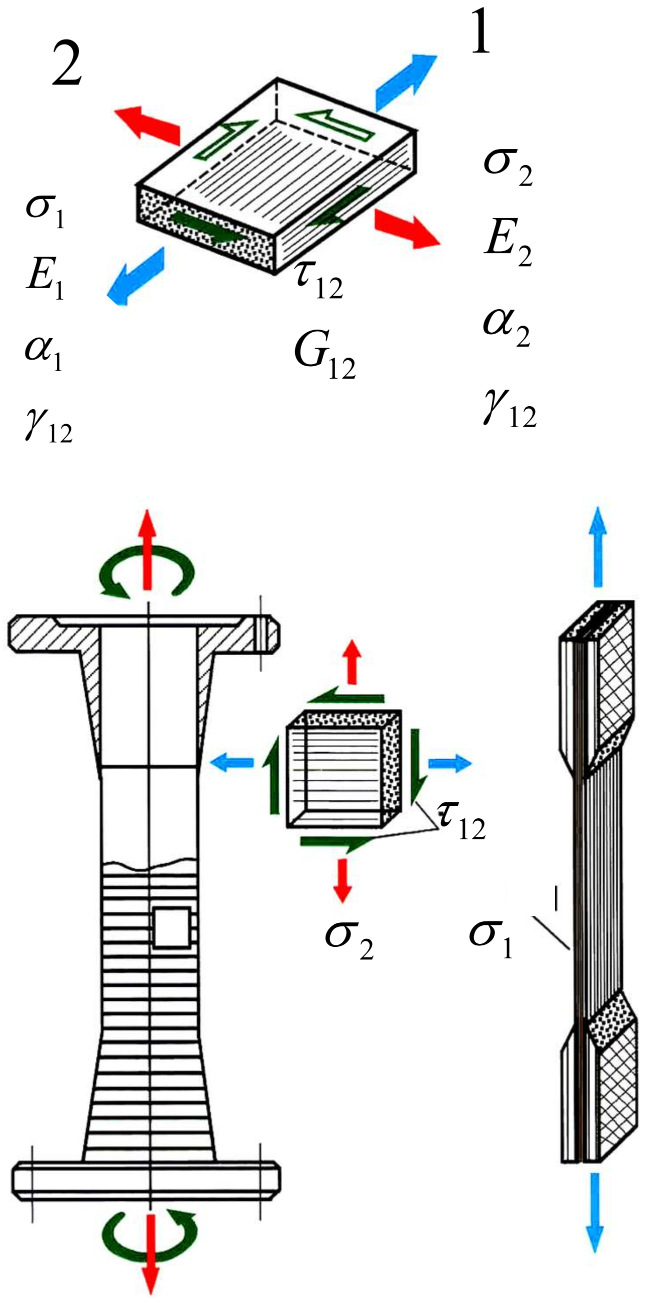
$$\begin{pmatrix}\varepsilon_{11}\\\varepsilon_{22}\\\gamma_{12}\end{pmatrix}=\begin{pmatrix}S_{11}&S_{12}&0\\S_{12}&S_{22}&0\\0&0&S_{66}\end{pmatrix}\begin{pmatrix}\sigma_{11}\\\sigma_{22}\\\tau_{12}\end{pmatrix}\Rightarrow\gamma_{12}=S_{66}\tau_{12}\longrightarrow G_{12}$$

⑤ Biaxial tension/torsion test

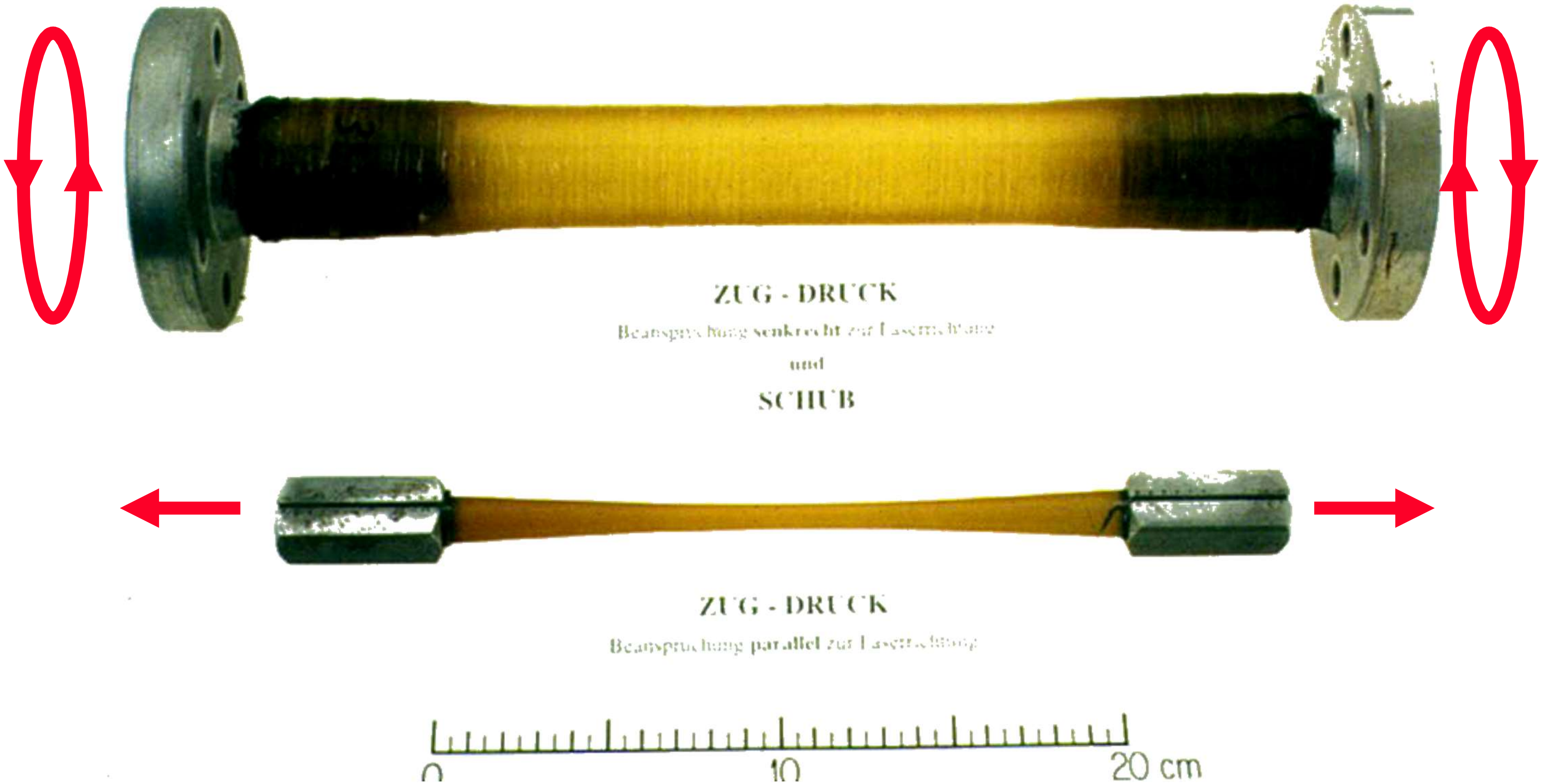




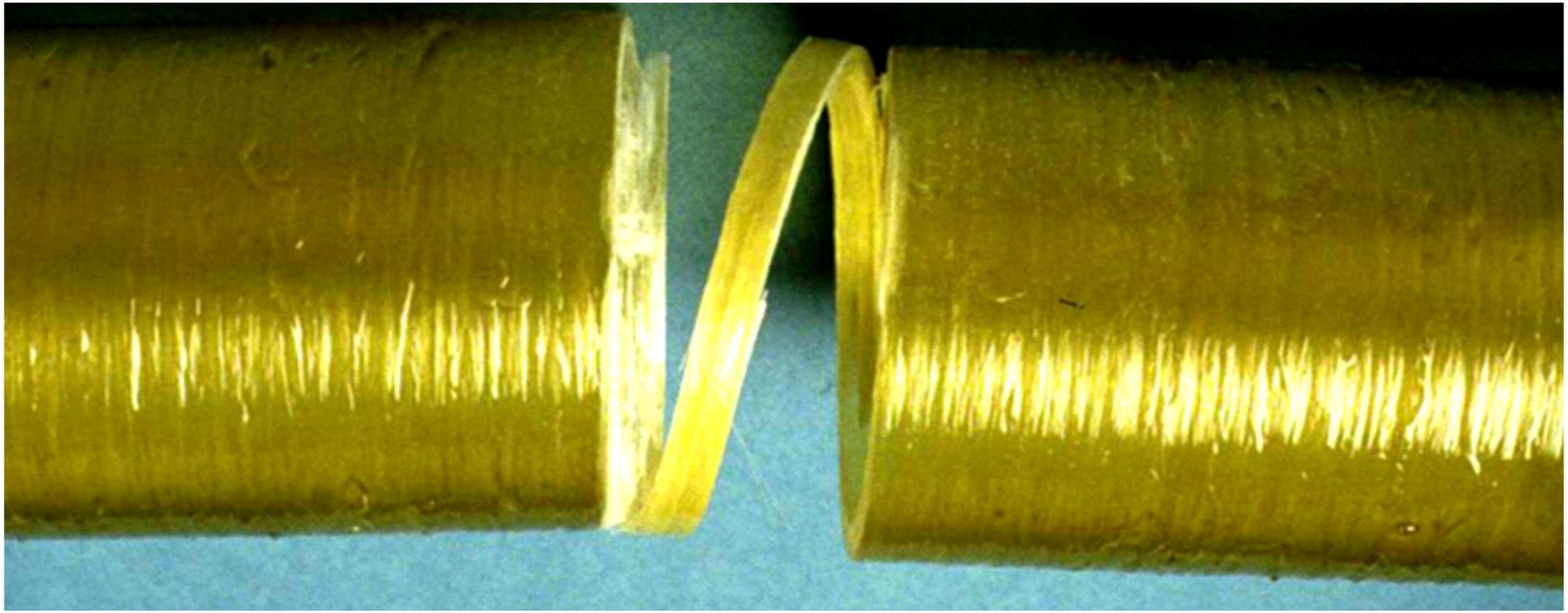
Experimental determination of the UD-Lamina properties:



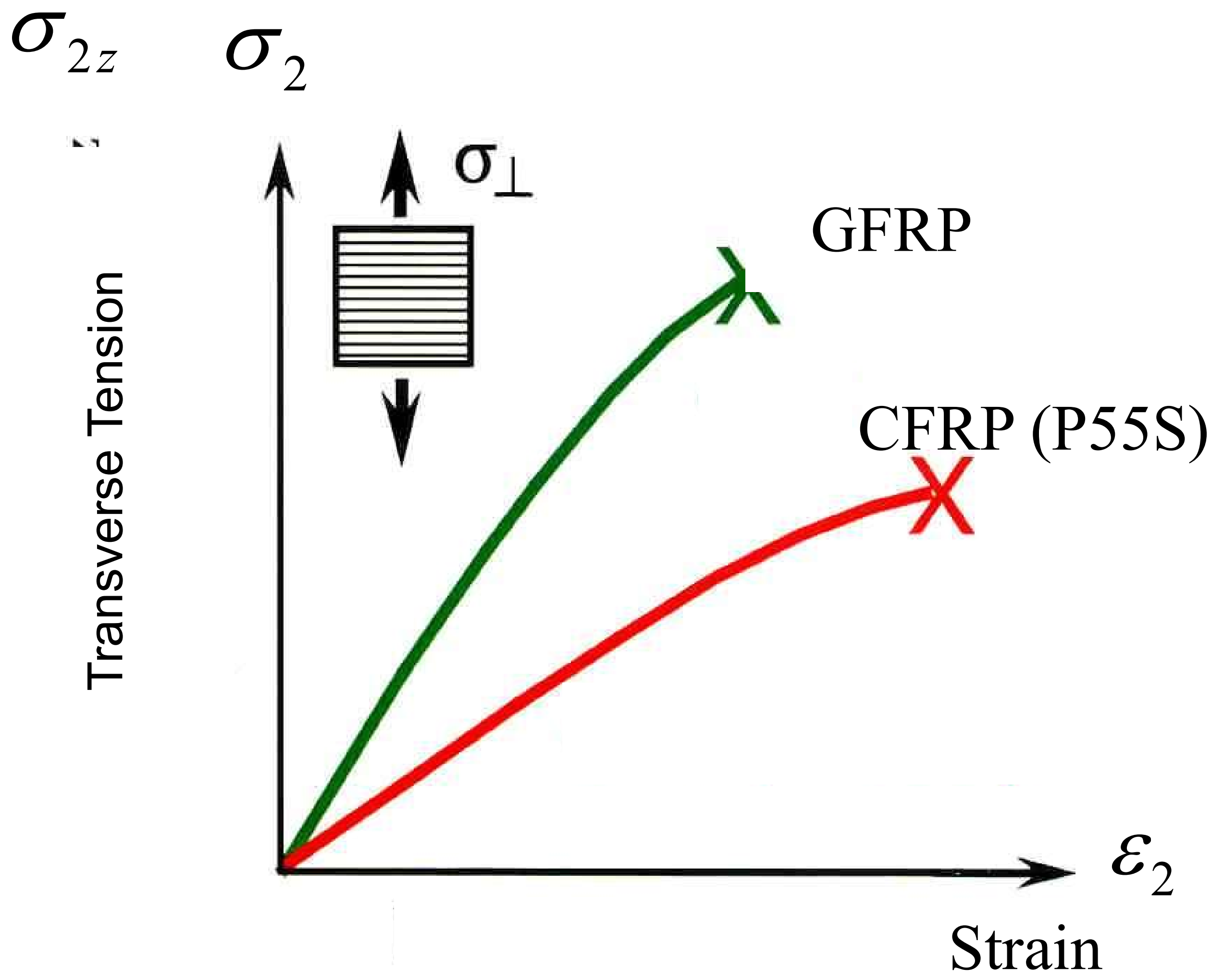
Torsion and Tensile Samples



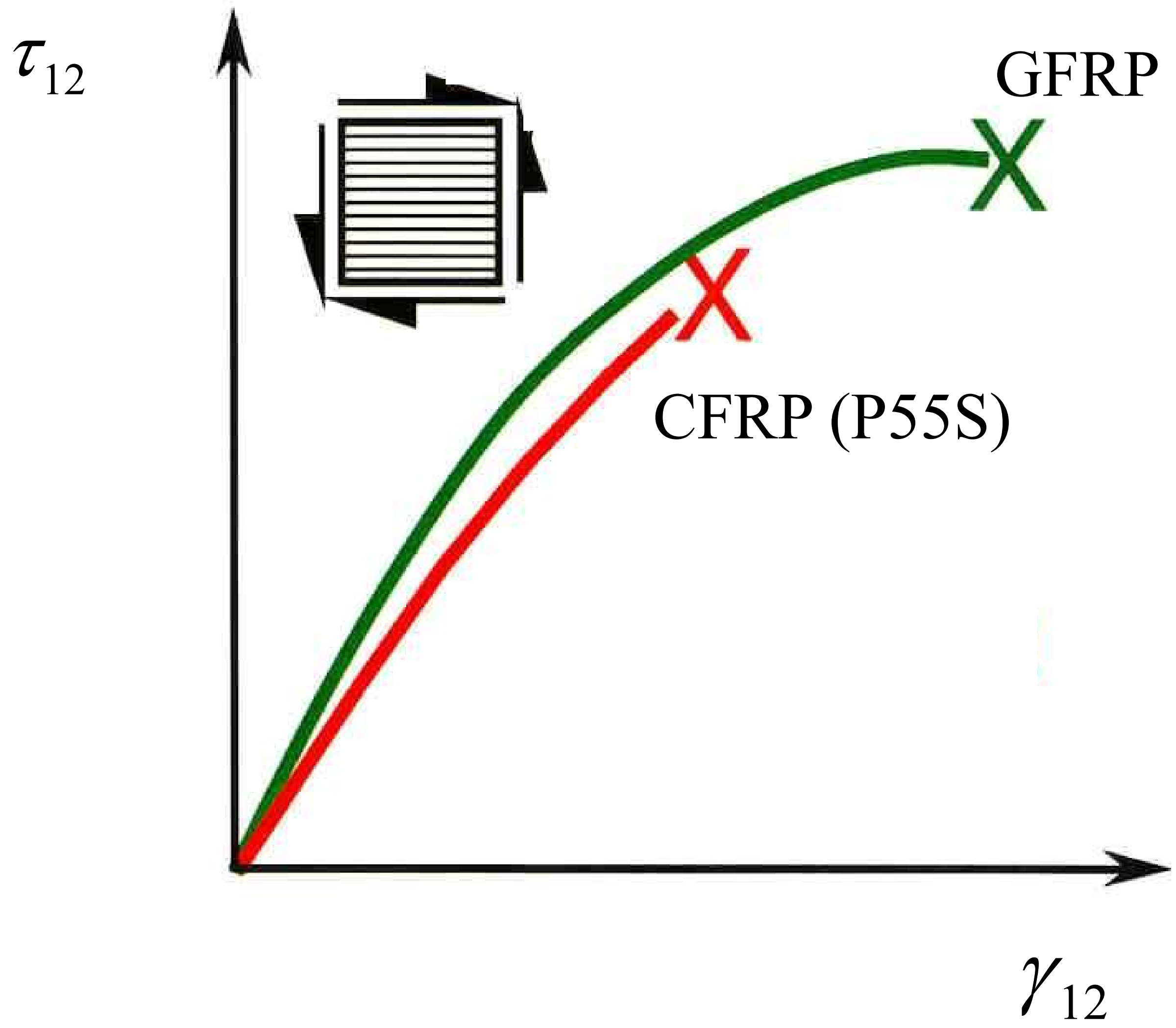
A torsion sample after the test



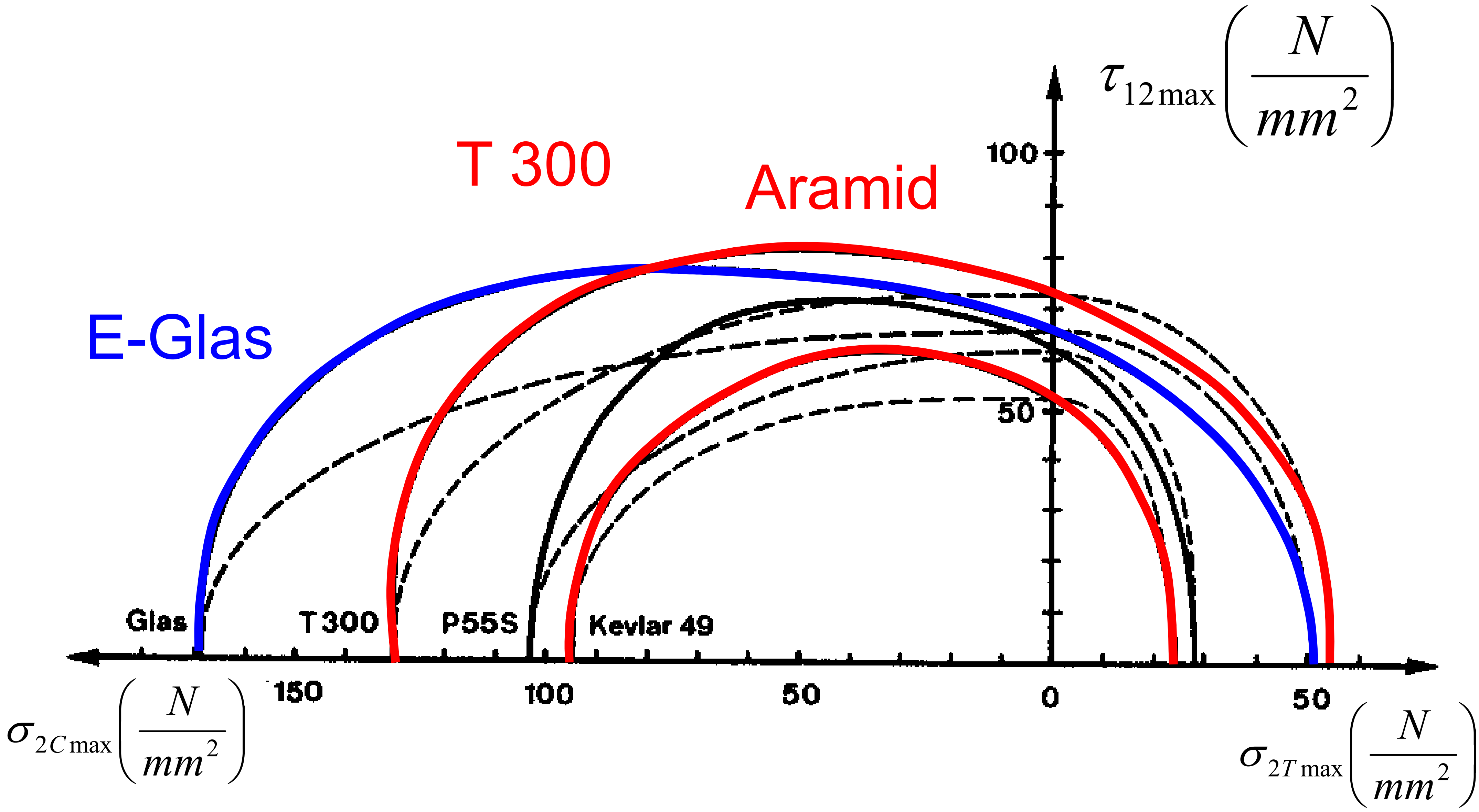
Transverse Tension



Shear



Combined shear and transverse stresses



Strength of some UD-Laminas

Lamina type	T 300/5208	B (4)/5505	AS/3501	Scotchply 1002	Kevlar 49 / Epoxy
$\sigma_{1Tmax}(N/mm^2)$ $\sigma_{1Cmax}(N/mm^2)$	1500 1500	1260 2500	1447 1447	1062 610	1400 235
$\sigma_{2Tmax}(N/mm^2)$ $\sigma_{2Cmax}(N/mm^2)$	40 246	61 202	51.7 206	31 118	12 53
$\tau_{12max}(N/mm^2)$	68	67	93	72	34

< Mechanics of a Lamina >

List of Symbols :

$\varepsilon_{11}, \varepsilon_{22}, \ldots \gamma_{23}, \gamma_{13}, \ldots$: normal and shear strains UD-Lamina; Local directions

$\sigma_{11}, \sigma_{22}, \ldots \tau_{23}, \tau_{13}, \ldots$: normal and shear stresses UD-Lamina; Local directions

$S_{11} = \frac{1}{E_{11}}$; $S_{12} = \frac{-\nu_{12}}{E_{11}}, \ldots$: Components of Compliance matrix
$E_{11}, E_{22}, \ldots G_{12}, \ldots$ E- and G-modulus in principal direction

$C_{11} = \frac{S_{22} S_{33} - S_{23} S_{23}}{S}$; $C_{12} \ldots$: Components of stiffness matrix

$\nu_{12}, \nu_{13}, \nu_{23}$: poisson's ratios

$\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}$: stiffness matrix of a UD-Lamina

$\begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix}$: Compliance matrix of a UD-Lamina

E_F, ν_F : fibre E-modulus and fibre Poisson's ratio

E_M, ν_F : Matrix E-modulus and Matrix Poisson's ratio

φ (or φ_f or V^f) : fibre volume fraction

$E_1 = E_{11}$; $E_2 = E_{22}$: UD - Elastic moduli

α_1 ; α_2 : UD Thermal expansion coefficients in Local direction

α_x ; α_y , α_{xy} : UD Thermal expansion Coefficients in global direction