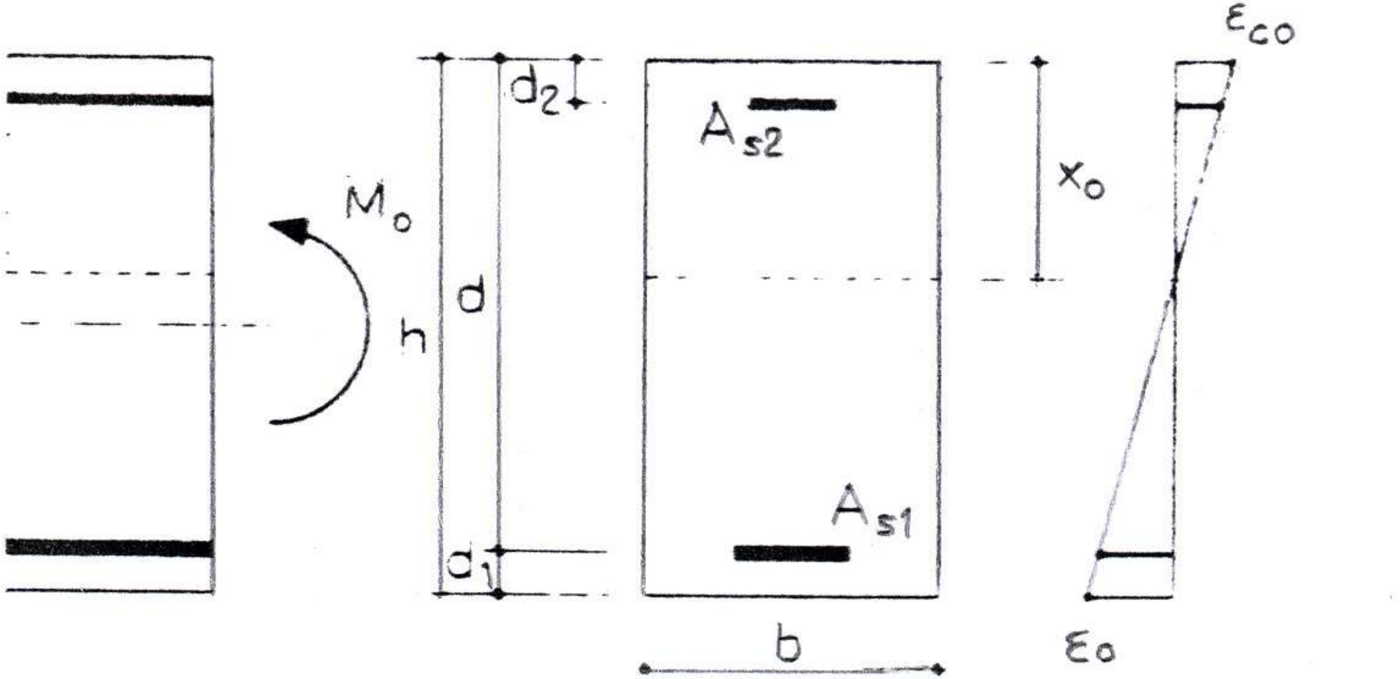
Table of content:

Flexural strengthening

Book Composite for Construction, L. C. Bank, Chapter 9

Initial situation prior to strengthening

The effect of the initial load prior to strengthening should be considered in the calculation of strengthened member. Based on the theory of elasticity and with M_0 the service moment (*no* load safety factors are applied) acting on the critical RC section during strengthening, the strain distribution of the member can be evaluated. As M_0 is typically larger than the cracking moment M_{cr}, the calculation is based on a cracked section.



If M0 is smaller than Mcr, its influence on the calculation of the strengthened member may easily be neglected.

Based on the transformed cracked section, the neutral axis depth x_0 can be solved from:

$$\frac{1}{2} b x_0^2 + (\alpha_s - 1) A_{s2} (x_0 - d_2) = \alpha_s A_{s1} (d - x_0)$$

Where:

$$\alpha_s = \frac{E_s}{E_c}$$

The concrete strain at the top fiber can be expressed as:

$$\varepsilon_{c0} = \frac{M_0 x_0}{E_c I_{02}}$$

Where I_{02} is the moment of inertia of the transformed cracked section:

$$I_{02} = \frac{b x_0^3}{3} + (\alpha_s - 1) A_{s2} (x_0 - d_2)^2 + \alpha_s A_{s1} (d - x_0)^2$$

Based on strain compatibility, the concrete strain at the extreme tension fiber can be derived as:

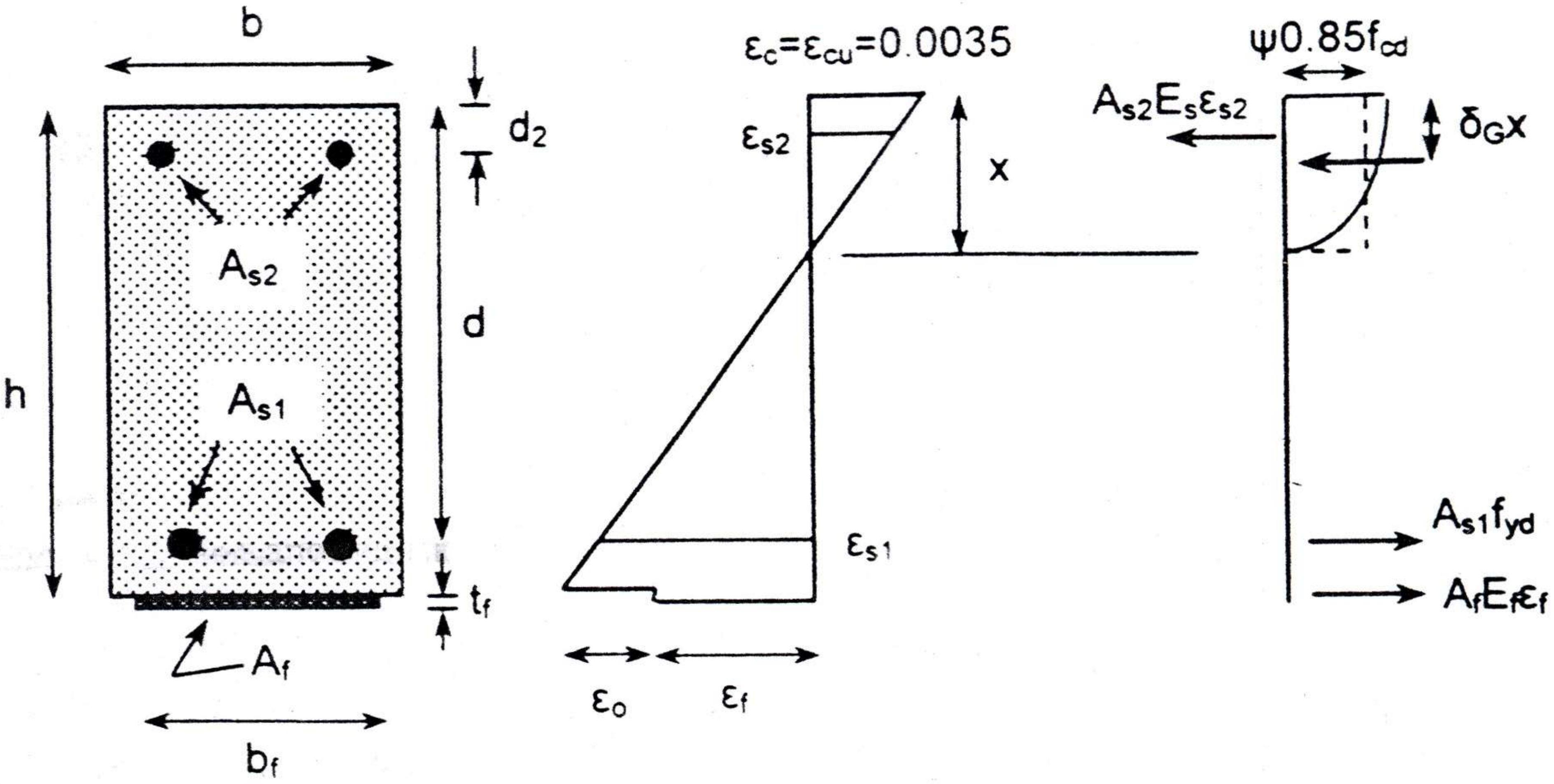
$$\varepsilon_0 = \varepsilon_{c0} \frac{h - x_0}{x_0}$$

This strain equals the initial axis strain at the level of the FRP, needed for the evaluation of the strengthened member.

Analysis of Ultimate Limit State (ULS)

Full composite action

Steel yielding followed by concrete crushing



Calculation of neutral axis depth, x:

$$0.85.\psi.f_{cd}bx + A_{s2}E_s\varepsilon_{s2} = A_{s1}f_{yd} + A_f E_{fu}\varepsilon_f$$

Where:

$$\psi = 0.8$$

and:

$$\varepsilon_{s2} = \varepsilon_{cu}\frac{x-d_2}{x}$$

($E_s\varepsilon_{s2}$ not to exceed f_{yd})

$$\varepsilon_f = \varepsilon_{cu}\frac{h-x}{x} - \varepsilon_0$$

Design bending moment capacity:

$$M_{Rd} = A_{s1} f_{yd} (d - \delta_G x) + A_f E_f \varepsilon_f (h - \delta_G x) + A_{s2} E_s \varepsilon_{s2} (\delta_G x - d_2)$$

Where:

$$\delta_G = 0.4$$

Check if

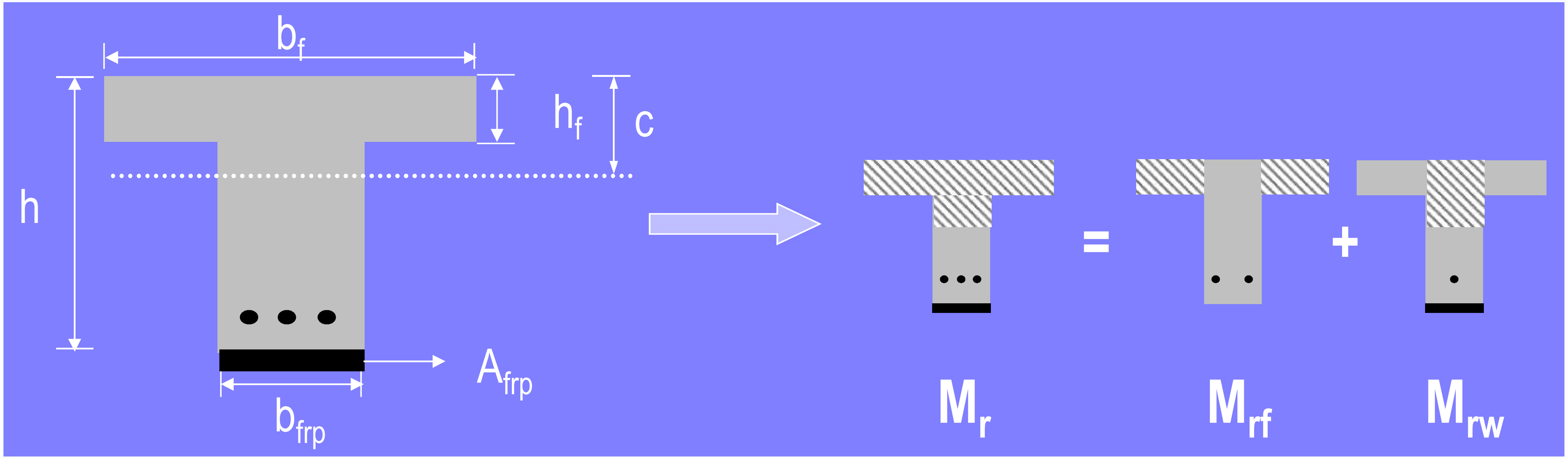
a) Yielding of tensile steel reinforcement:

$$\varepsilon_{s1} = \varepsilon_{cu} \frac{d - x}{x} \geq \frac{f_{yd}}{E_s}$$

b) Straining of the FRP is limited to the ultimate strain:

$$\varepsilon_{f} = \varepsilon_{cu} \frac{h - x}{x} - \varepsilon_{0} \leq \varepsilon_{fud}$$

Tee Beams

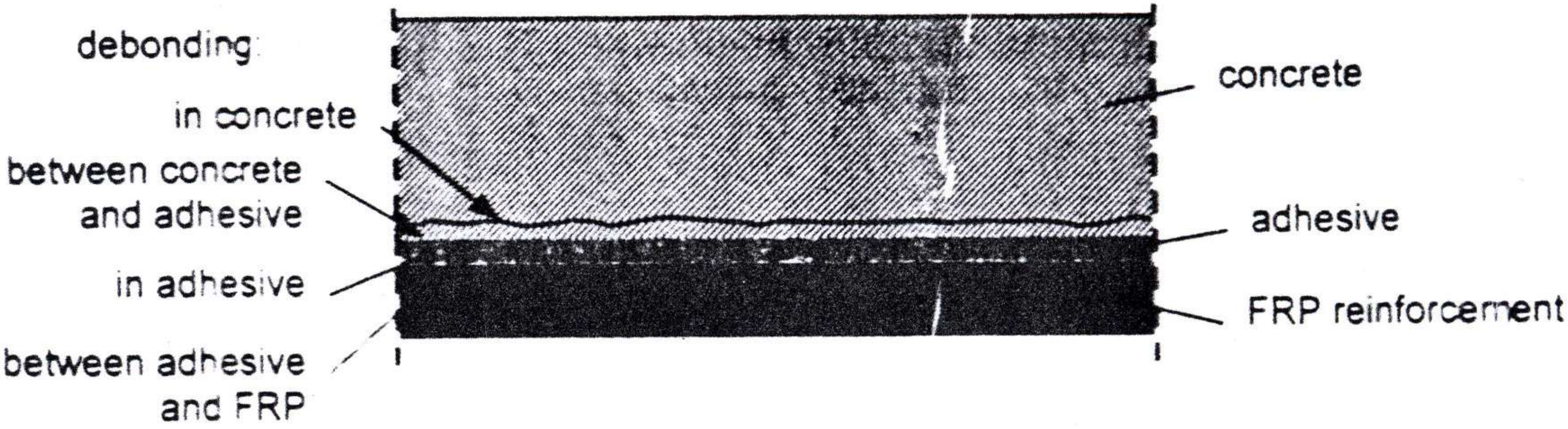


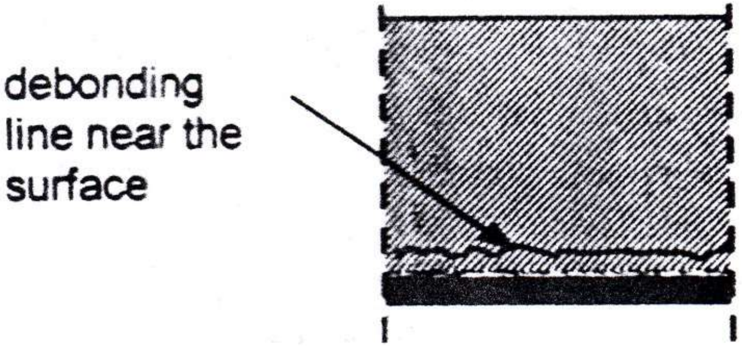
Neutral axis in flange: treat as rectangular section

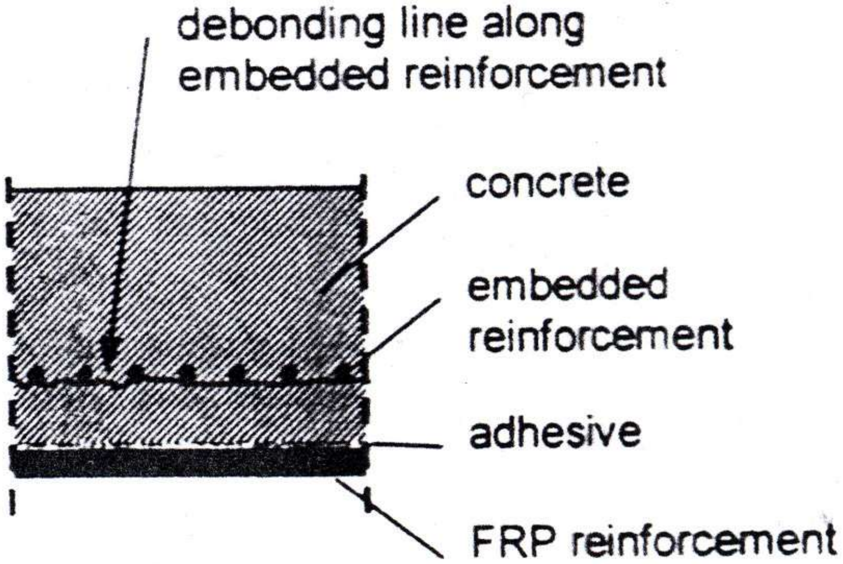
Neutral axis in web: treat as tee section

Debonding and bond failure modes

- Debonding in the concrete near the surface or along a weakened layer, e.g. along the line of the embedded steel reinforcement.
- Debonding in the adhesive (cohesion failure).
- Debonding at the interfaces between concrete and adhesive or adhesive and FRP (adhesion failure).
- Debonding inside the FRP (interlaminar shear failure).

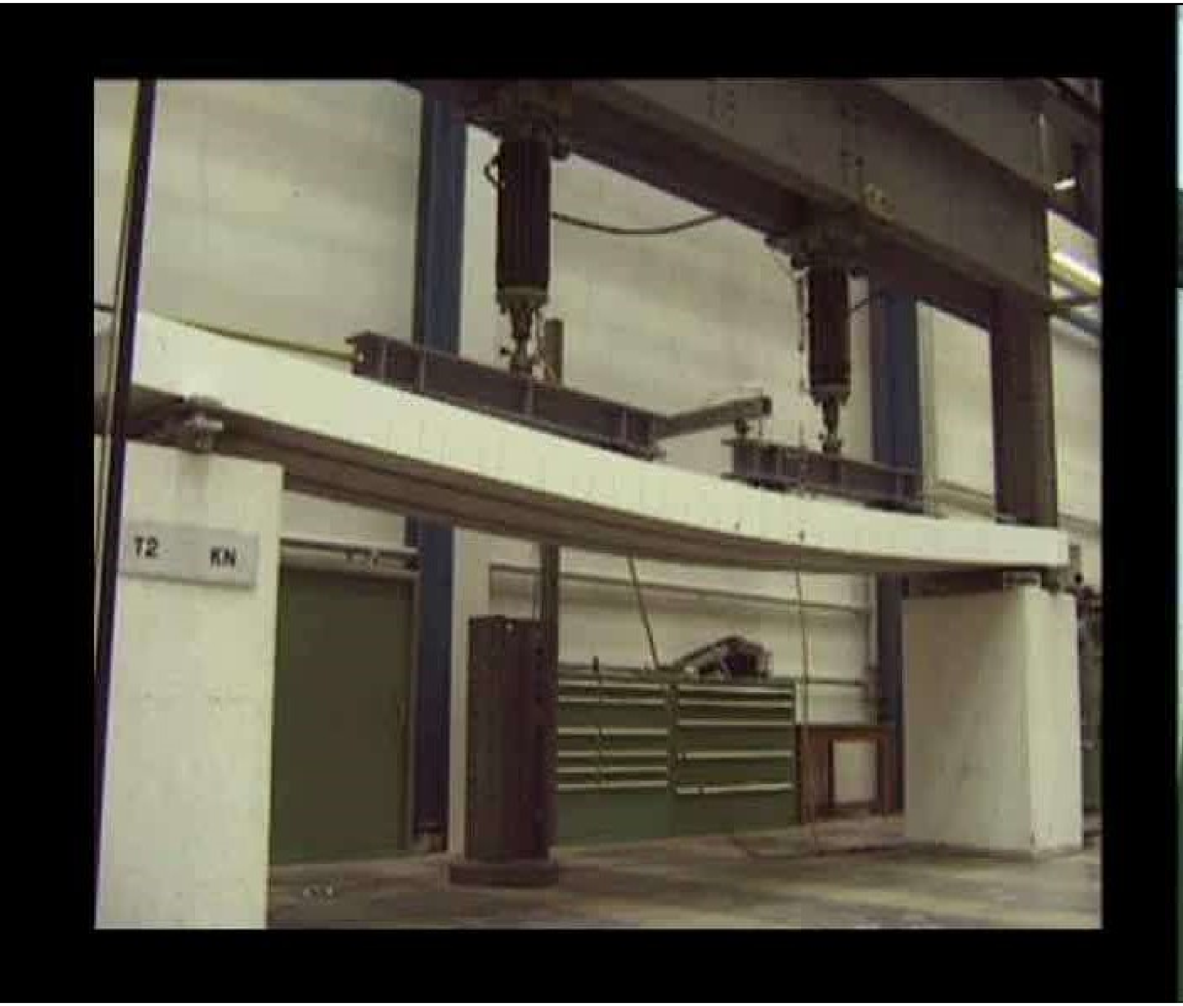

debonding:
in concrete
between concrete and adhesive
in adhesive
between adhesive and FRP
concrete
adhesive
FRP reinforcement


debonding line near the surface


debonding line along embedded reinforcement
concrete
embedded reinforcement
adhesive
FRP reinforcement

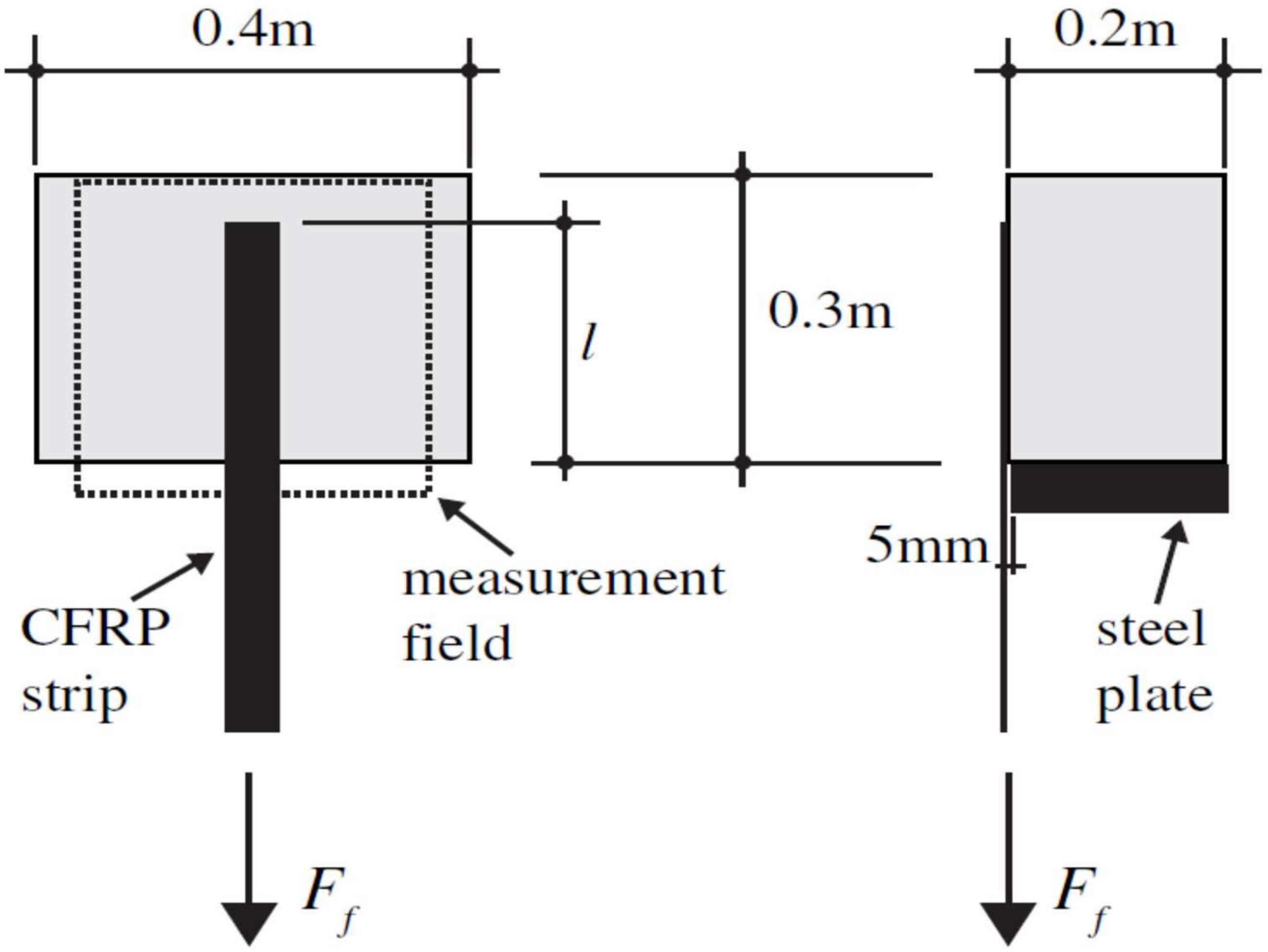
Debonding Video Clips

Non Prestressed CFRP

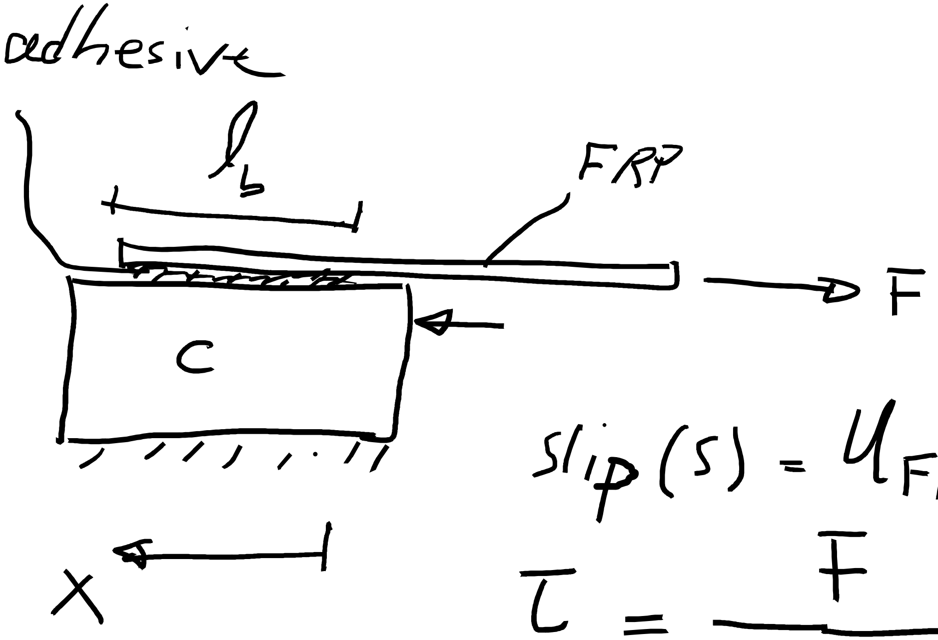


Prestressed CFRP

Lap Shear Test

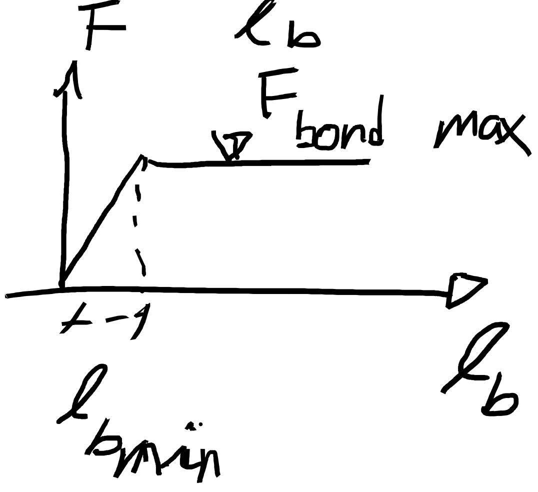


Lap shear test (Bond test)



①

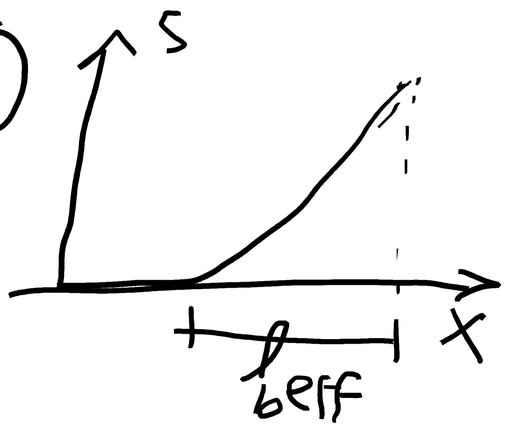
several test with different l_b



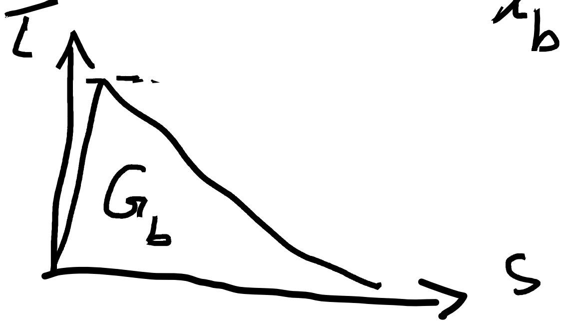
$$\text{slip}\,(s) = u_{FRP} - u_c$$

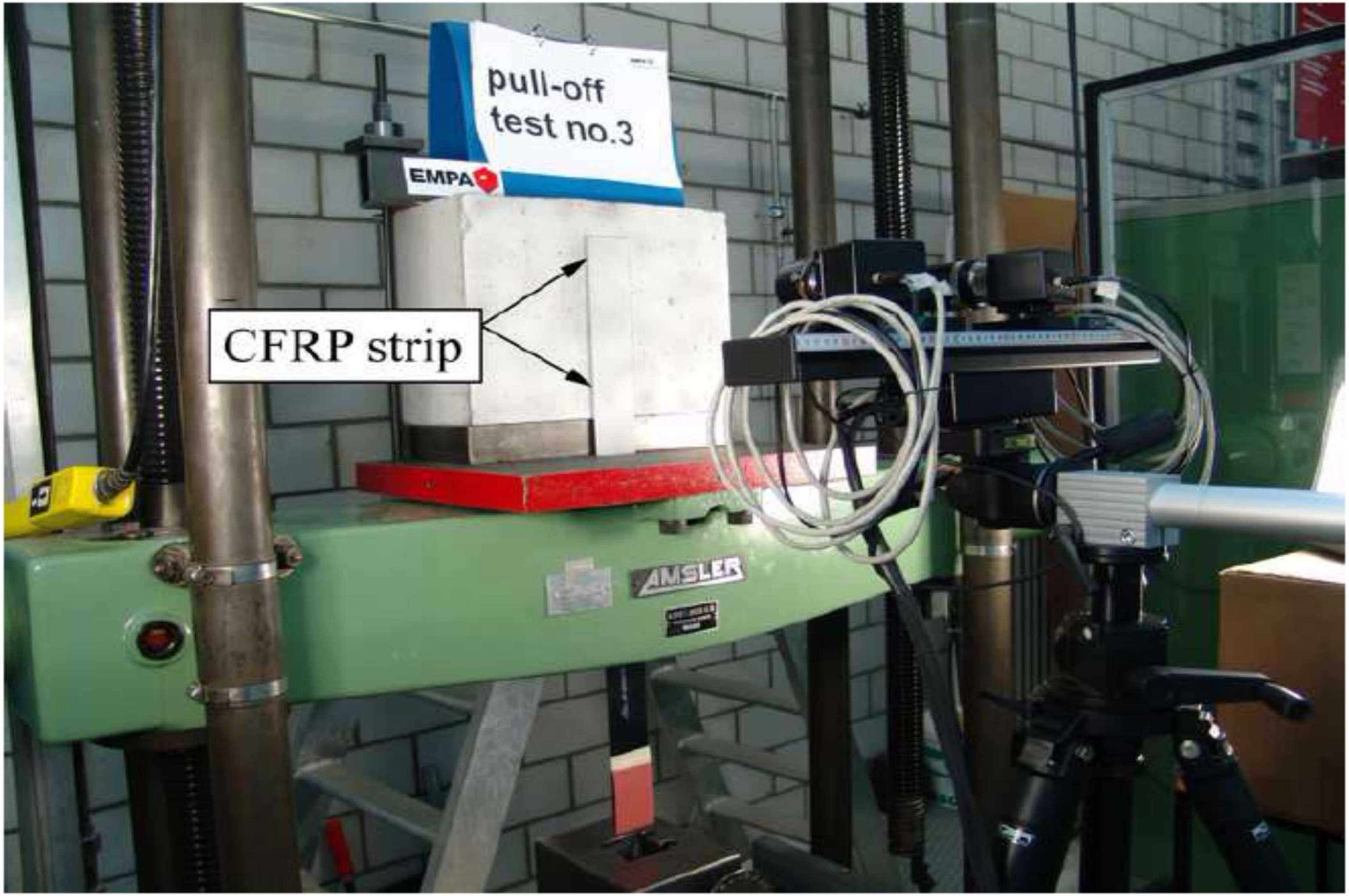
$$\tau = \frac{F}{l_{b\,eff} \cdot W}$$

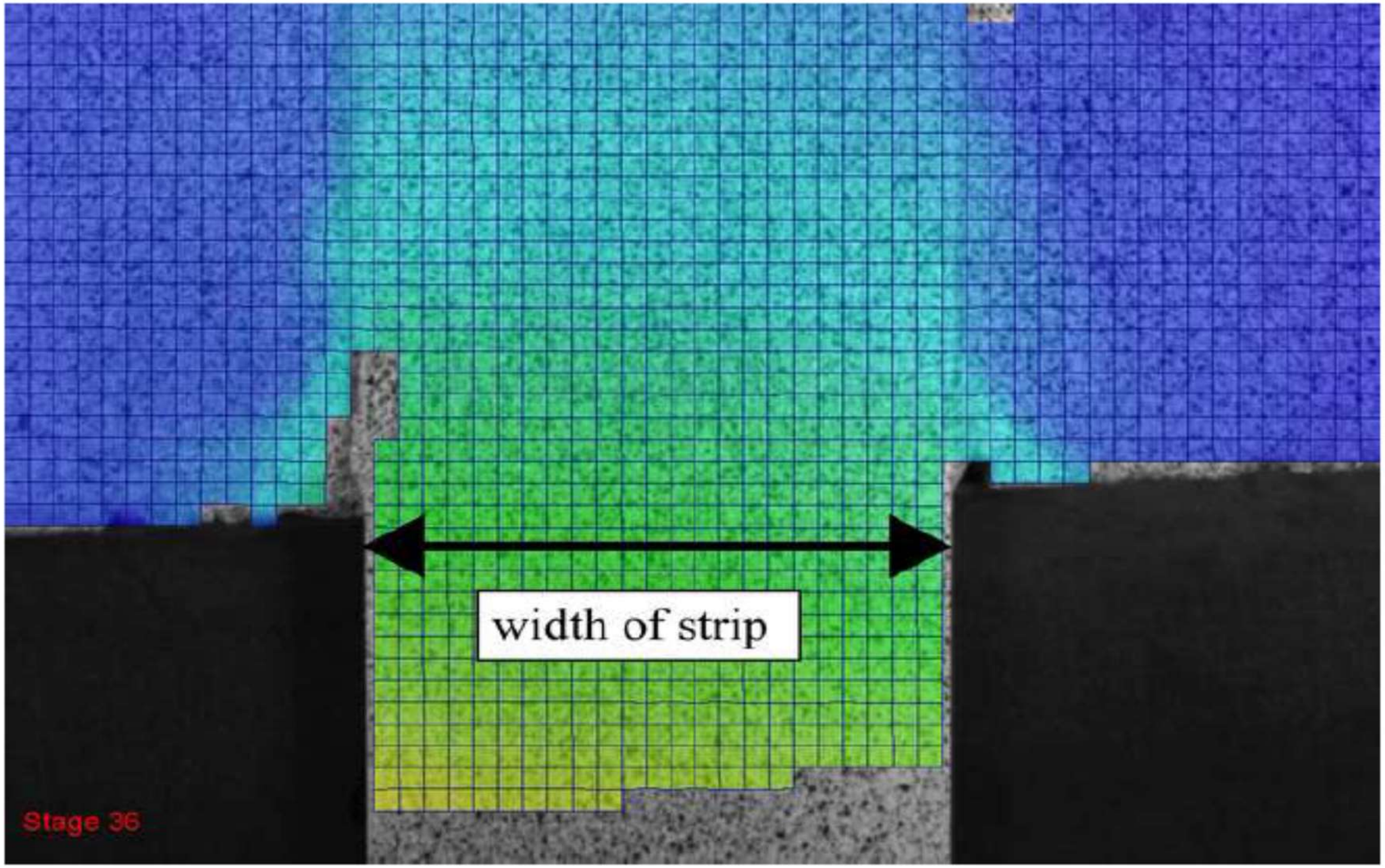
②

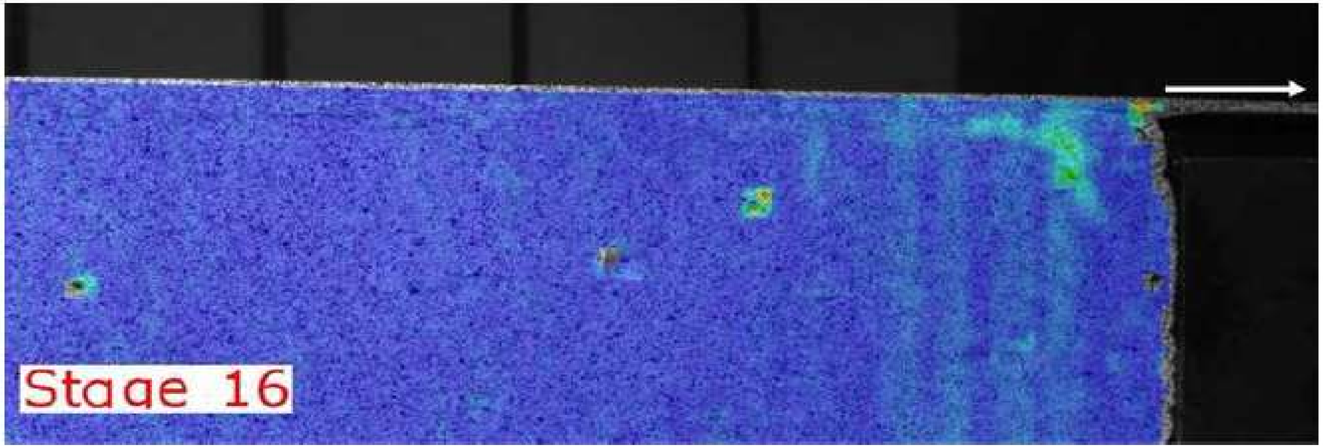


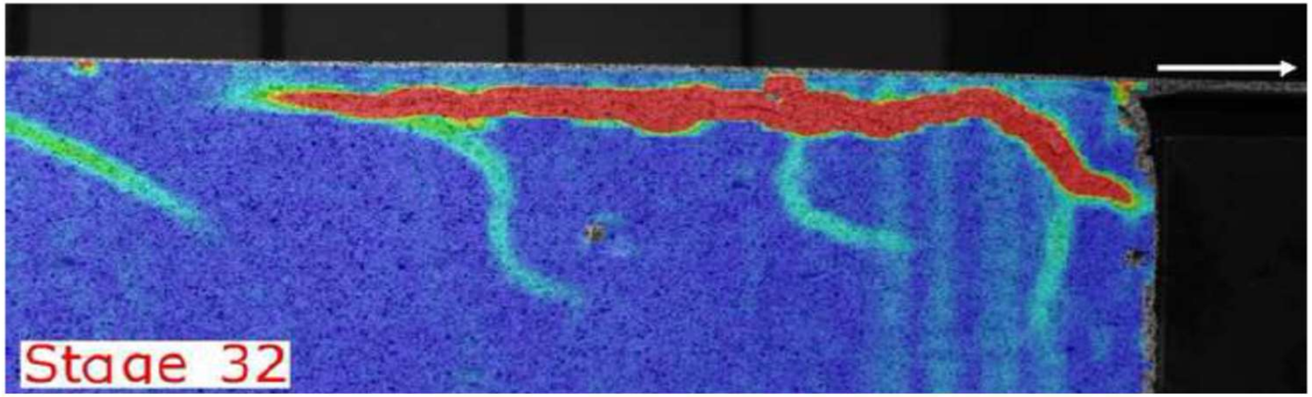
③

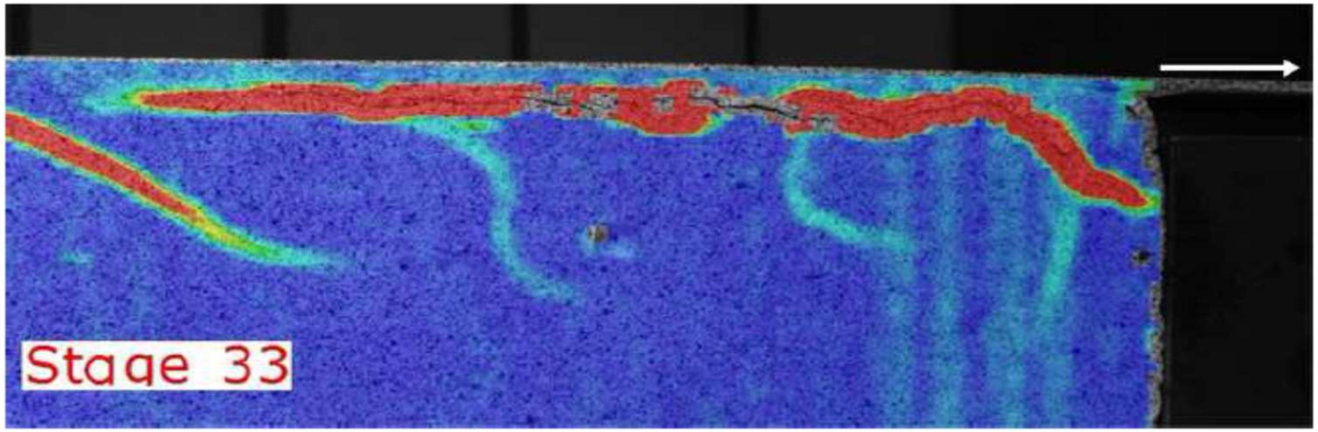


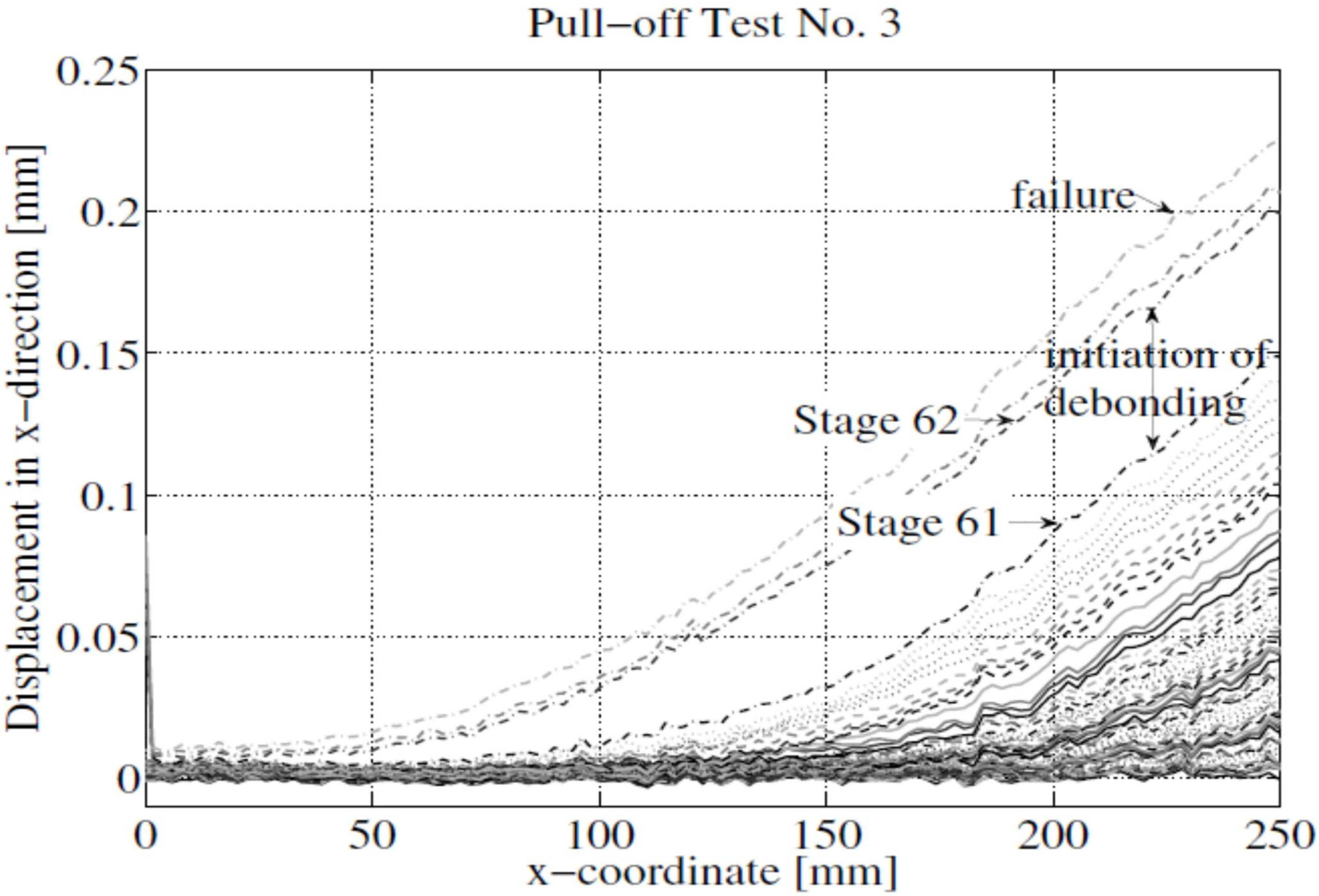

pull-off
test no.3
EMPA
CFRP strip
AMSLER

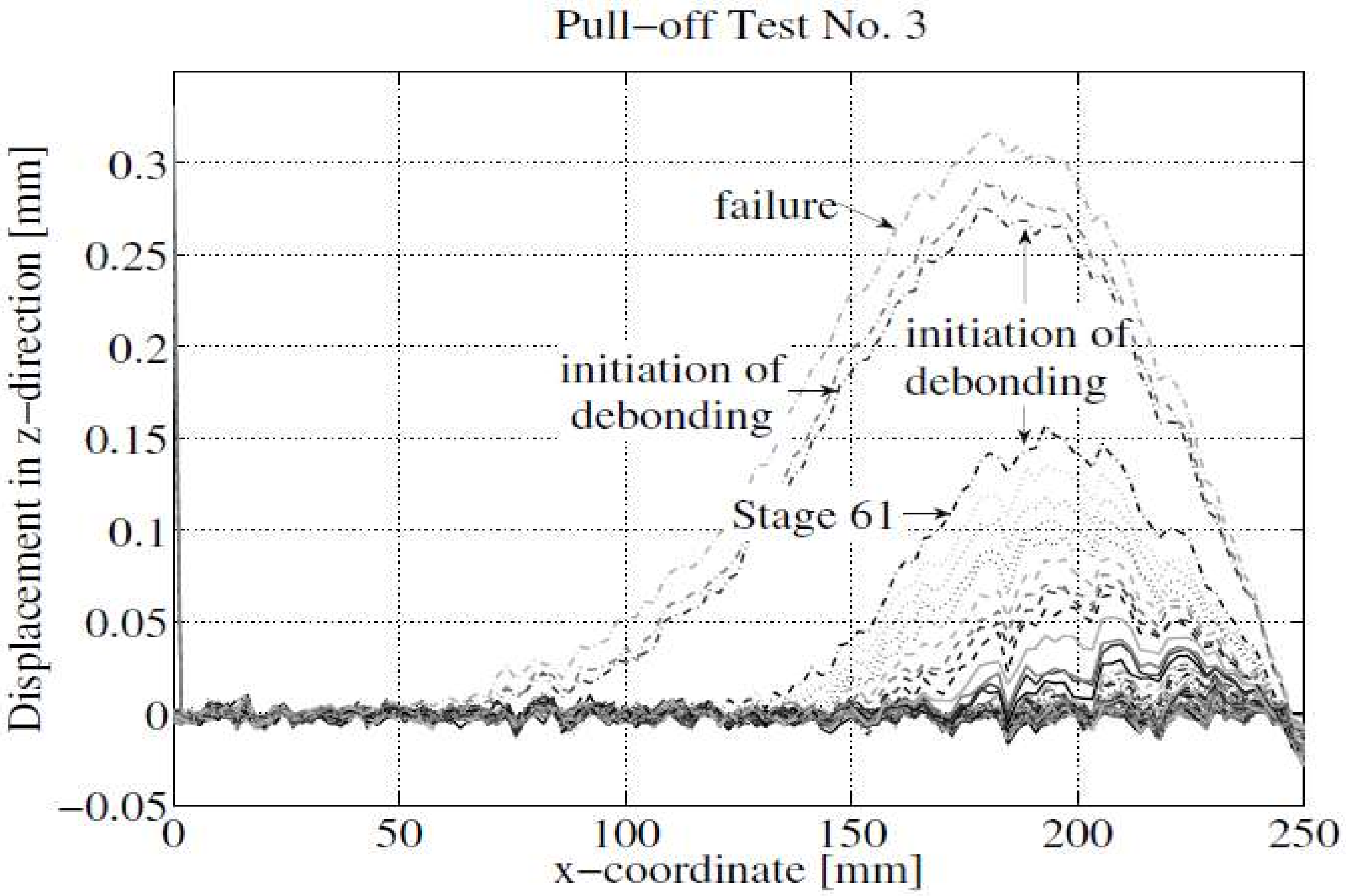

width of strip
Stage 36

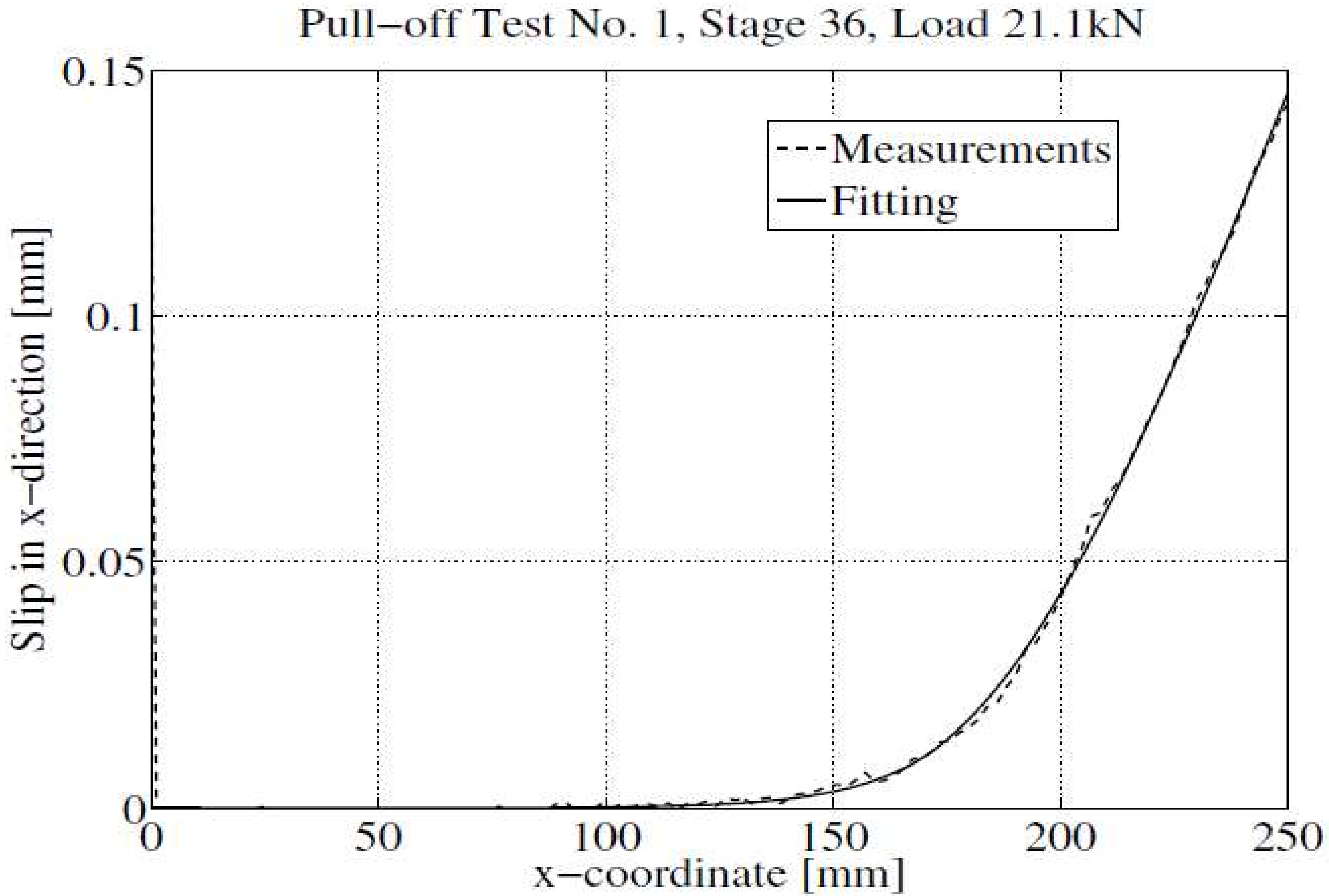

Stage 16

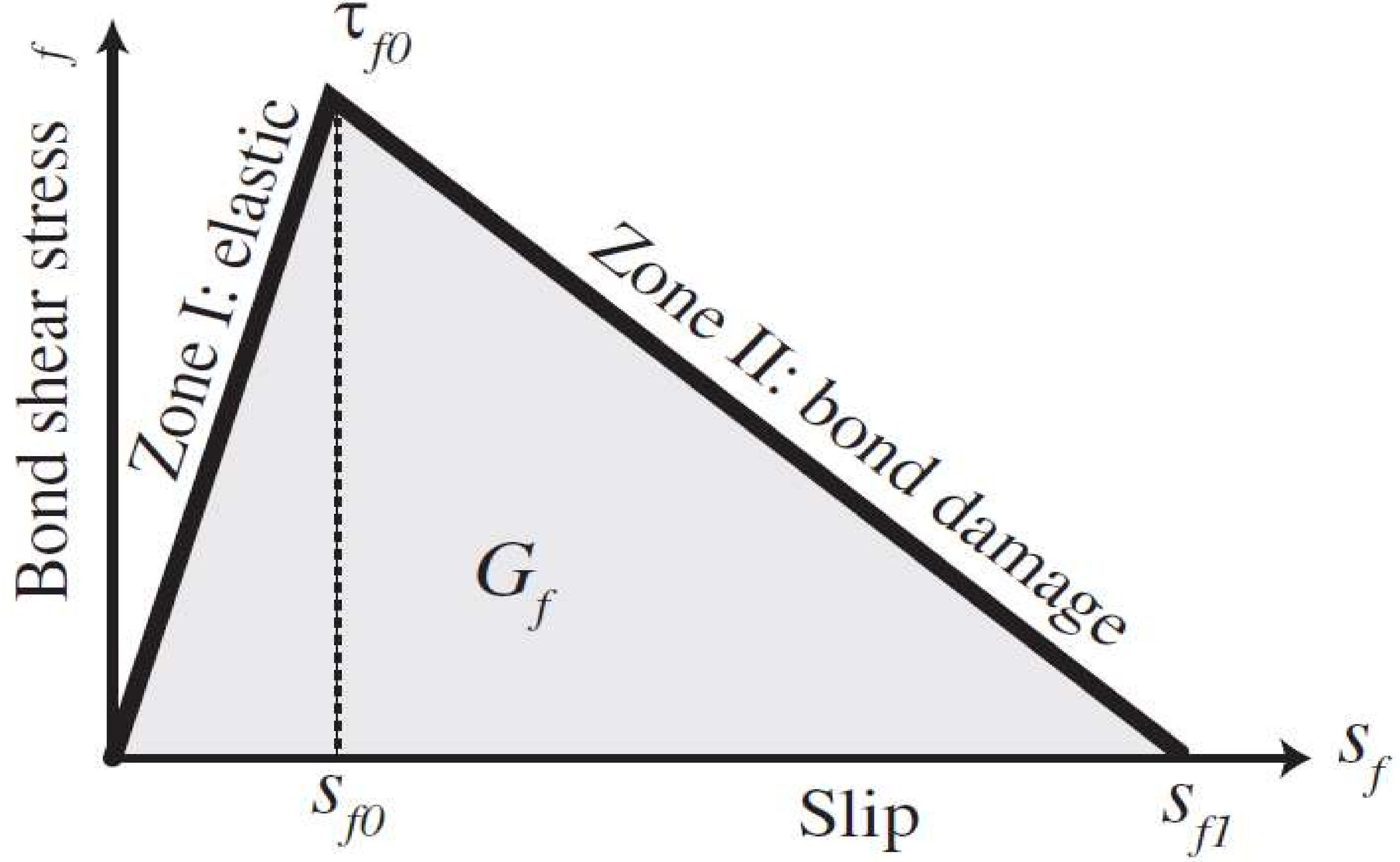

Stage 32


Stage 33


Pull–off Test No. 3
Displacement in x–direction [mm]
0.25
0.2
0.15
0.1
0.05
0
x–coordinate [mm]
0
50
100
150
200
250
failure
initiation of debonding
Stage 62
Stage 61

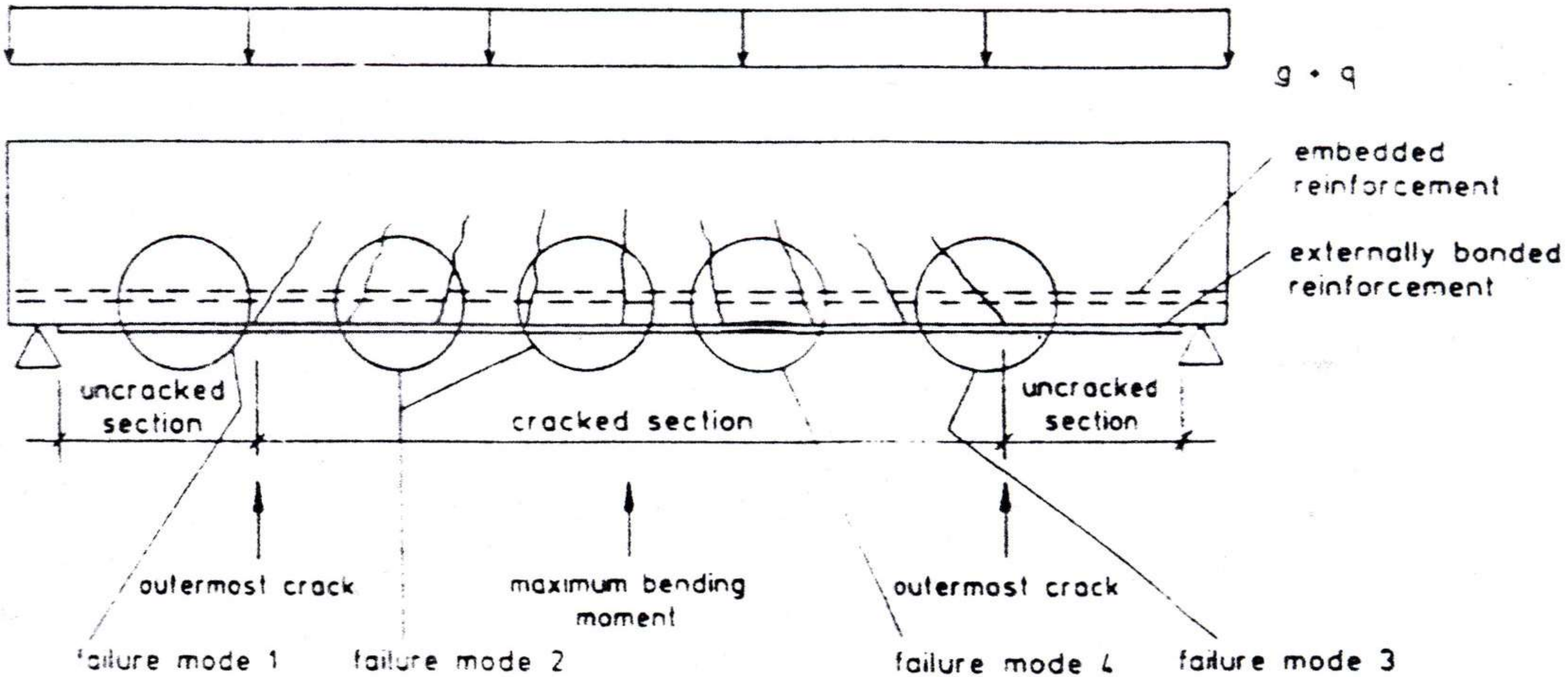

Pull−off Test No. 3
Displacement in z−direction [mm]
x−coordinate [mm]
0.3
0.25
0.2
0.15
0.1
0.05
0
−0.05
0
50
100
150
200
250
failure
initiation of debonding
initiation of debonding
Stage 61


Pull−off Test No. 1, Stage 36, Load 21.1kN
Slip in x−direction [mm]
0.15
0.1
0.05
0
0
50
100
150
200
250
x−coordinate [mm]
Measurements
Fitting


Bond shear stress τ_f
τ_{f0}
Zone I: elastic
Zone II: bond damage
G_f
s_f
s_{f0}
Slip
s_{f1}

Bond failure of RC members strengthened with FRP:

See next lecture given by Dr. Christoph Czaderski



Summary of the three Swiss Code (SIA 166) verifications

See next lecture given by Dr. Christoph Czaderski

1. End strip debonding failure at the last crack

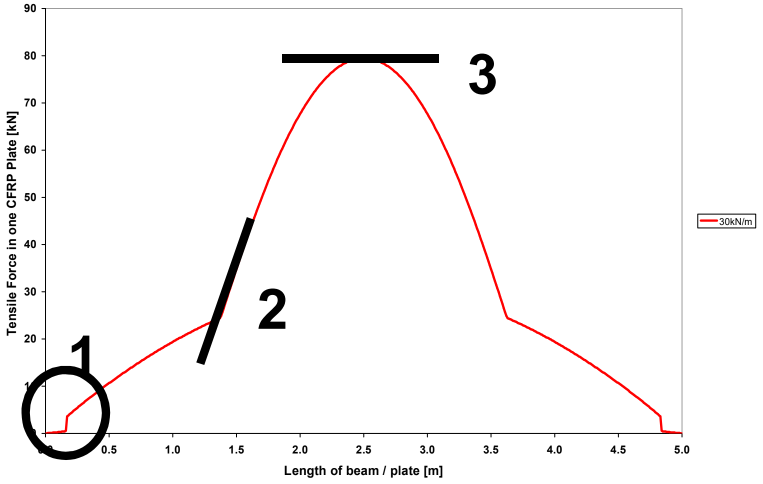
$$\boxed{F_{fcr} \le F_{b,R}}$$

2. Debonding at strong strain increase in strip

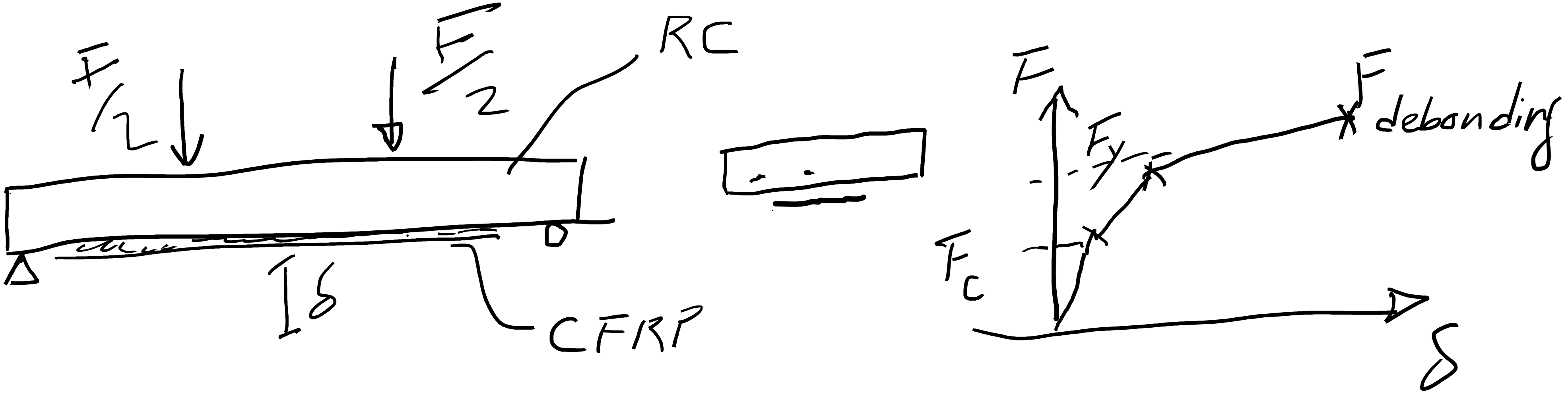
$$\boxed{\left(\frac{\Delta F_f}{\Delta x}\right) \le \left(\frac{\Delta F_f}{\Delta x}\right)_R}$$

3. Debonding at flexural cracks

$$\boxed{\varepsilon_f \le \varepsilon_{f,lim,d} = 8\text{‰}}$$



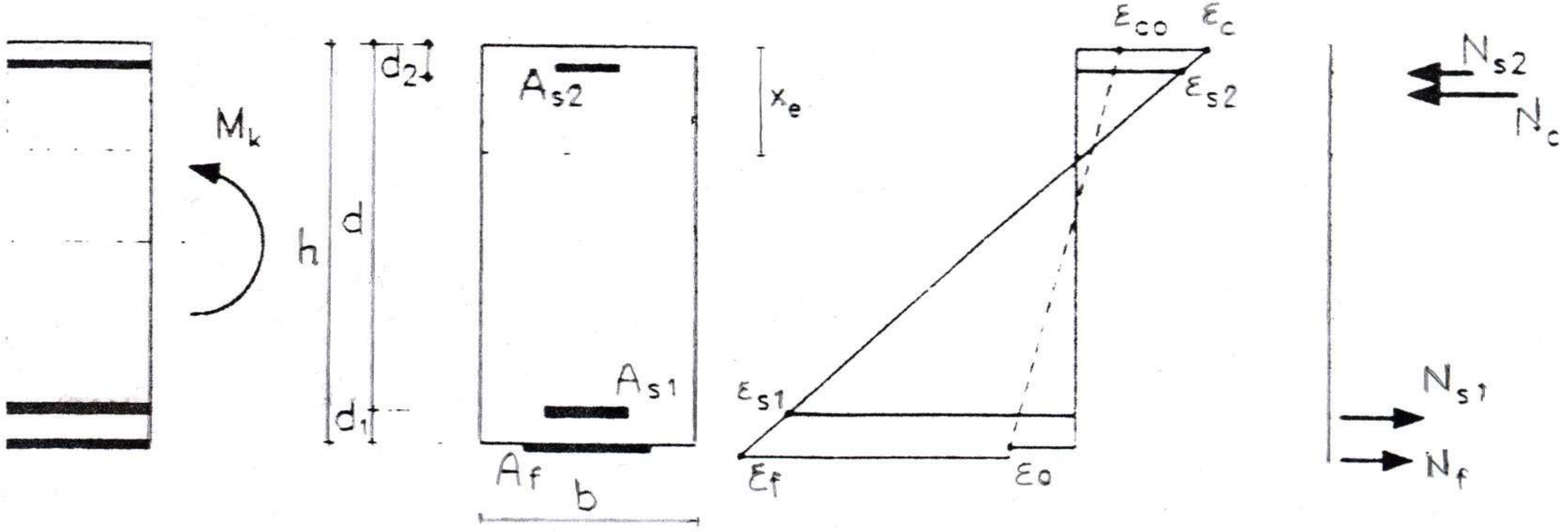
4-Point Bendig test, RC beam strengthened with a CFRP strip



Debonding: check the 3 criterias in SIA 166

Serviceability Limit State (SLS)

- linear elastic material behavior
- cracked section analysis



Calculation of neutral axis x_e:

$$\frac{1}{2} b x_e^2 + (\alpha_s - 1) A_{s2} (x_e - d_2) = \alpha_s A_{s1} (d - x_e) + \alpha_f A_f \left[h - (1 + \frac{\varepsilon_0}{\varepsilon_c}) x_e \right]$$

Where: $$\alpha_f = \frac{E_f}{E_c}$$

$$\alpha_s = \frac{E_s}{E_c}$$

And the cracking moment for rectangular beams:

$$M_{cr} \approx f_{ctm} . \frac{bh^2}{6}$$

Stress limitation

limit stresses in the concrete, steel and FRP to prevent

- damage or excessive creep of the concrete
- steel yielding
- excessive creep or creep rupture of the FRP

$\sigma_c \leq 0.60 f_{ck}$ under the rare load combination

$\sigma_c \leq 0.45 f_{ck}$ under the quasi-permanent load combination

where: $\sigma_c = E_c \varepsilon_c$

To prevent yielding of the steel at service load:

$$\sigma_s = E_s . \varepsilon_c . \frac{d - x_e}{x_e} \leq 0.80 f_{yk}$$ rare load combination

FRP stress under service load should be limited as:

$$\sigma_f = E_f.(\varepsilon_c.\frac{h - x_e}{x_e} - \varepsilon_0) \leq \eta.f_{fk}$$ quasi-permanent load combination

Where $$\eta = \begin{cases} 0.8 : CFRP \\ 0.5 : AFRP \\ 0.3 : GFRP \end{cases}$$

Verification of deflections

The mean deflection, a, is calculated from:

$$a = a_1 .(1 - \zeta_b) + a_2 .\zeta_b$$

Where a_1 and a_2 are the deflections in the uncracked and the fully cracked state, respectively and the distribution coefficient is:

$$\zeta_b = 0 \ldots\ldots\ldots M_k < M_{cr}$$

$$\zeta_b = 1 - \beta_1 .\beta_2 .(\frac{M_{cr}}{M_k})^{n/2} \ldots\ldots\ldots M_k > M_{cr}$$

- Where β1 is a coefficient taking into account the bond characteristics of the reinforcement and equals 0.5 and 1 for smooth and deformed steel, respectively;

- β2 is a coefficient taking into account the loading type and equals 0.5 and 1 for long-term and short term loading, respectively.

- The power n equals 2. For high strength concrete more accuracy is obtained with n equal to 3.

The deflection in the **uncracked state, a1**, and in the **fully cracked state, a2,** can be calculated by classical elasticity analysis, referring to a flexural stiffness in the **uncracked state E_cI_1** and in the **fully cracked state E_cI_2**, respectively.

Verification of crack widths

Neglecting the tension stiffening effect (ζ = 1) and assuming $\varepsilon_0 \approx 0$

$$w_k = 2.1\rho_{c,eff} \cdot \frac{M_k}{E_s d \rho_{eq}} \cdot \frac{1}{(u_s + 0.694 u_f)}$$

Where the ratio of the effective area in tension is:

$$\rho_{c,eff} = \frac{A_{c,eff}}{bd}$$

ρ_{eq} is the equivalent reinforcement ratio and u_s and u_f is the bond perimeter of the steel and FRP reinforcement.

Summary of design procedure:

- Before strengthening: check ULS and SLS (just to compare with the strengthened member!).
- From the service moment M_0 prior to strengthening determine ε_0 at the extreme tension fiber.
- Assume full composite action and from the design moment after strengthening determine the required FRP cross section to fulfill the ULS. Verify the ductility requirements.
- Calculate the deflections in the SLS. If allowable deflection is exceeded, determine the required FRP cross section.

- Calculate the stresses in the concrete, steel and FRP and verify the allowable stresses.

- Verify that the provided FRP bond width is sufficient to control crack widths in the SLS. Increase the FRP width, if necessary, or, given a maximum width, increase the amount (thickness) of FRP.

- Verify the resisting shear force at which bond failure due to shear cracks occurs (ULS).

- Verify that bond failure at the anchorage does not occur. Otherwise mechanical anchorage should be provided.

- Verify that FRP end shear failure is avoided. Provide shear strengthening at the ends if required.

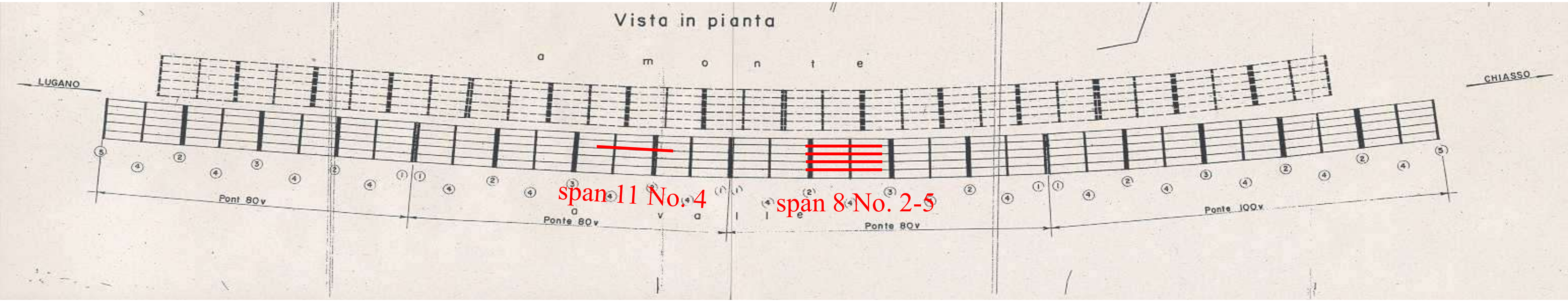
- Verify the accidental situation.

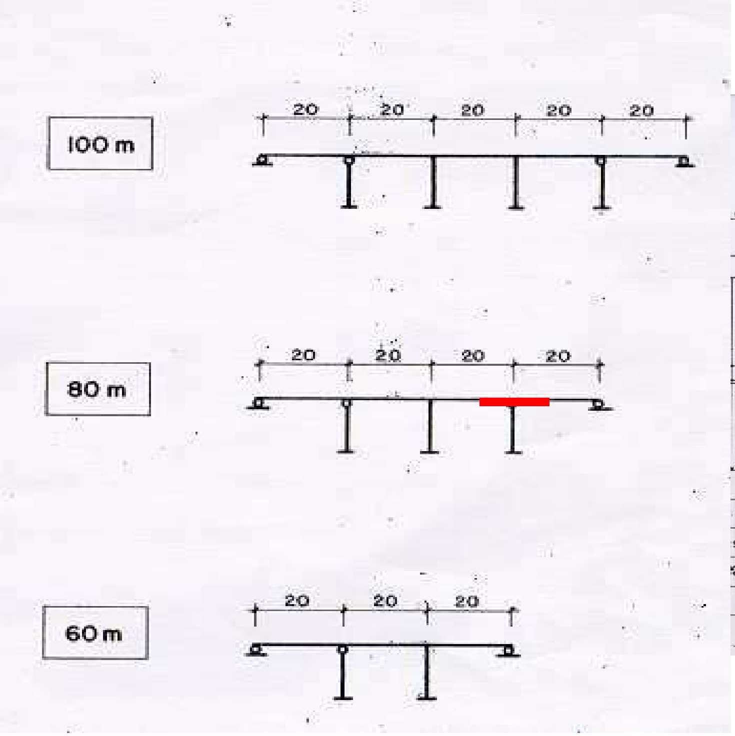
- Verify the shear design resistance of the strengthened member. If needed shear strengthening should be provided.

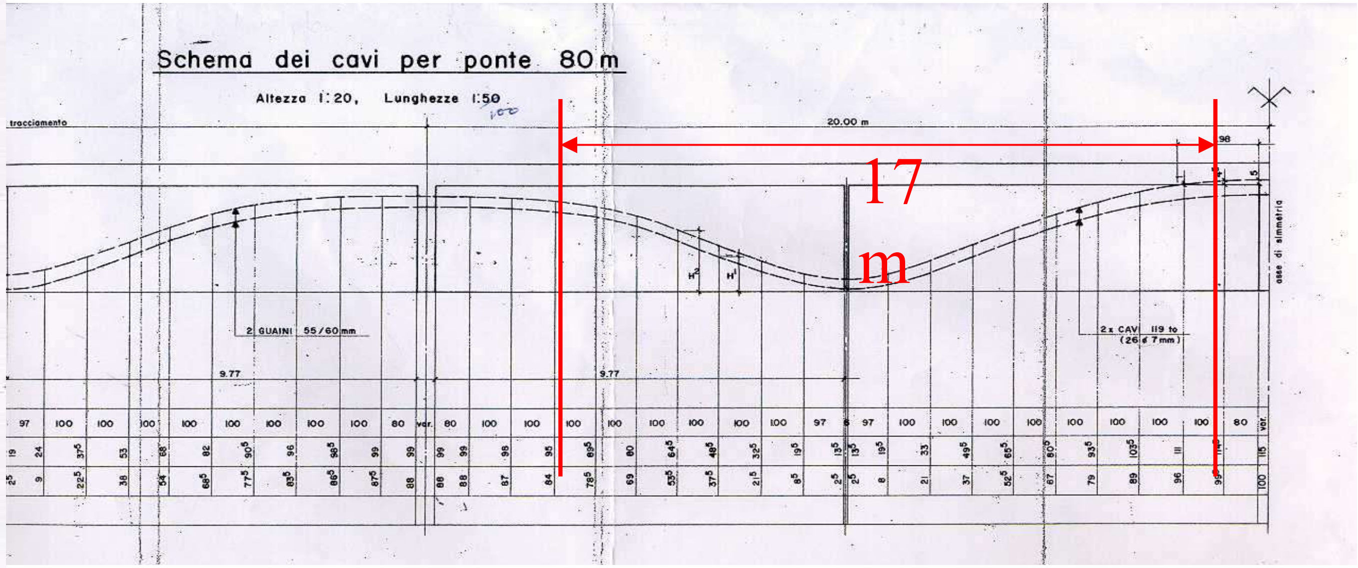
Strengthening of a Large Scale Pre-Stressed Bridge Girder Using Carbon Fibre Reinforced Polymers:
Comparision between Non Prestressed and Prestressed CFRP Plates


P7
CAT

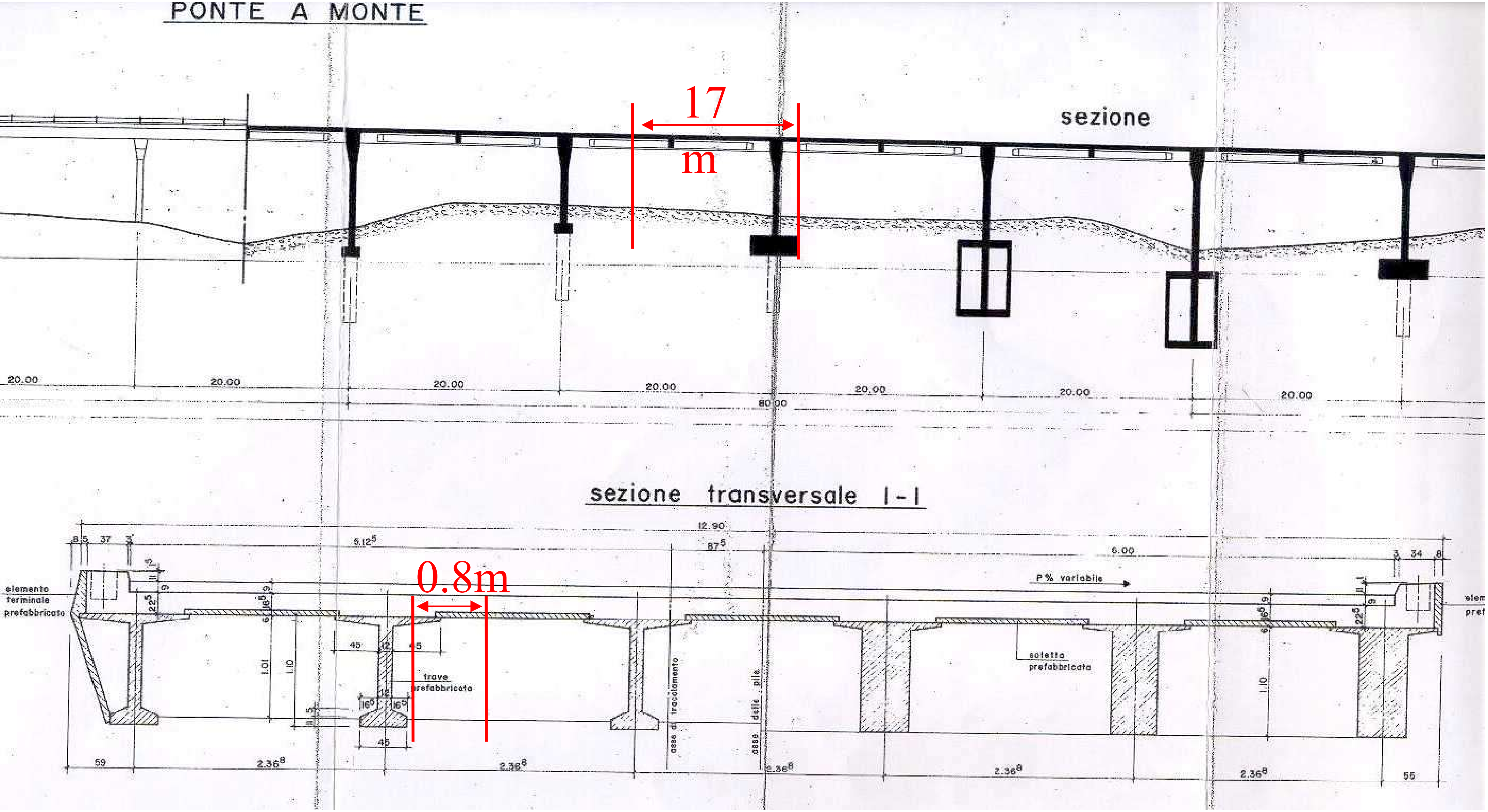
Bridge „Viadotto delle Cantine a Capolago“







Bridge „Viadotto delle Cantine a Capolago“




Trasporti - Speciali
sabesa
sabesa
SCANIA
124c
420

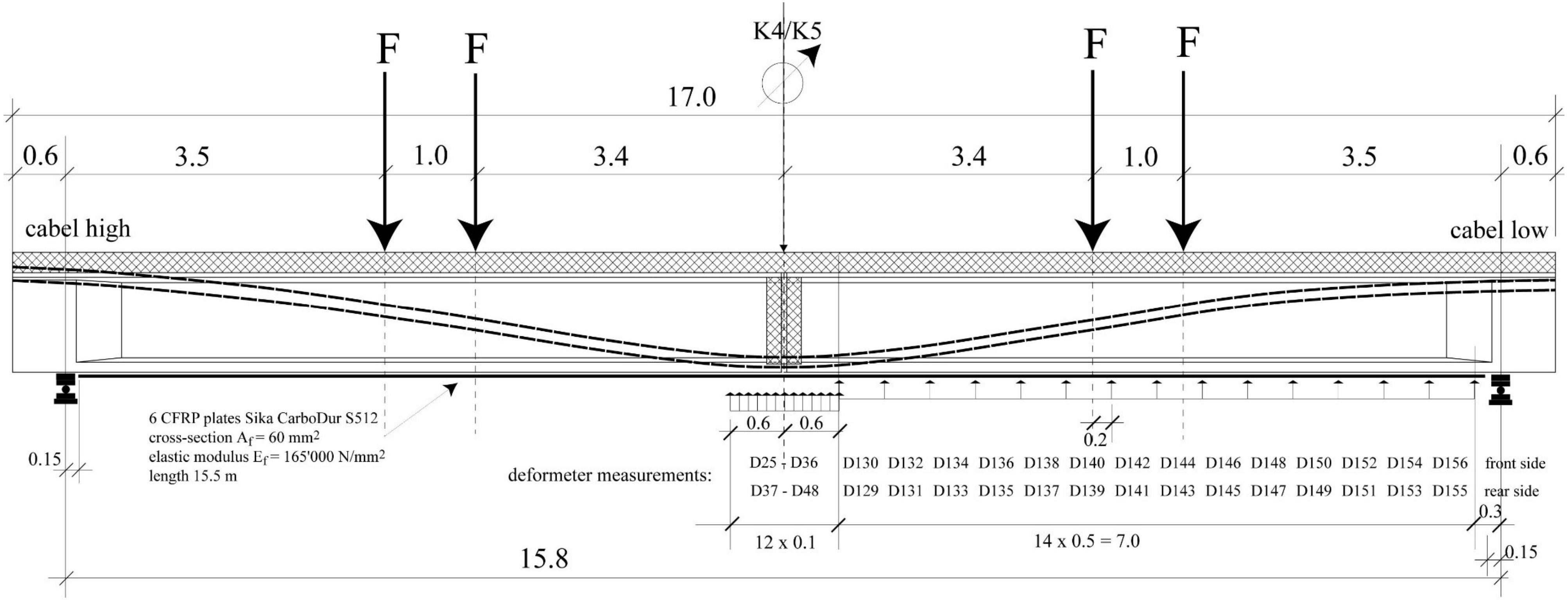
Overview

- Reference beam

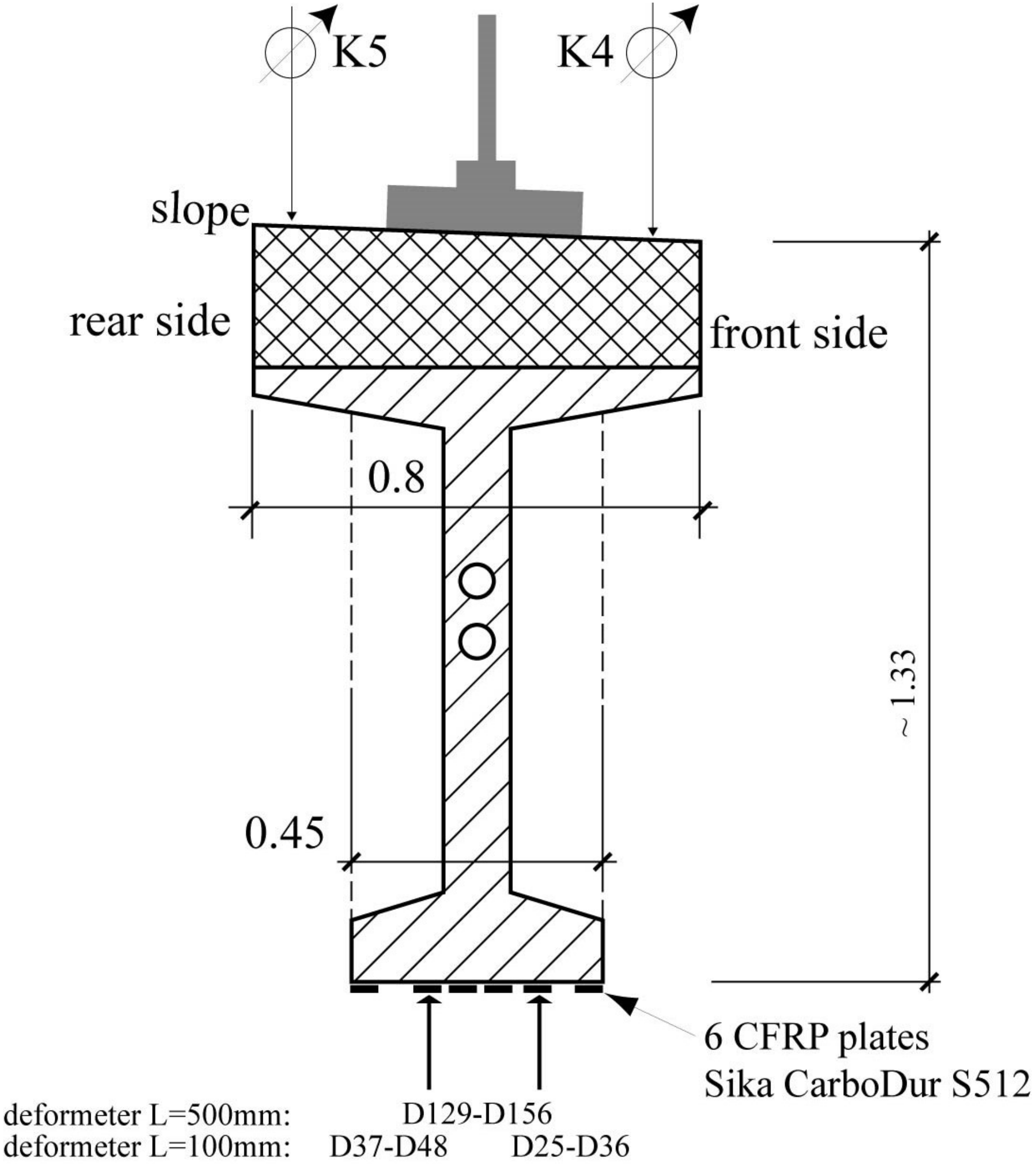
- Beam strengthened with *non prestressed* CFRP plates
  - 6 Sika CarboDur 512 plates, each 15.5 m long

- Beam strengthened with *prestressed* CFRP plates
  - the same type and number of plates
  - each plate prestressed approx. 1000 MPa (60 kN)
  - anchorage: Empa gradient method

Strengthened with non prestressed CFRP plates

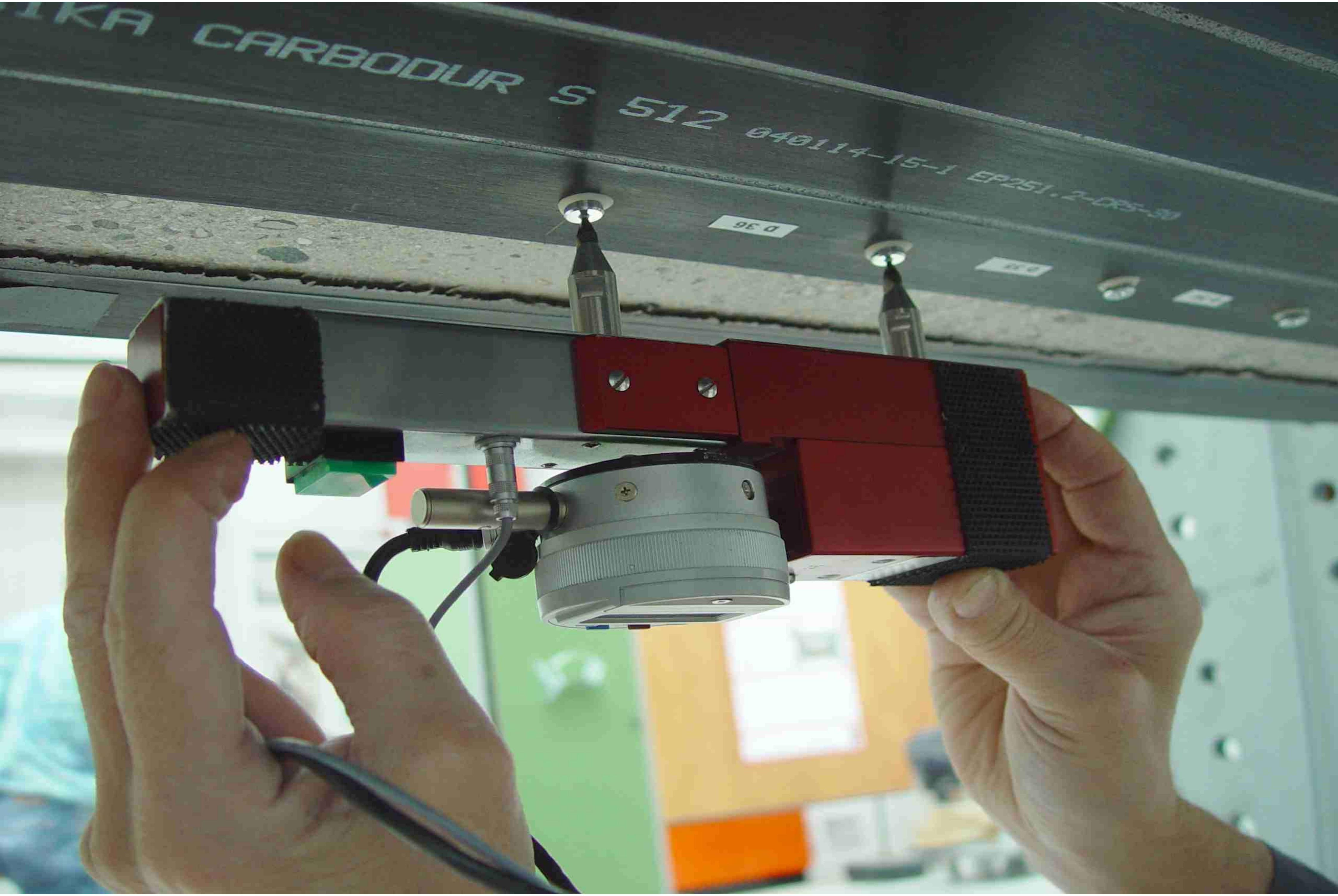


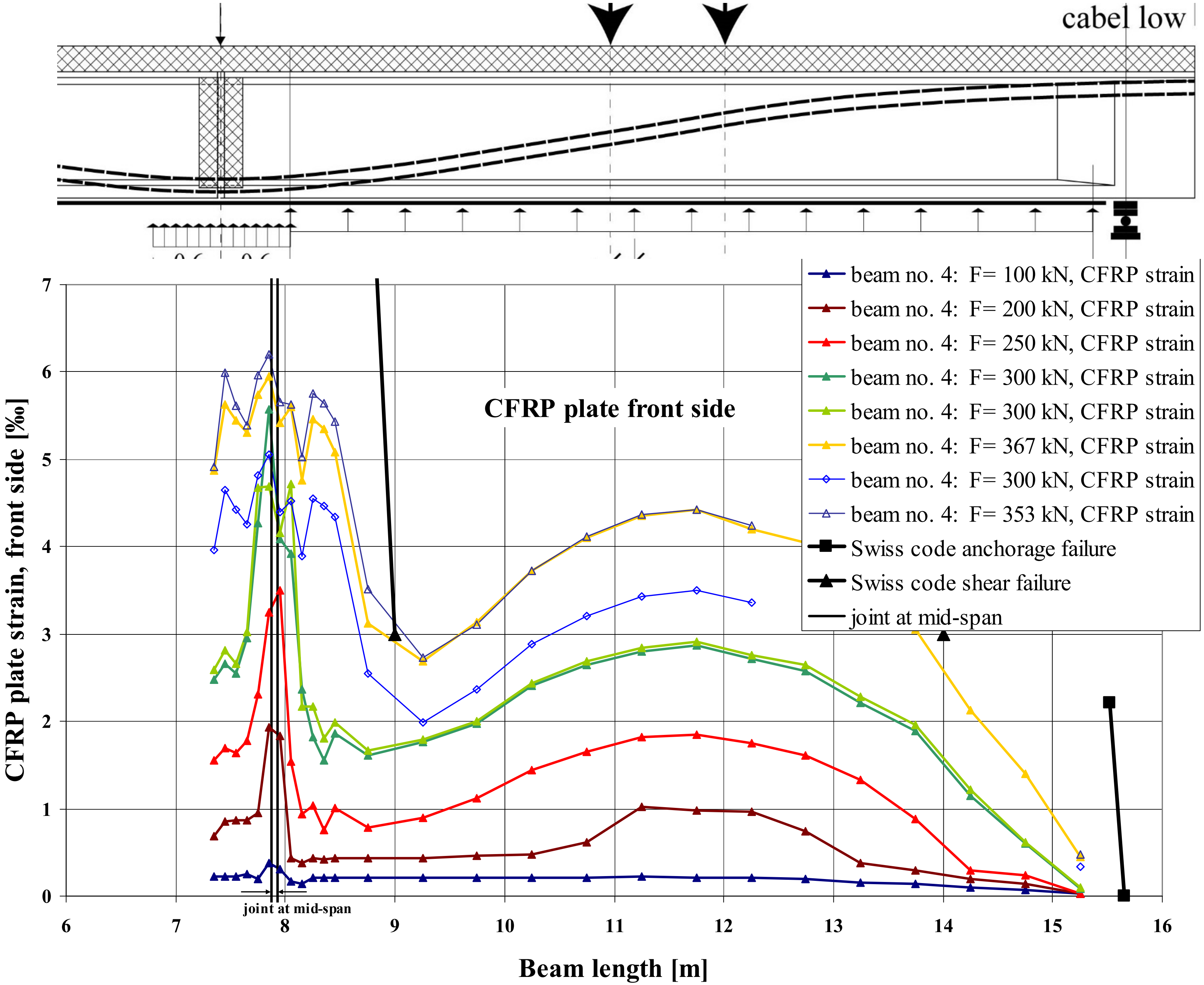
6 CFRP plates Sika CarboDur S512
cross-section $A_f = 60\ \text{mm}^2$
elastic modulus $E_f = 165'000\ \text{N/mm}^2$
length 15.5 m


K5
K4
slope
rear side
front side
0.8
~1.33
0.45
6 CFRP plates
Sika CarboDur S512
deformeter L=500mm:
D129-D156
deformeter L=100mm:
D37-D48
D25-D36

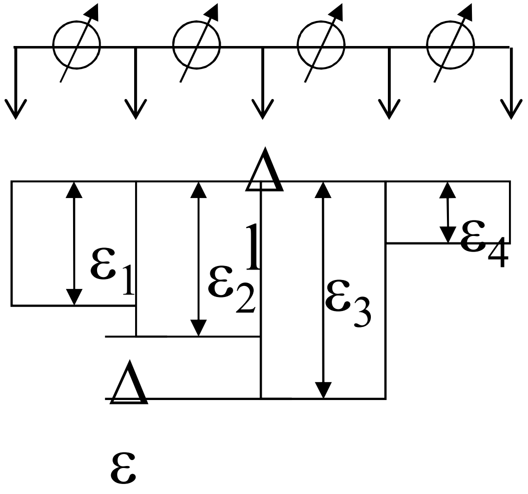

EMPA
4
EMPA

Deformeter Measurements


IKA CARBODUR S 512 040114-15-1 EP251.2-DRS-30
D 36

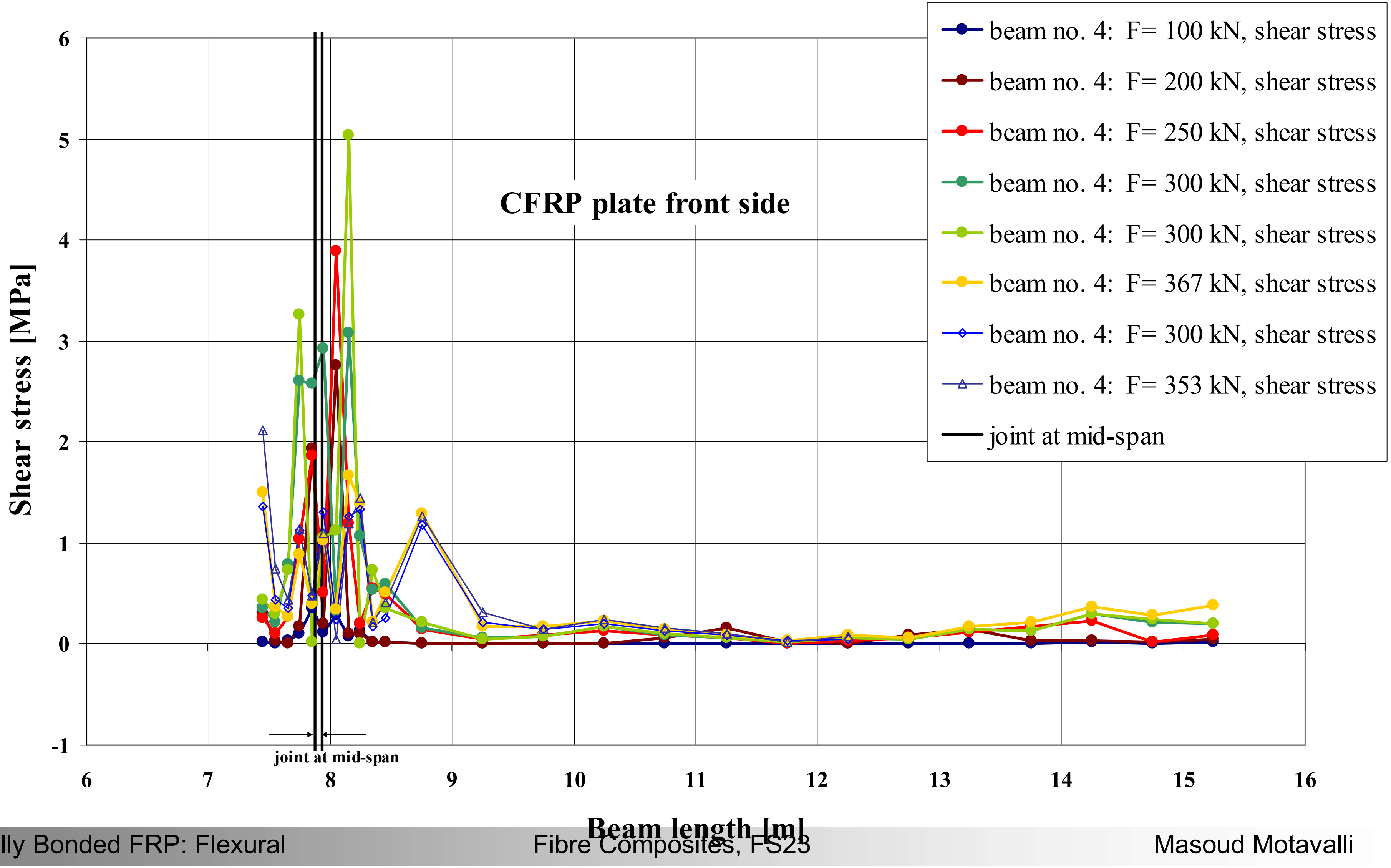

cabel low
CFRP plate front side
beam no. 4: F= 100 kN, CFRP strain
beam no. 4: F= 200 kN, CFRP strain
beam no. 4: F= 250 kN, CFRP strain
beam no. 4: F= 300 kN, CFRP strain
beam no. 4: F= 300 kN, CFRP strain
beam no. 4: F= 367 kN, CFRP strain
beam no. 4: F= 300 kN, CFRP strain
beam no. 4: F= 353 kN, CFRP strain
Swiss code anchorage failure
Swiss code shear failure
joint at mid-span
joint at mid-span
CFRP plate strain, front side [‰]
0
1
2
3
4
5
6
7
Beam length [m]
6
7
8
9
10
11
12
13
14
15
16

Shear stress from Deformeter-measurement



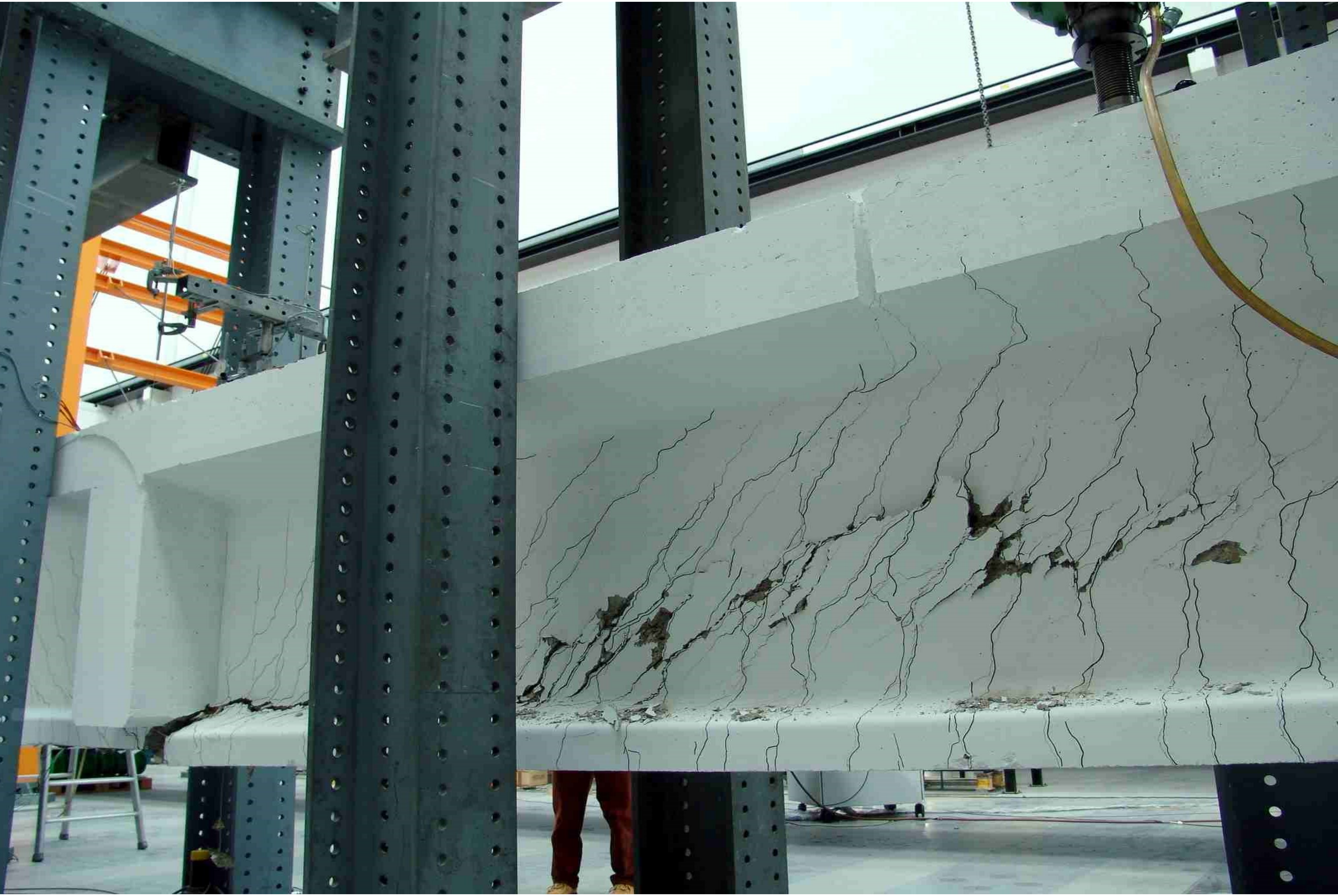
$$\tau_1 = \frac{\Delta F}{b_1 \cdot \Delta l} = \frac{\Delta \varepsilon \cdot E_1 \cdot A_1}{b_1 \cdot \Delta l}$$

Behavior during loading




P1


Reference
beam
No. 3 (2/8)
EMPA
F= BRUCH
4.PHASE



Summary of the three SIA 166 verifications

See next lecture given by Dr. Christoph Czaderski

1. End strip debonding failure at the last crack

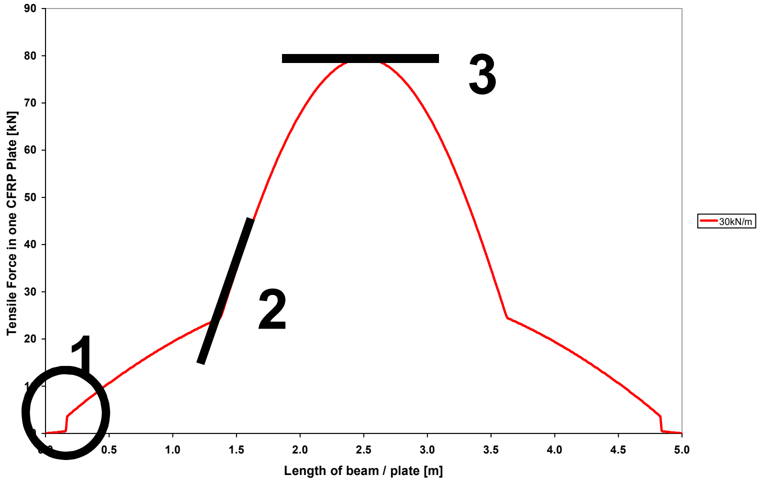
$$\boxed{F_{fcr} \le F_{b,R}}$$

2. Debonding at strong strain increase in strip

$$\boxed{\left(\frac{\Delta F_f}{\Delta x}\right) \le \left(\frac{\Delta F_f}{\Delta x}\right)_R}$$

3. Debonding at flexural cracks

$$\boxed{\varepsilon_f \le \varepsilon_{f,lim,d} = 8‰}$$

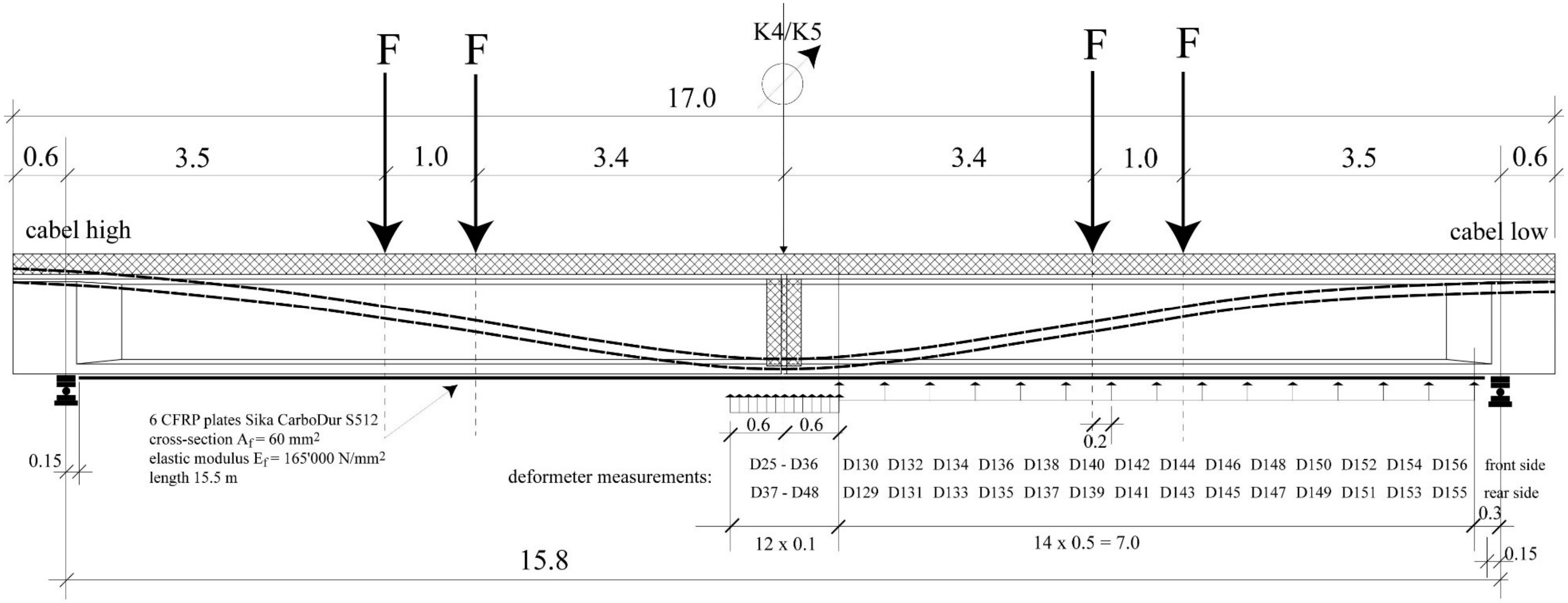


SIA166 “Externally bonded reinforcement”

	measurement in the experiment	Swiss code SIA166
Shear failure	**5.0 MPa** (maximum value)	**5.0 MPa**

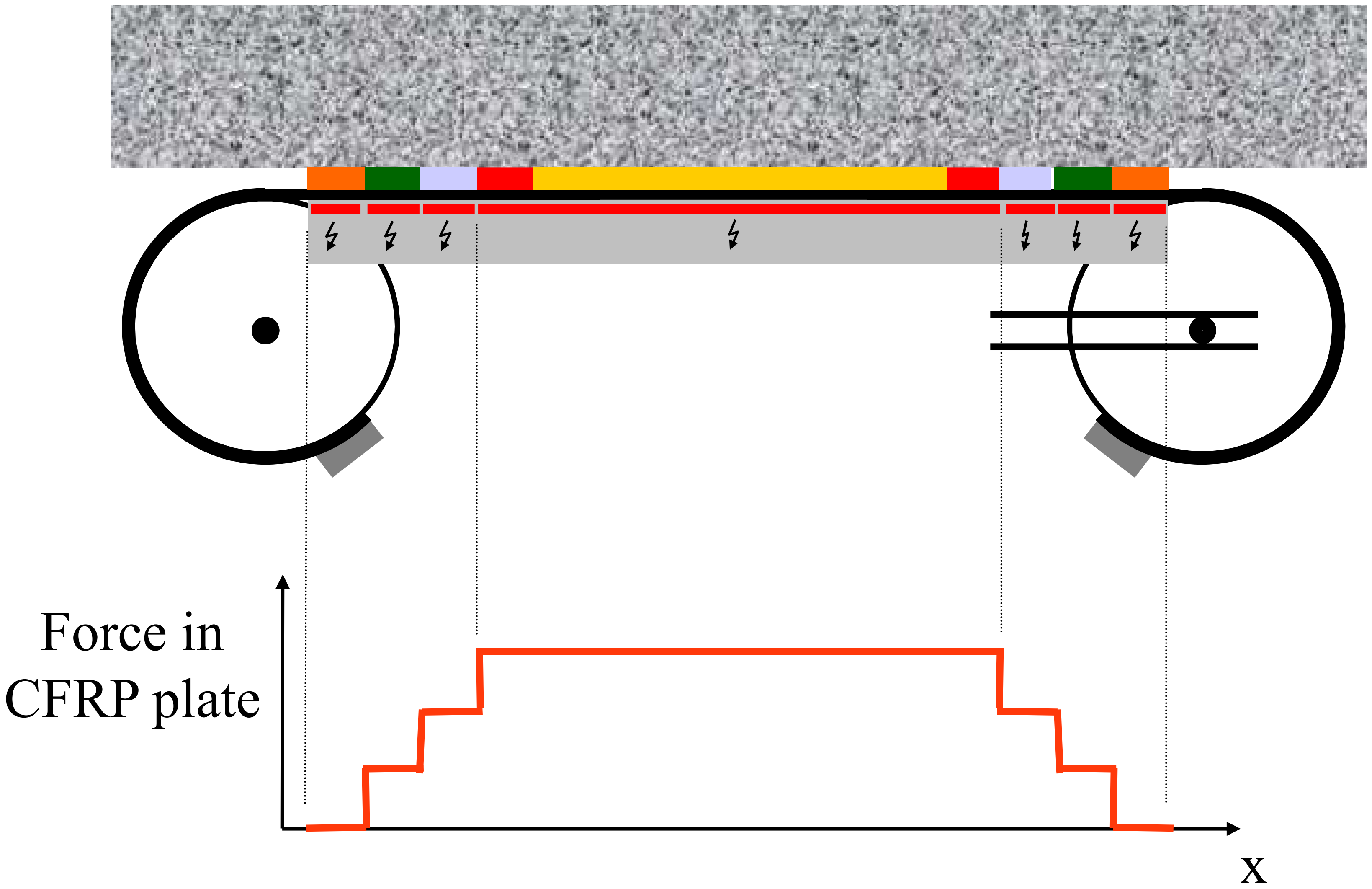
$$\tau_{1,\lim} = 2.5 \cdot \tau_c = 2.5 \cdot 2.0 = 5.0\mathrm{MPa}$$

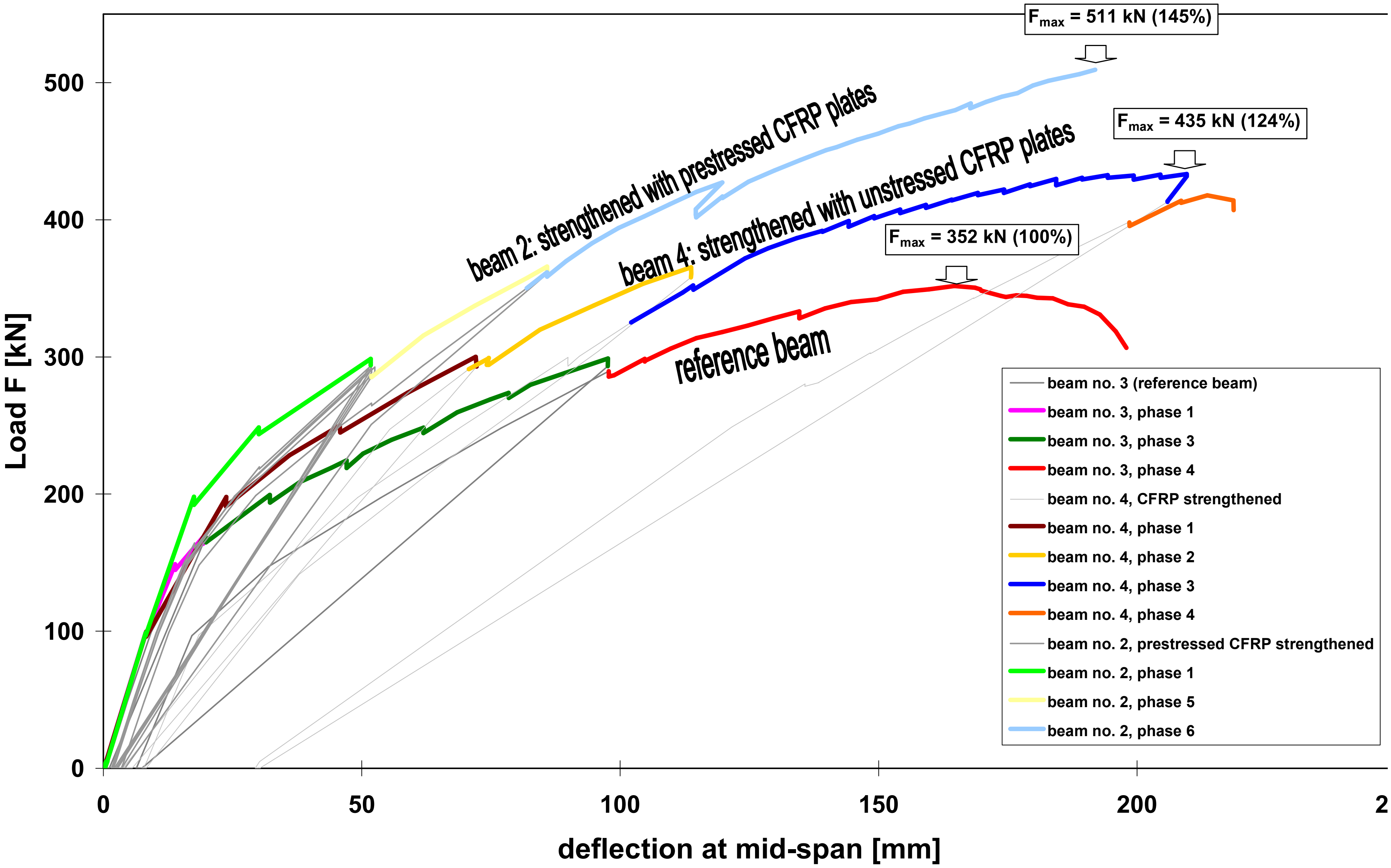
Strengthened with prestressed CFRP plates



CFRP plates prestressed approx. 1000 MPa (60 kN)

Prestressing using Gradient-method




Fmax = 511 kN (145%)
Fmax = 435 kN (124%)
Fmax = 352 kN (100%)
beam 2: strengthened with prestressed CFRP plates
beam 4: strengthened with unstressed CFRP plates
reference beam
500
400
300
200
100
0
Load F [kN]
0
50
100
150
200
2
deflection at mid-span [mm]
beam no. 3 (reference beam)
beam no. 3, phase 1
beam no. 3, phase 3
beam no. 3, phase 4
beam no. 4, CFRP strengthened
beam no. 4, phase 1
beam no. 4, phase 2
beam no. 4, phase 3
beam no. 4, phase 4
beam no. 2, prestressed CFRP strengthened
beam no. 2, phase 1
beam no. 2, phase 5
beam no. 2, phase 6

"List of Symbols"

< Flexural strengthening >

M_0 : service moment

M_{cr} : cracking moment

A_{s1} : steel cross-section ~~at~~ (tensile reinforcement)

A_{s2} : steel cross-section (compression reinforcement)

x_0 : Position of the neutral axis prior to strengthening

b : cross-section width

d : " debth ; $h = d + d_1$

α_s : $\frac{E_s}{E_c}$ $\begin{cases} E_s : \text{steel E-modulus} \\ E_c : \text{Concrete "} \end{cases}$

I_{02} : moment of interhia of the transformed cracked section prior to strengthening

l_b : bond length

s_f : slip (= s_e

ψ : 0.8 if "steel yielding followed by concrete crushing"

A_f : FRP cross-section, or (A_{FRP})

x (also c) : position of the neutral axis after strengthening

$\delta_G x$: position of the concrete compression force at ULS, ($\delta_G = 0.4$) if "steel yielding followed by concrete crushing"

h_f : Height of the flange

b_f : width of " "

b_{FRP} : " " the FRP

$\rho_{eq} = \dfrac{A_s + A_f \frac{E_f}{E_s}}{bd}$: equivalent reinforcement ratio

$V_R = \tau_R \cdot b \cdot d$: V_R : shear resistance ; τ_R : shear strength

ρ_s : steel reinforcement ratio

ρ_f : FRP reinforcement ratio

"List of Symbols" -2-

< Flexural strengthening >

$N_{fa,max}$: maximum FRP force, which can be anchored

$l_{b,max}$: maximum anchorage length

f_{ctm} : mean concrete tensile strength

t_f : FRP thickness

x_e : position of neutral axis at SLS

$\sigma_f \leq \eta \cdot f_{fk}$: FRP stress under service load, where

$$\eta = \begin{cases} 0.8 : \text{CFRP} \\ 0.5 : \text{AFRP} \\ 0.3 : \text{GFRP} \end{cases}$$

a; a_1; a_2 : mean deflection; deflection in uncracked state; deflection in fully cracked state

ξ_b : the distribution coefficient to calculate the deflection

$E_c I$: flexural stiffness in the uncracked state

$E_c I_2$: " " " fully cracked state

w_k : crack width at SLS

u_s : bond perimeter of steel

u_f : " " of FRP

τ_{fl} : maximum shear stress at FRP end at SLS

G_a, t_a : shear modulus, thickness of adhesive