Empa
Materials Science and Technology

Flexural strengthening of reinforced concrete
Swiss Code 166 and other codes/guidelines

ETH Lecture 101-0167-01L
Fibre Composite Materials in Structural Engineering

Yunus Harmanci
Christoph Czaderski

25. October 2023

Program overview of the lectures and laboratory work

- Wednesday 25.10.2023, 15:45-17:30 (ETH Hönggerberg, HIL E7), Lecturer Yunus Harmanci
  - Lecture on Flexural Strengthening
  - Preparations for laboratory competition (Beam) and second written intermediate exam

- Wednesday 29.11.2023, 15:45-17:30 (Empa Dübendorf) Responsible Ali Jafarabadi
  - Meeting point at ETH Hönggerberg 15:30!!
  - Application of Externally Bonded FRP Reinforcement (Confinement) for laboratory competition
  - Video of the beam failure test
  - Empa structural laboratory tour (if time available)

- Wednesday 13.12.2023, 15:45 – ca.18:00 (Empa Dübendorf) Responsible Ali Jafarabadi
  - Meeting point at ETH Hönggerberg 15:30!!
  - Laboratory experiments and awarding of lab competition
  - Second written interim exam

Overview of the lecture

- Information about the second written intermediate exam
- Introduction
- Bond between CFRP strip and concrete
  - Lap-shear test
  - Slip, Bond shear stress and Bond shear stress-Slip-Relation
  - Simplified modeling
- Debonding failure modes according to SIA 166
  - End-anchorage
  - Debonding at shear cracks
  - Debonding at flexural cracks
- Summary, debonding failure modes treated in SIA 166
- Example
- Video of CFRP strip application on a beam, competition on prediction of failure load
- Several additional topics according to SIA 166

Content of the second exam on 13.12.2023, 15:45 - ca. 18:00!!! at Empa Dübendorf

Topics:

- Design of FRP profiles and all FRP structures
  Lecture from 18.10.2023, Lecturer Prof. Dr. M. Shahverdi
- Flexural strengthening of RC according to SIA166
  Lecture from 01.11.2023, Lecturer Dr. Y. Harmanci
- Column confinement of RC
  Lecture from 8.11.2023, Lecturer Prof. Dr. M. Motavalli
- Externally bonded FRP reinforcement for metallic structures
  Lecture from 15.11.2023, Lecturer Dr. H. Heydarinouri

- Conceptual questions on the topics which were presented in the mentioned lectures. Furthermore, some calculations have to be performed.

- Time: 60 Minutes
- No laptops, tablets, smart phones etc.
- Only a calculator
- One A4 – Summary (both sides or two pages one side)

Introduction


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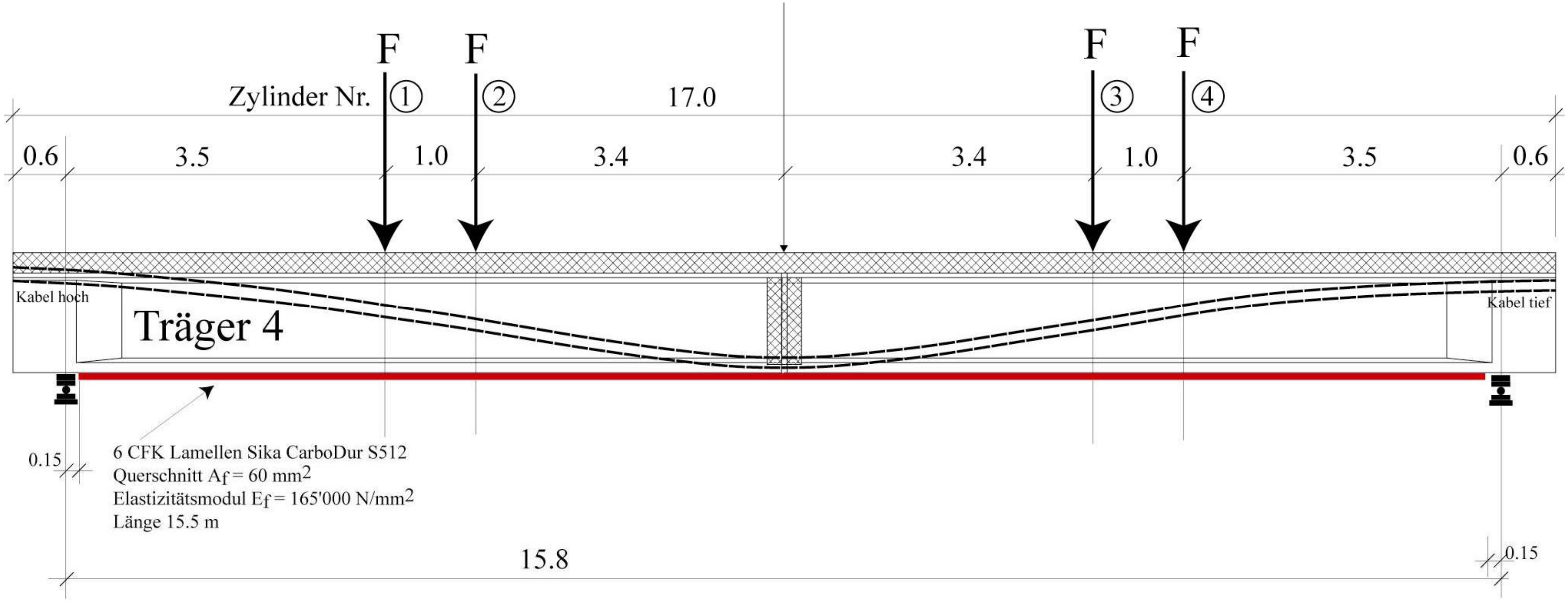
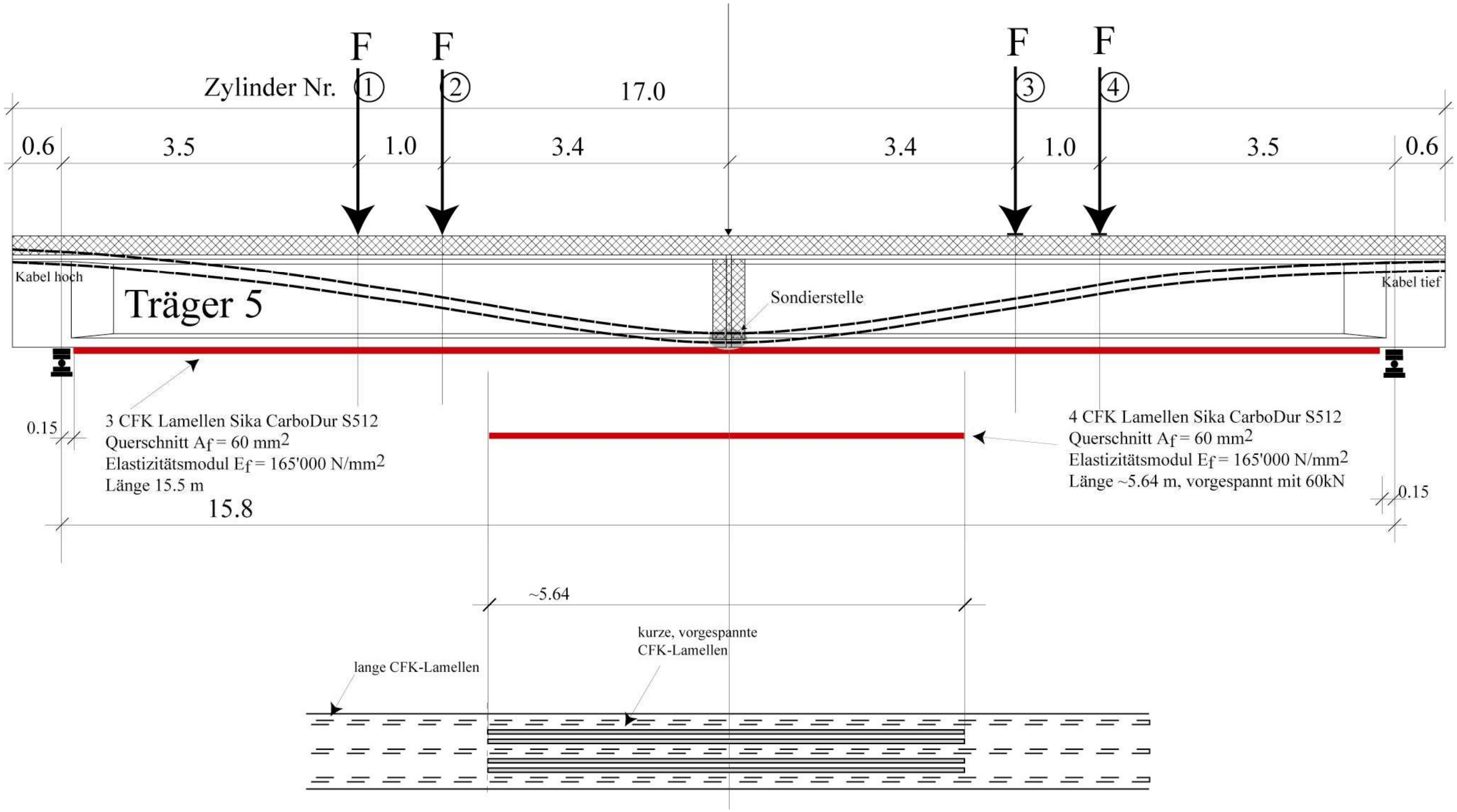
4
F = 421 KN



6 CFK Lamellen Sika CarboDur S512
Querschnitt $A_f = 60$ mm^2
Elastizitätsmodul $E_f = 165'000$ N/mm^2
Länge 15.5 m


F = 421 KN

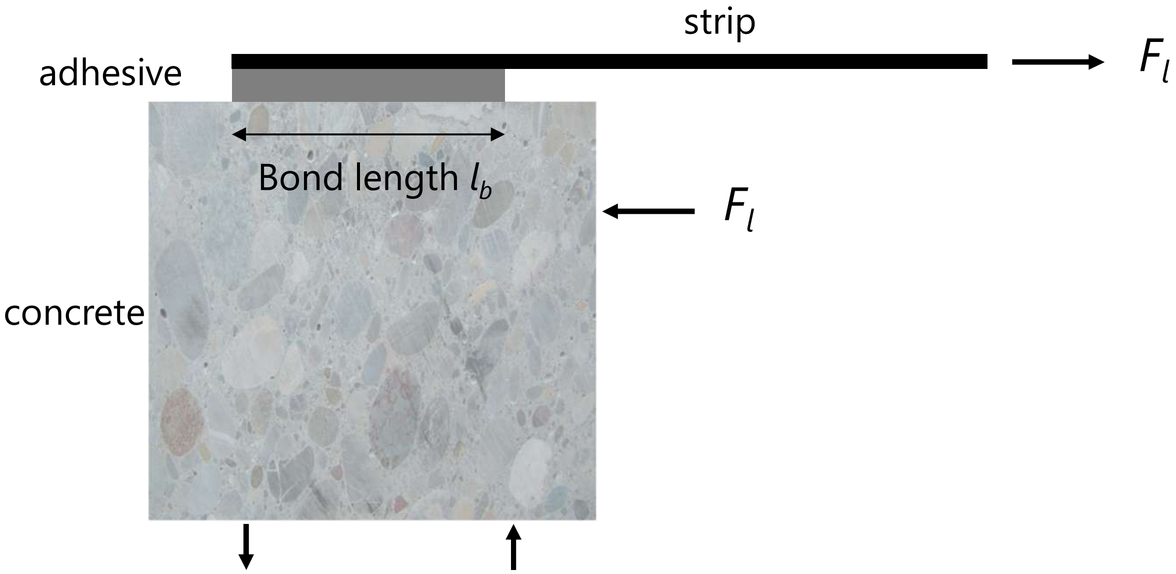

2 (4/11)
EMPA


F
F
F
F
Zylinder Nr. ①
②
③
④
17.0
0.6
3.5
1.0
3.4
3.4
1.0
3.5
0.6
Kabel hoch
Träger 5
Sondierstelle
Kabel tief
0.15
3 CFK Lamellen Sika CarboDur S512
Querschnitt $A_f = 60\ mm^2$
Elastizitätsmodul $E_f = 165'000\ N/mm^2$
Länge 15.5 m
15.8
4 CFK Lamellen Sika CarboDur S512
Querschnitt $A_f = 60\ mm^2$
Elastizitätsmodul $E_f = 165'000\ N/mm^2$
Länge ~5.64 m, vorgespannt mit 60kN
0.15
~5.64
kurze, vorgespannte CFK-Lamellen
lange CFK-Lamellen


EMPA

Bond between CFRP strip and concrete

Lap-shear test

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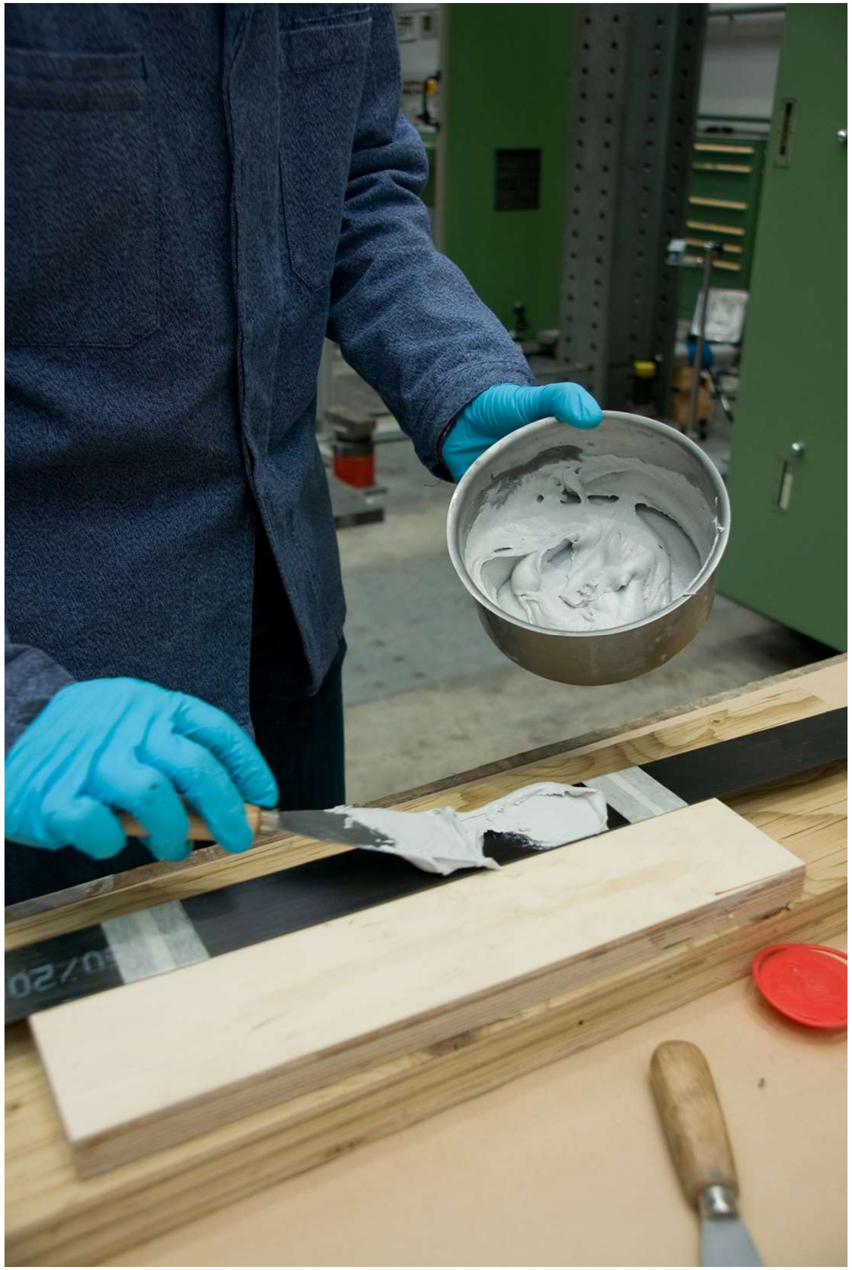


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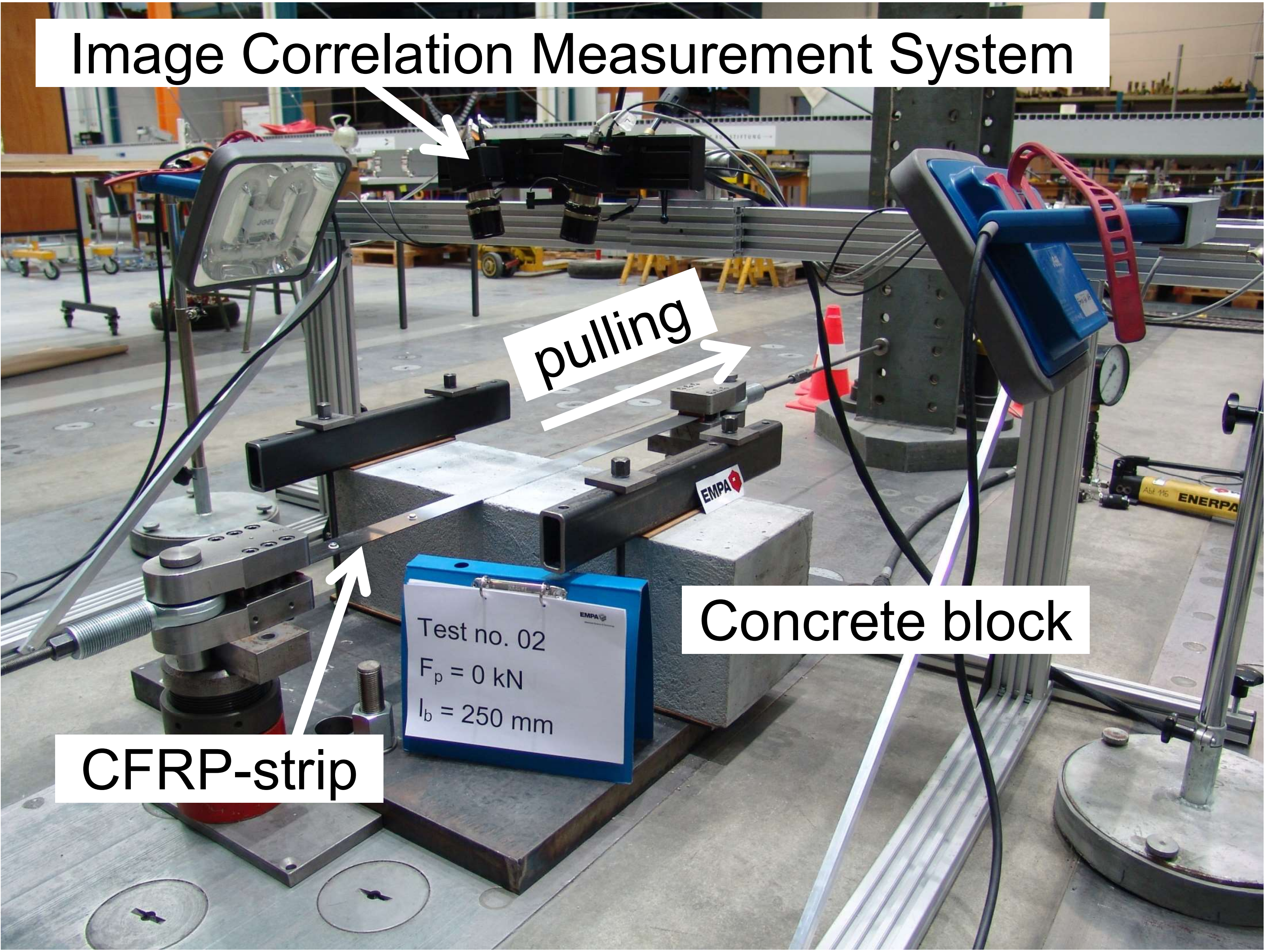
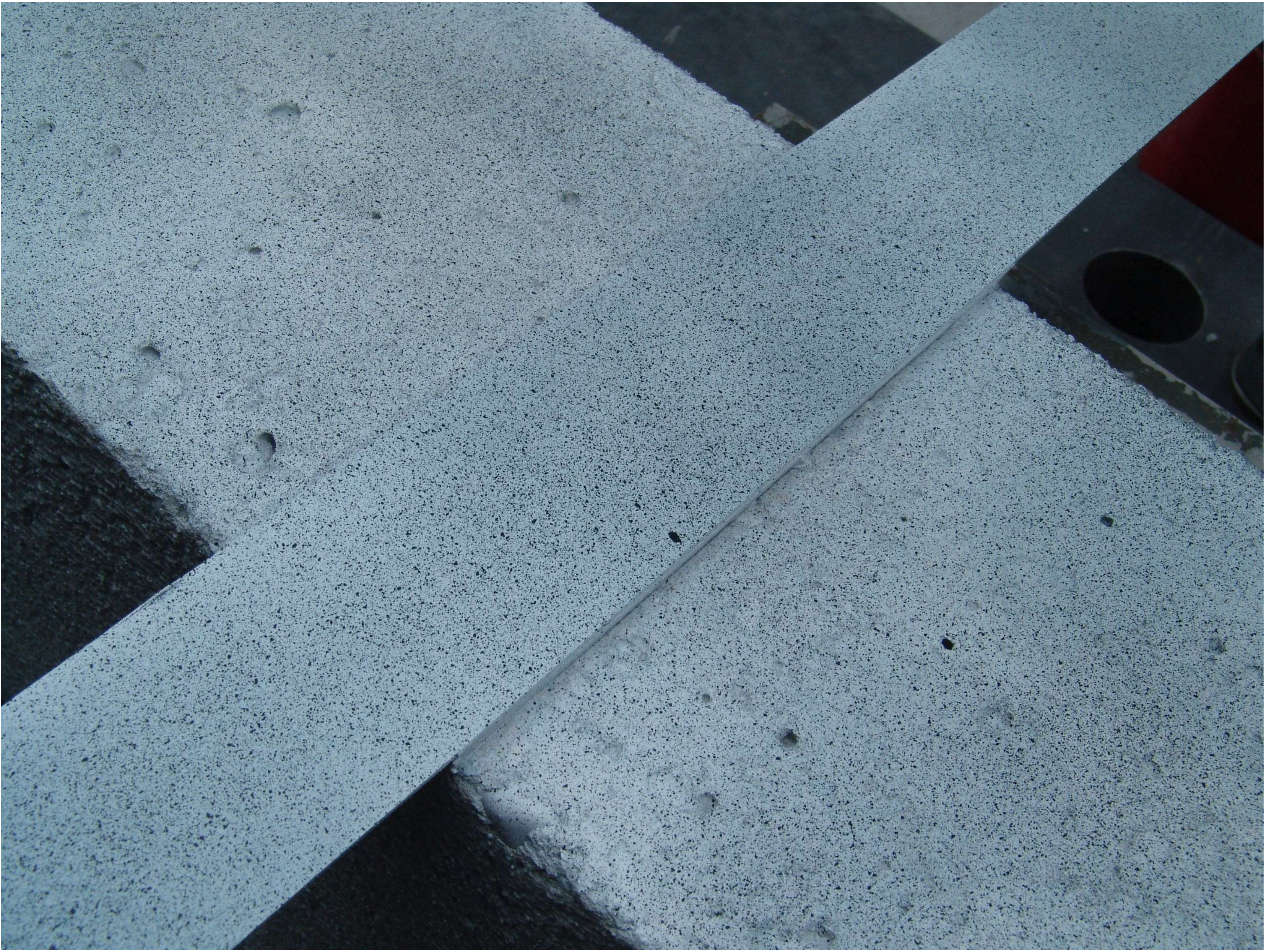


Image Correlation Measurement System

pulling

Concrete block

CFRP-strip



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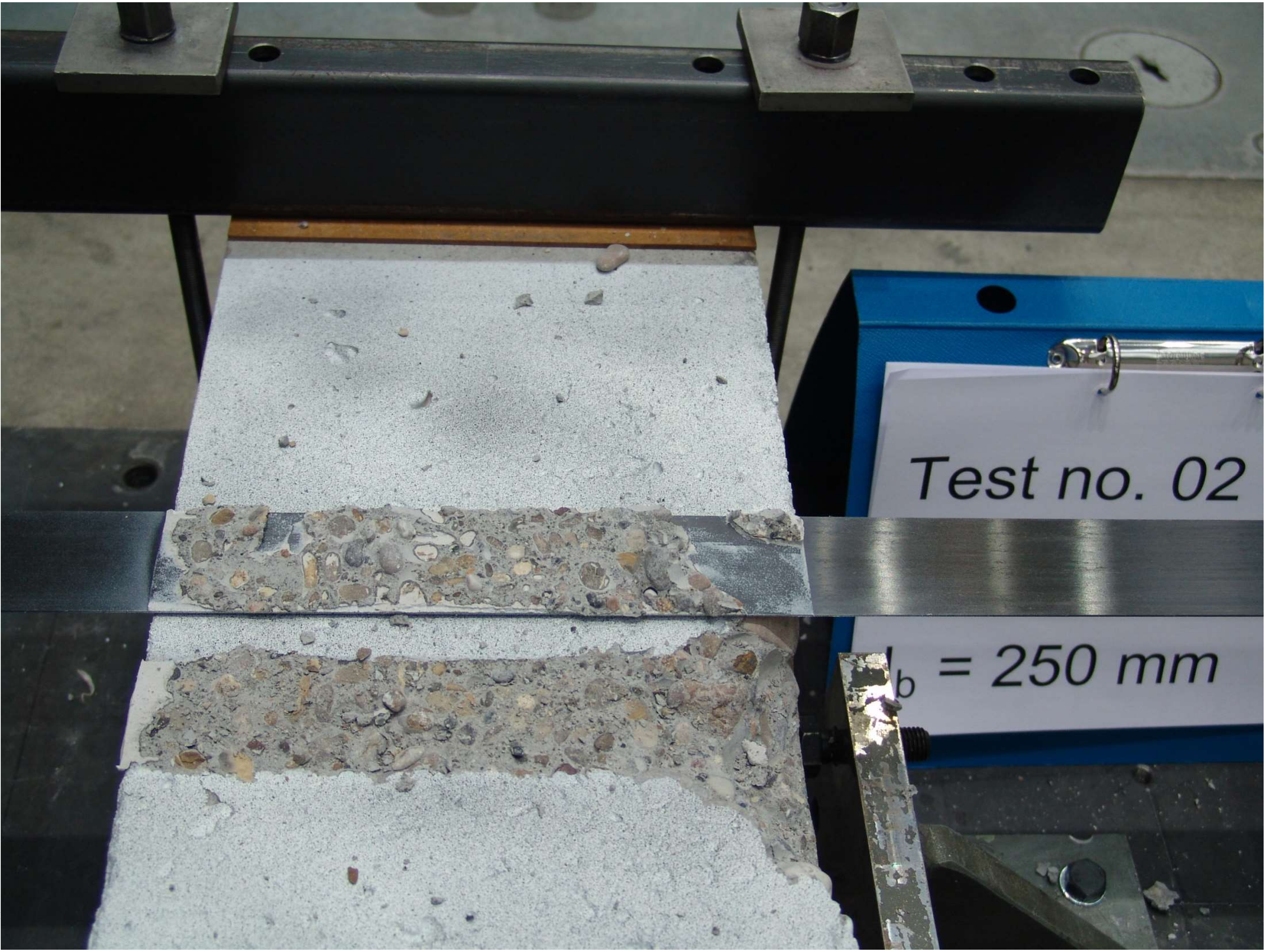

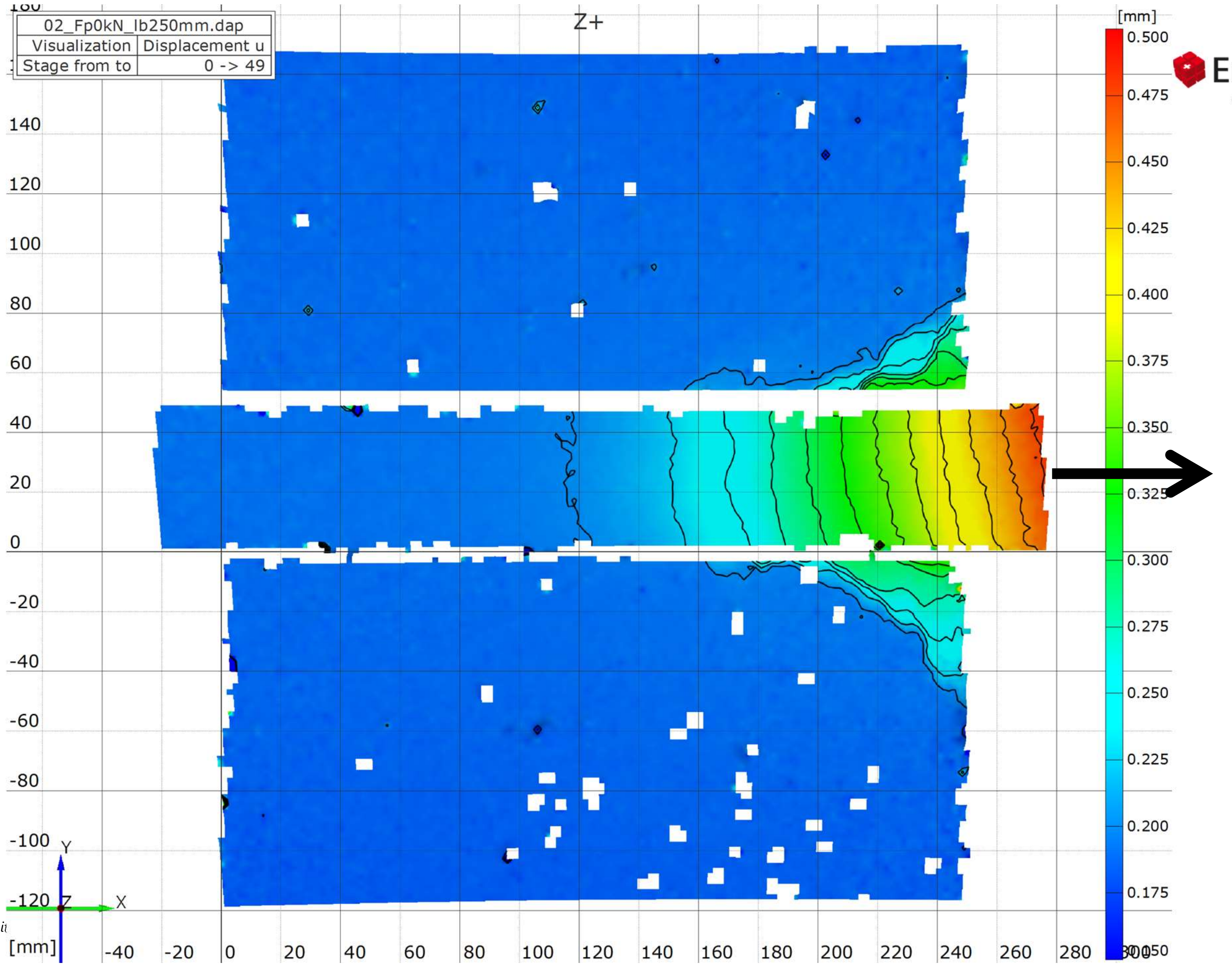
02_Fp0kN_lb250mm.dap
Visualization | Displacement u
Stage from to | 0 -> 49
Z+
[mm]
0.500
0.475
0.450
0.425
0.400
0.375
0.350
0.325
0.300
0.275
0.250
0.225
0.200
0.175
0.150
180
140
120
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
[mm]
-40
-20
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
Y
Z
X

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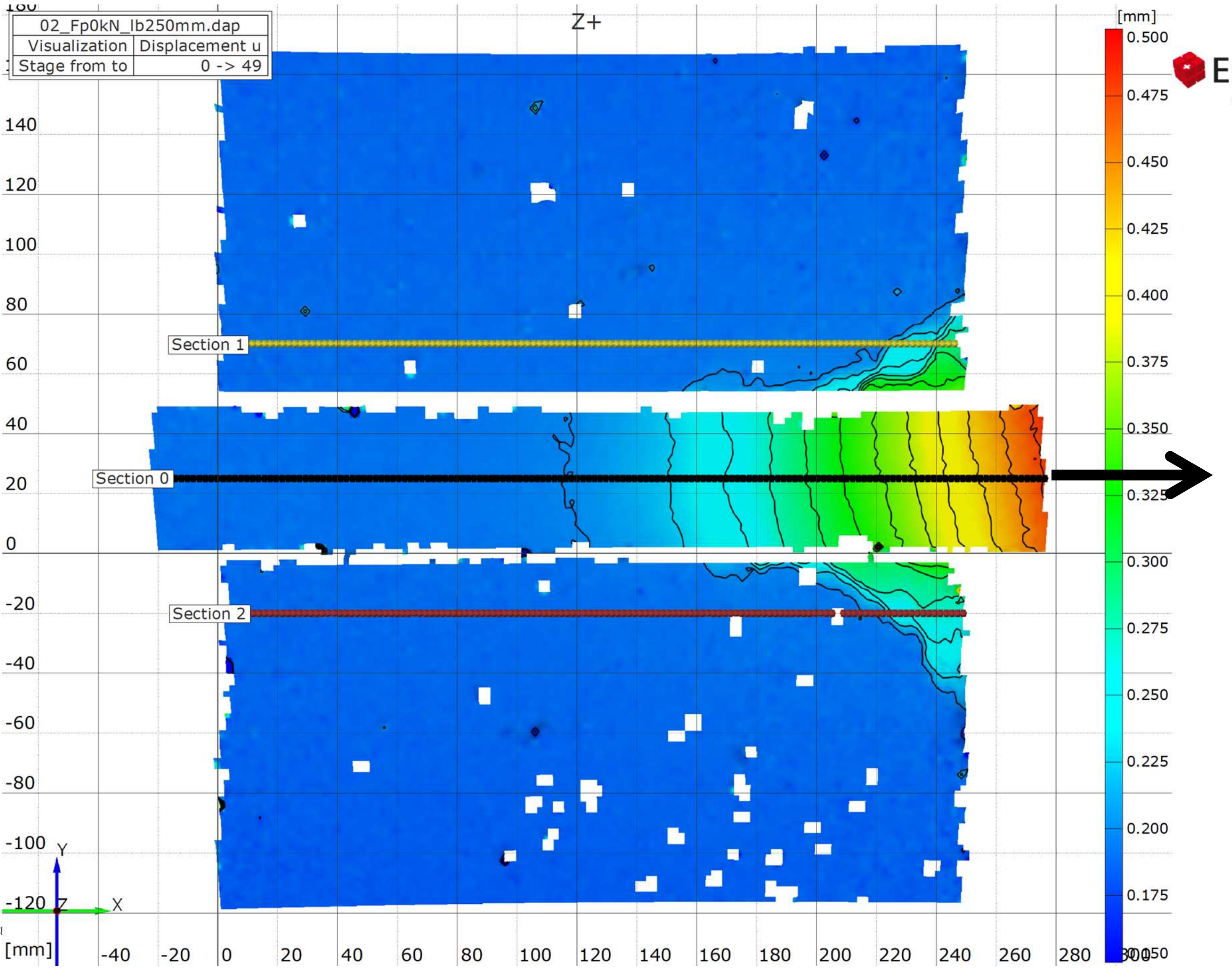
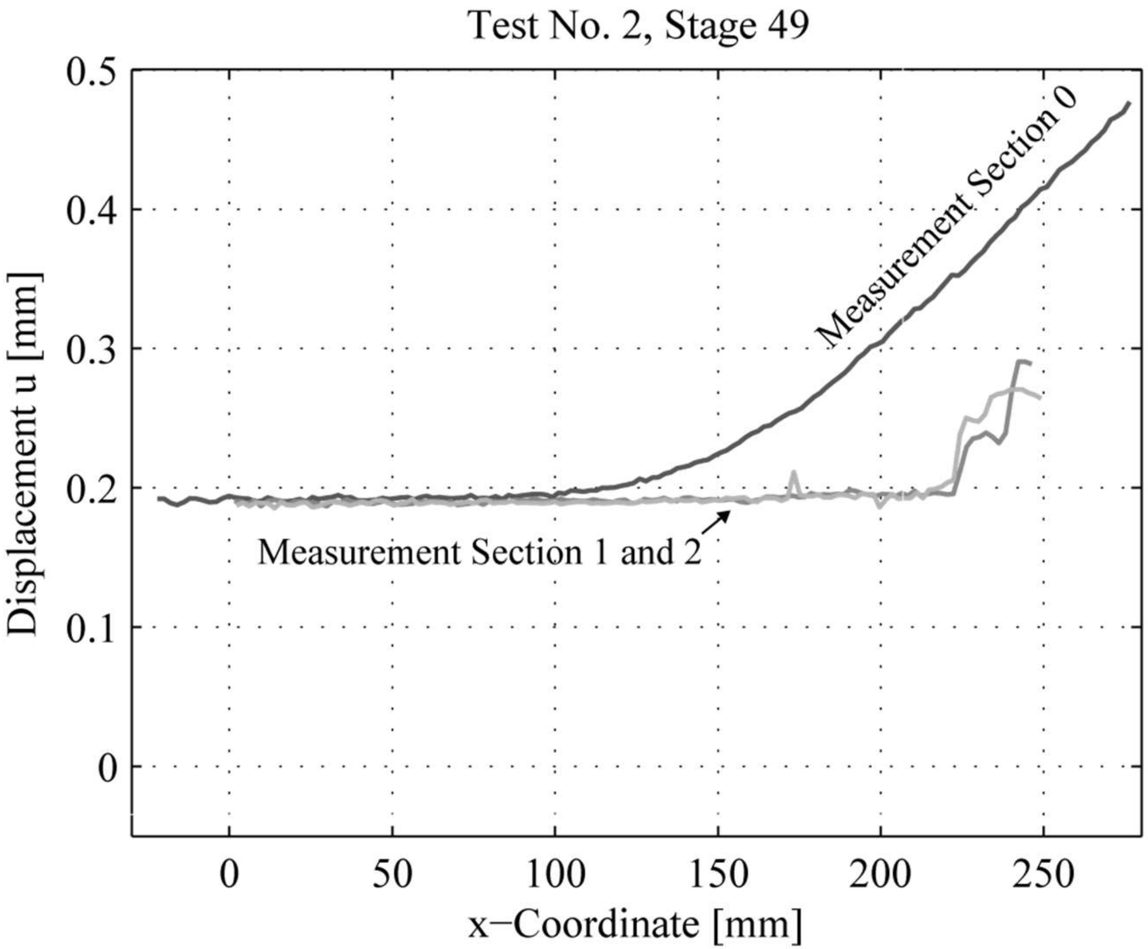
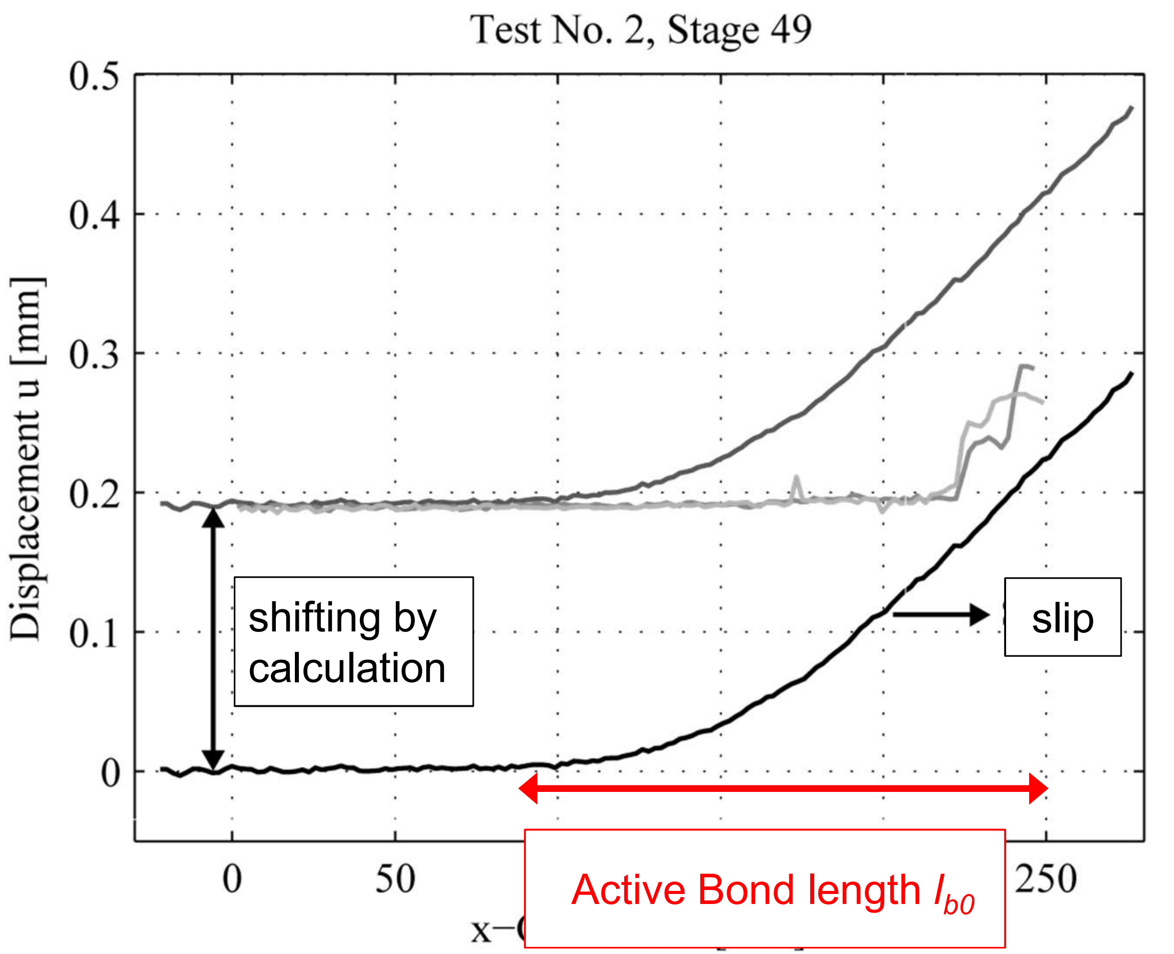



Test No. 2, Stage 49
Displacement u [mm]
0.5
0.4
0.3
0.2
0.1
0
0
50
100
150
200
250
x−Coordinate [mm]
Measurement Section 0
Measurement Section 1 and 2



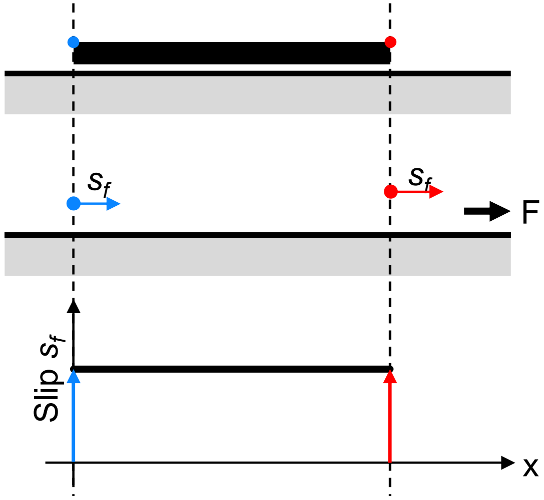
shifting by calculation

slip

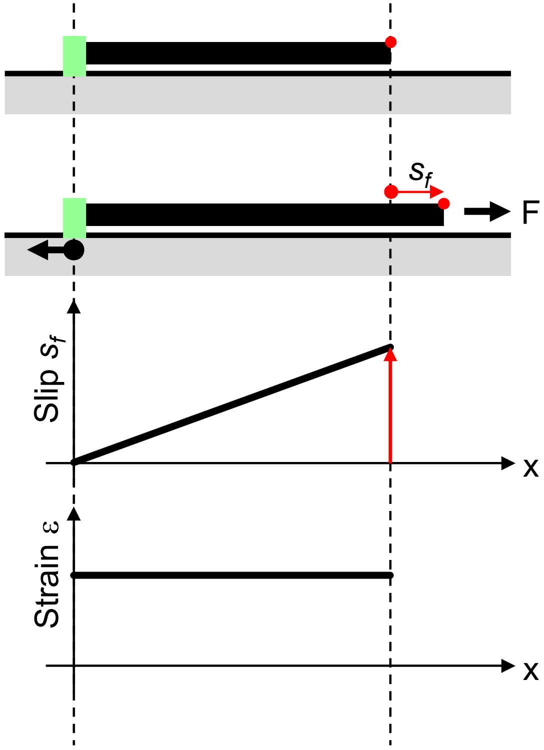
Active Bond length l_{b0}

Active bond length: the length which is actively involved in the force transfer from the strip to the concrete.

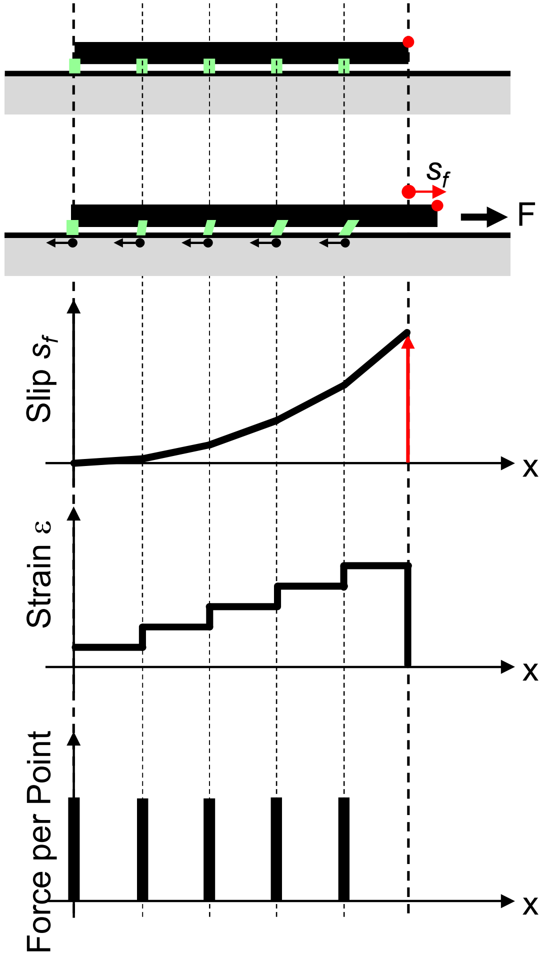
without connection

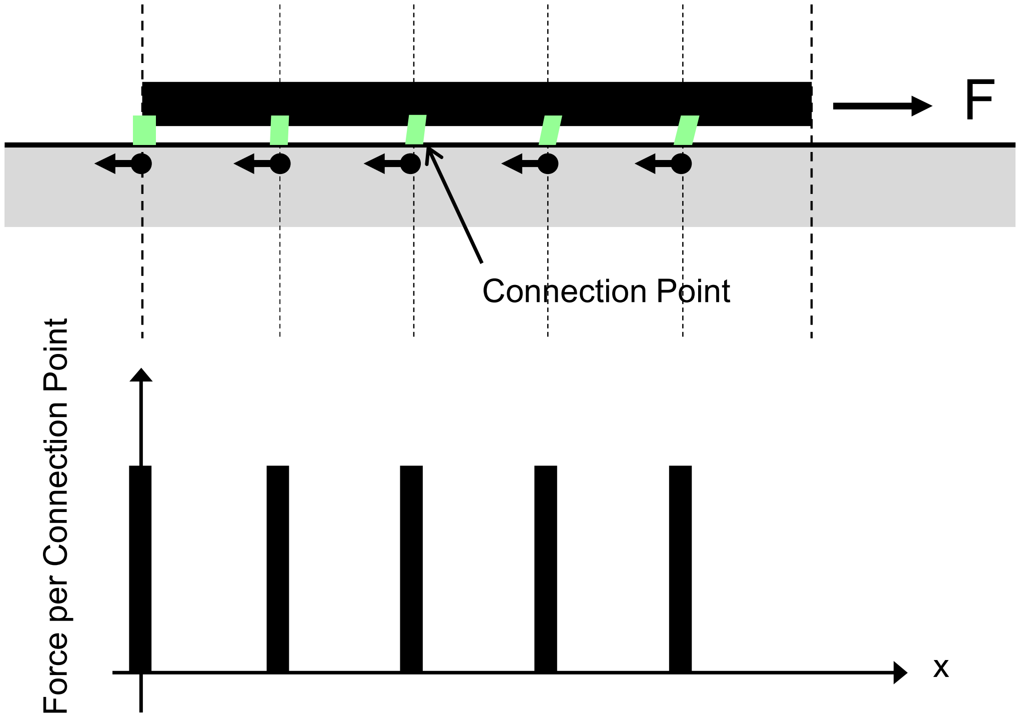


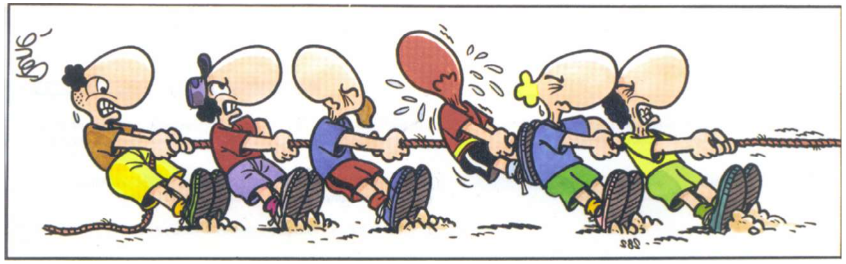
connection in one point



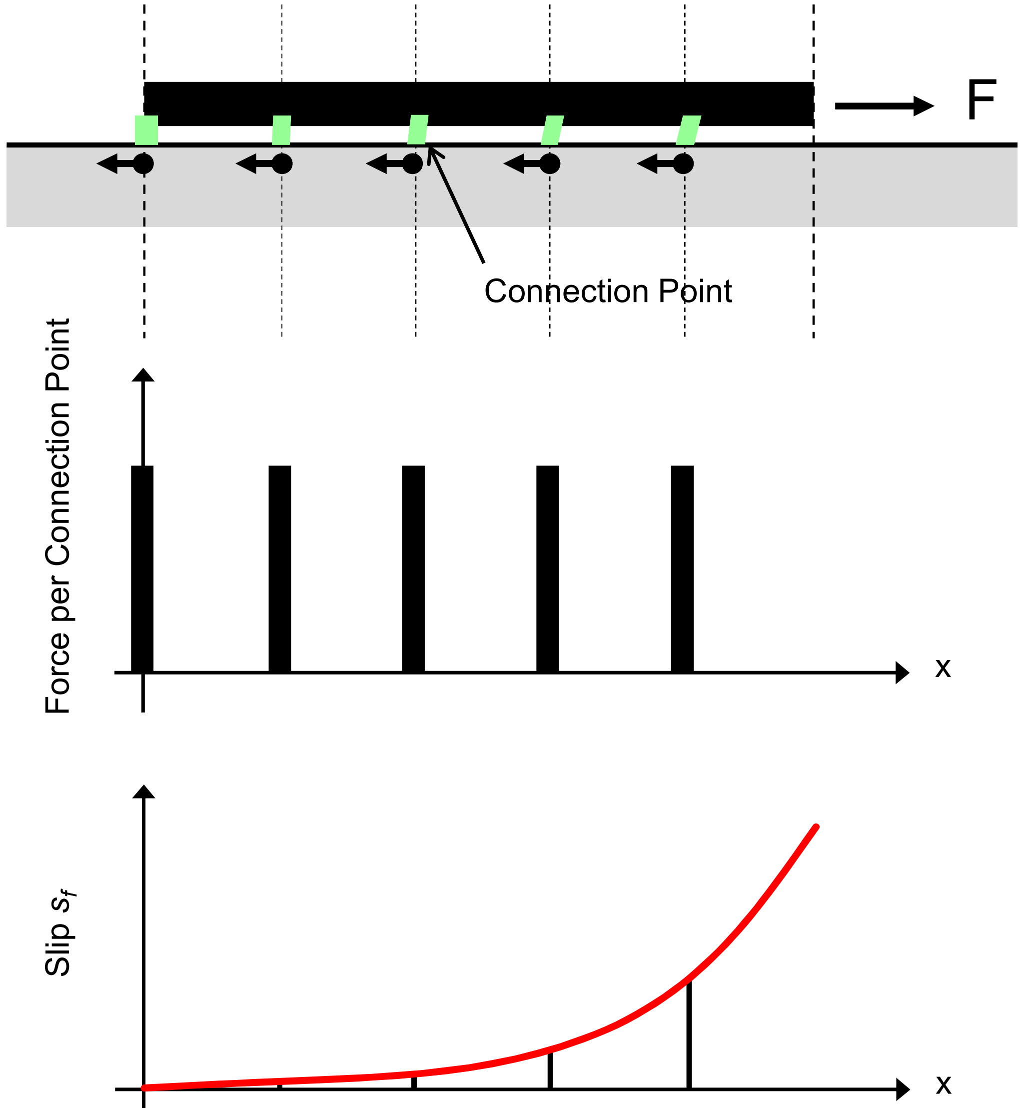
Connection in several points



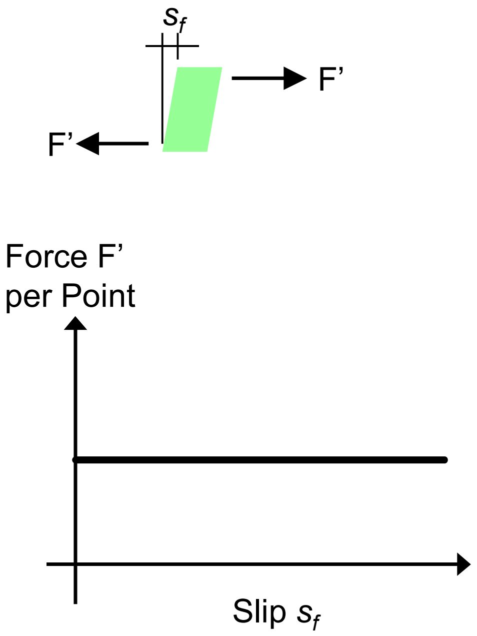


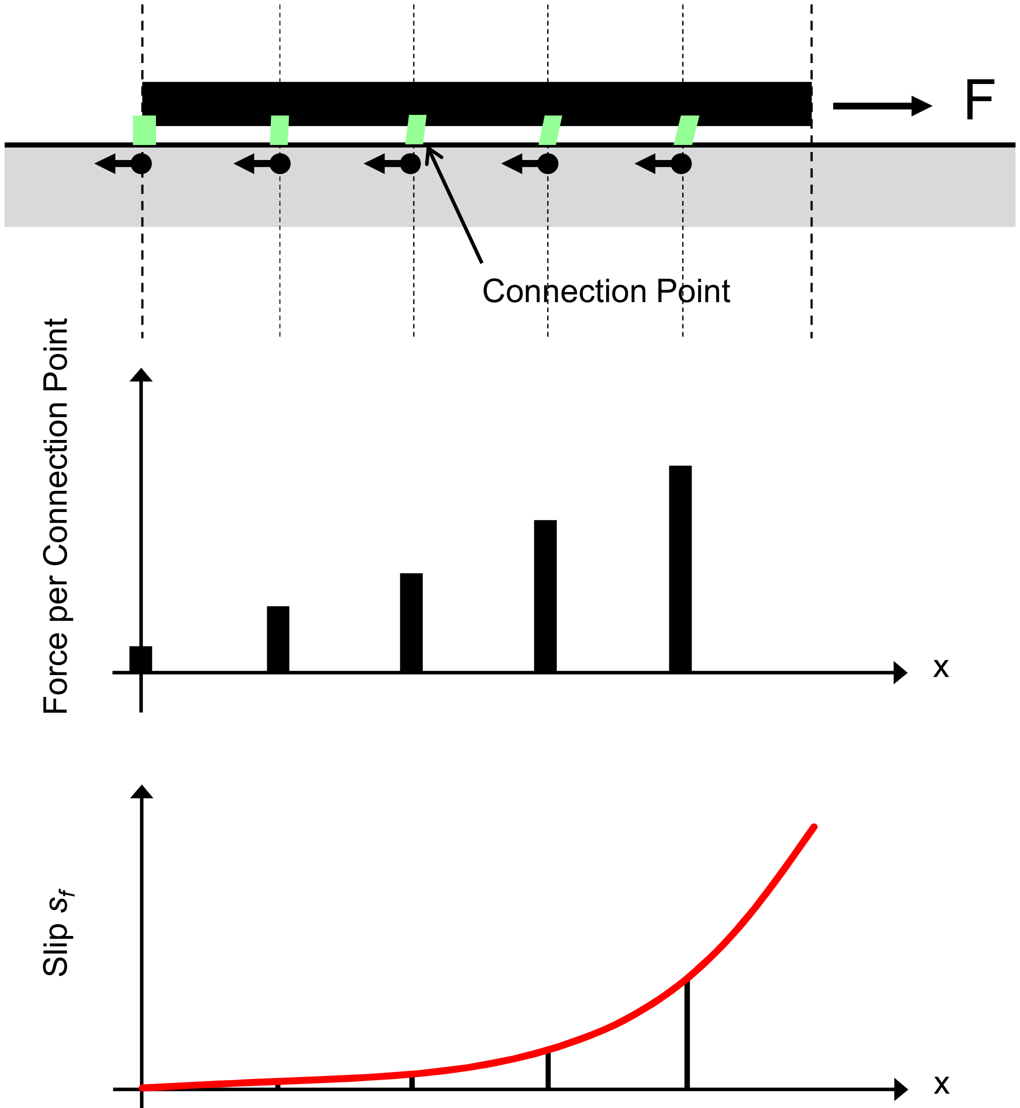


Picture taken from http://www.seilziehclub-gonten.ch/images/diverse/Comic1.JPG (displayed mirrored)

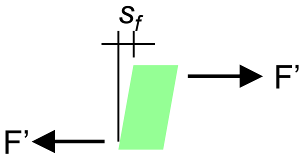


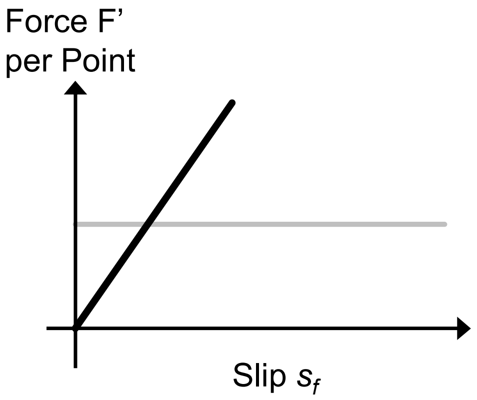
Behavior of the Connection Point:

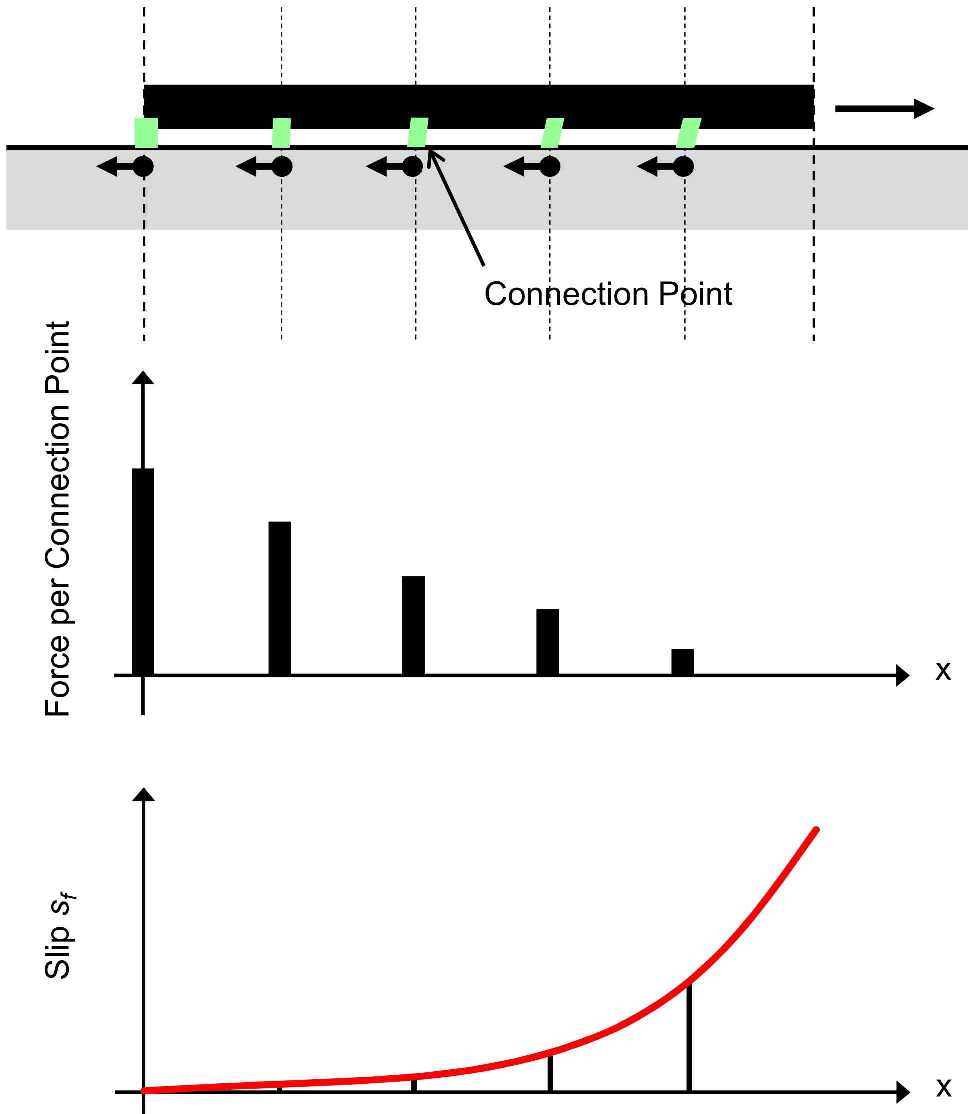




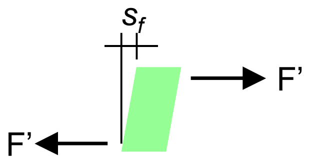
Behavior of the Connection Point:

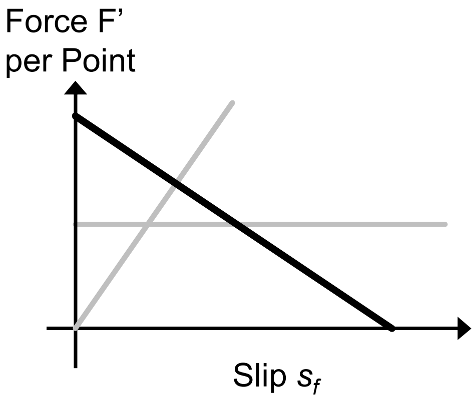


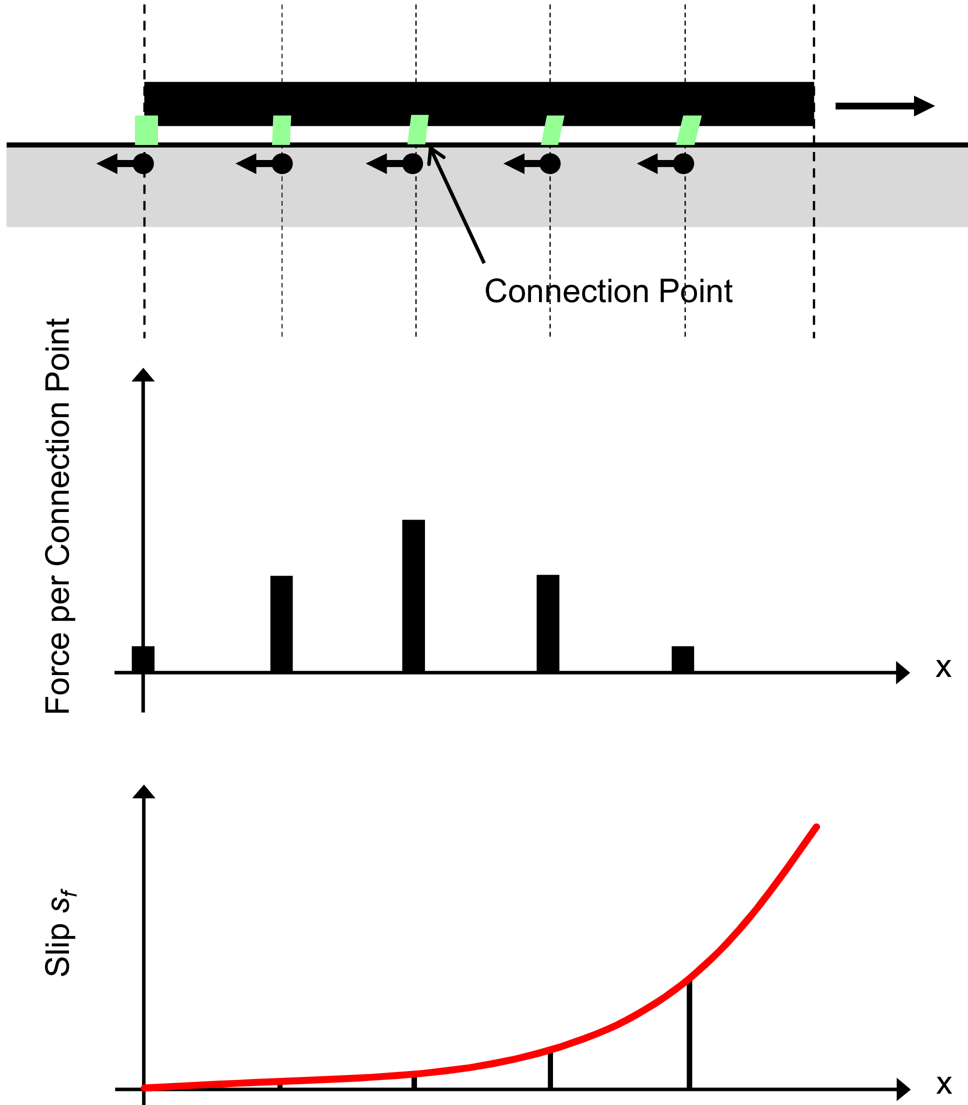




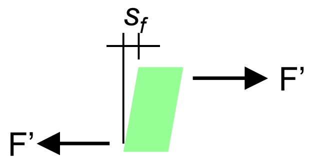
Behavior of the Connection Point:

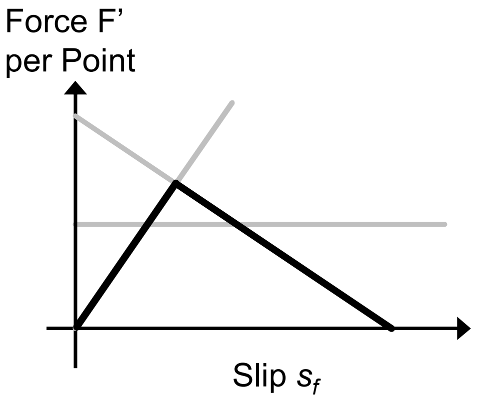


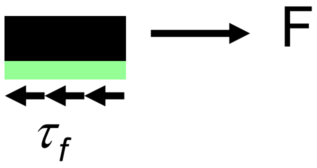




Behavior of the Connection Point:

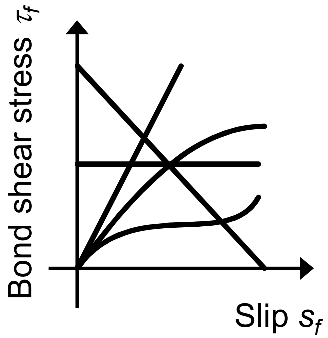






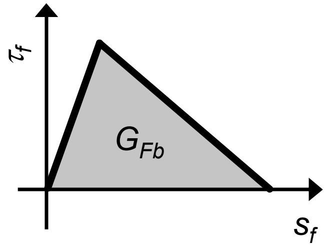
$$\tau_f = \frac{Force}{unit\ area}\ \frac{N}{mm^2}$$

$$\tau_f = Bond\ shear\ stress$$

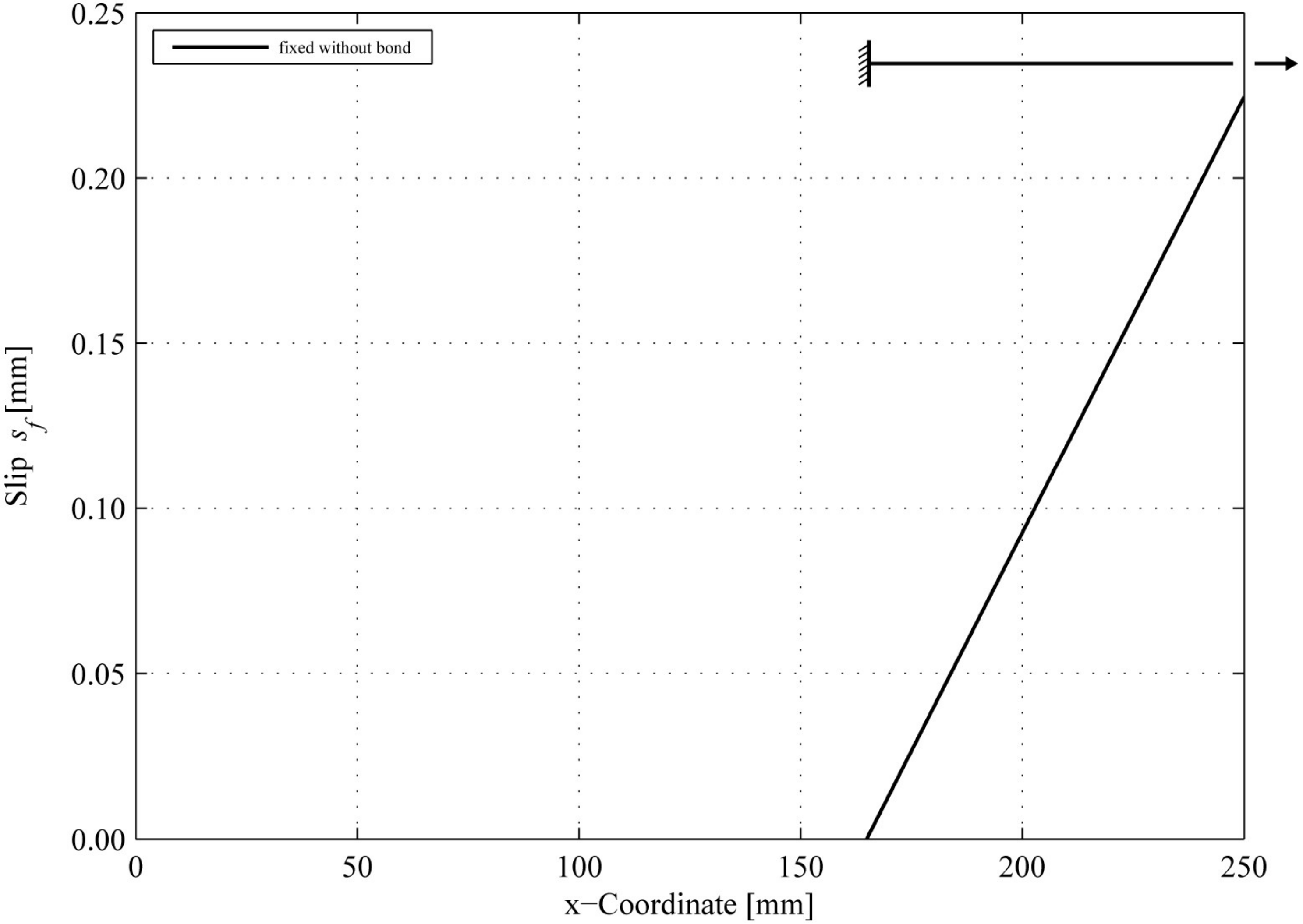


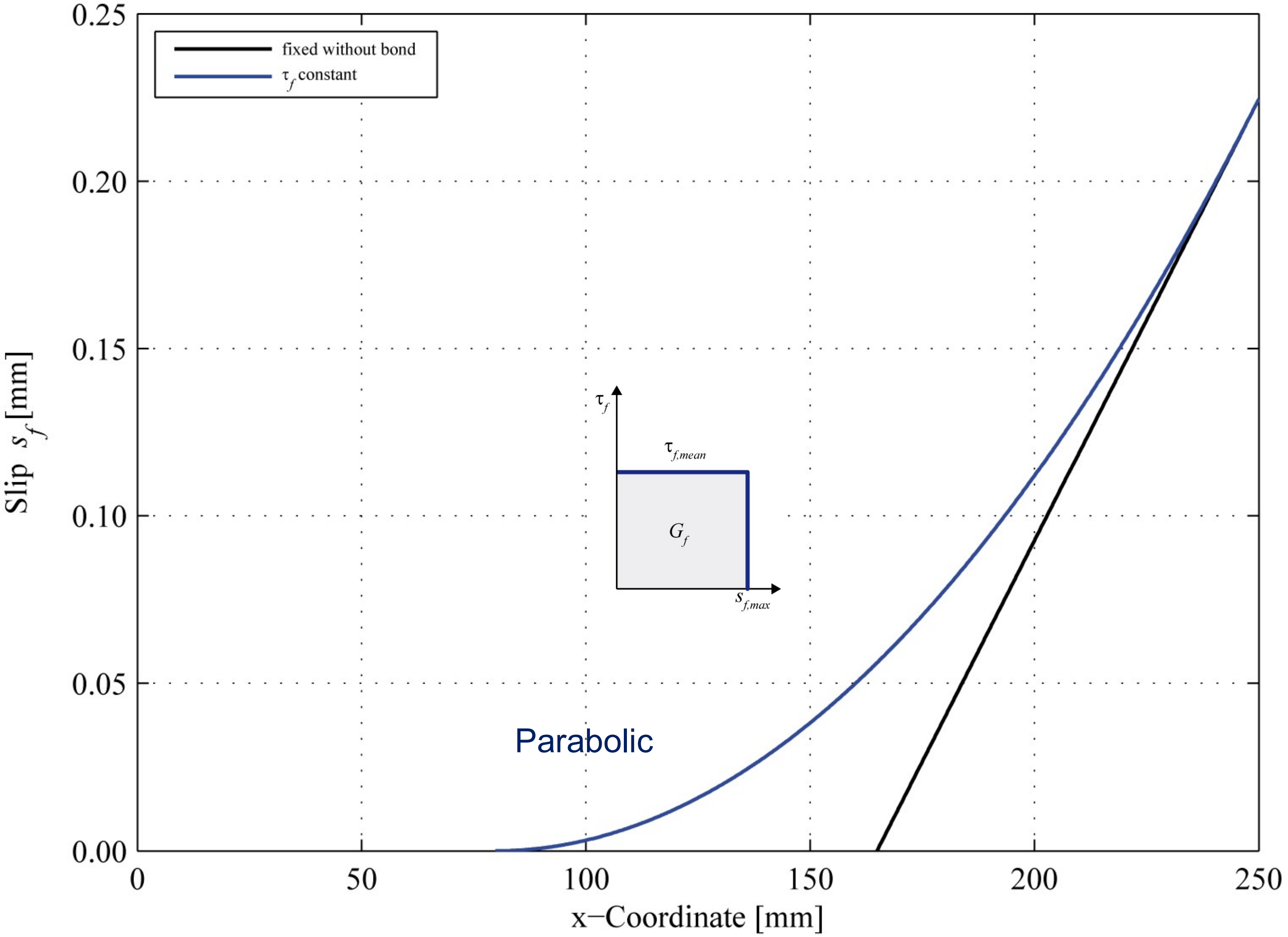
Bond shear stress – Slip – Relation

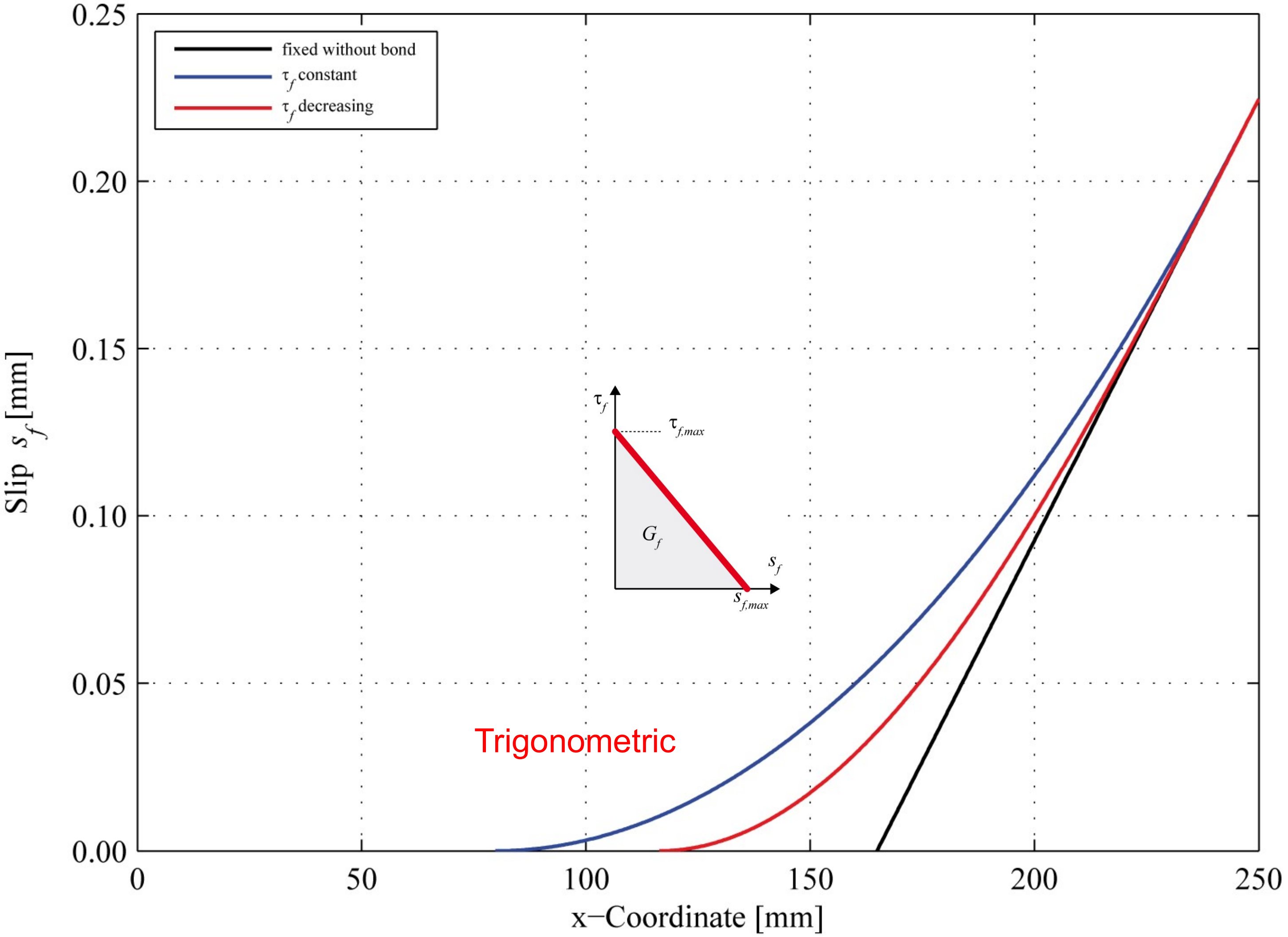
This relation characterizes the bond behavior.



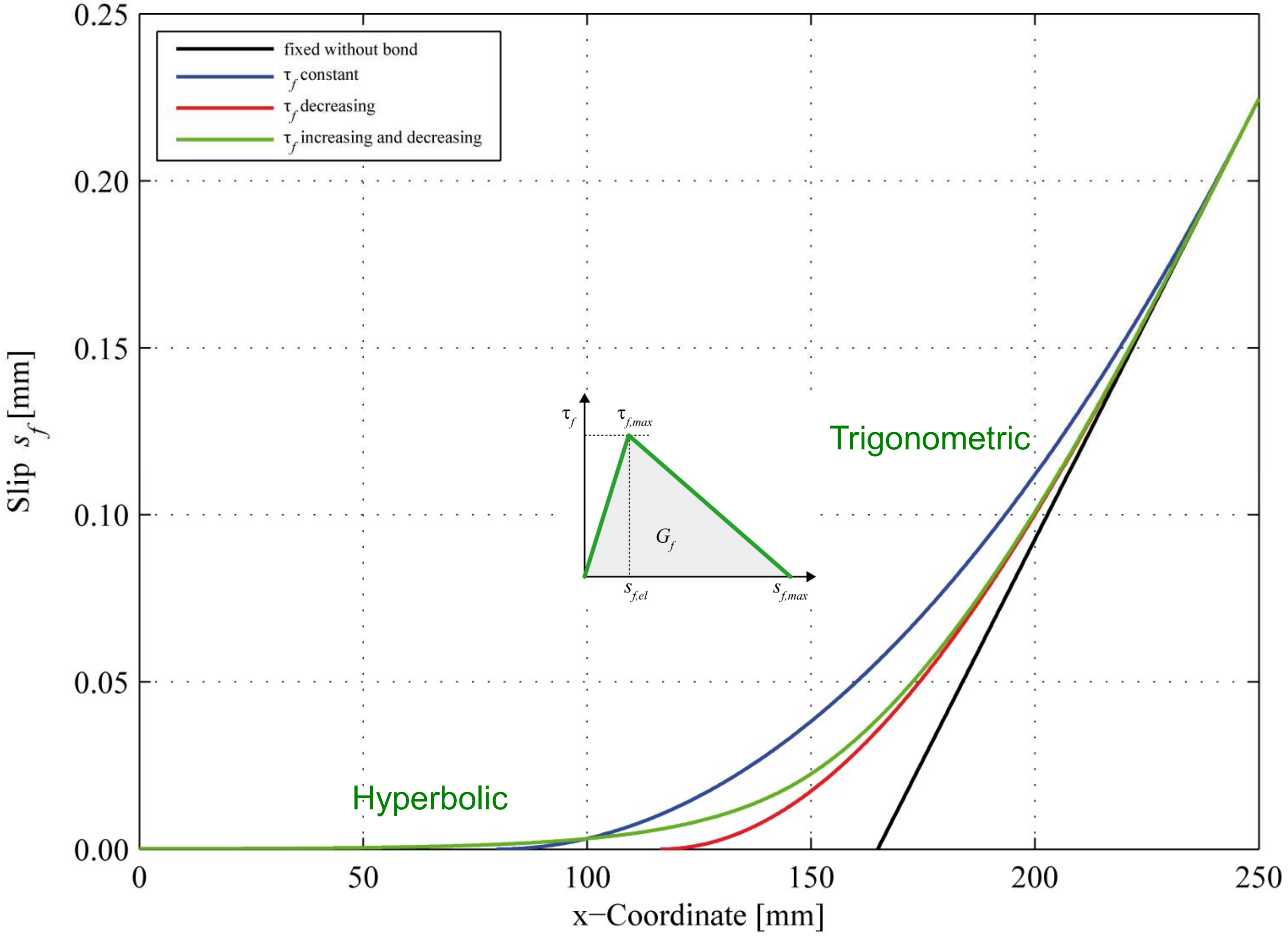
The area under the bond shear stress – slip – relation corresponds to the specific fracture energy G_{Fb}.


fixed without bond
Slip s_f [mm]
0.25
0.20
0.15
0.10
0.05
0.00
0
50
100
150
200
250
x−Coordinate [mm]

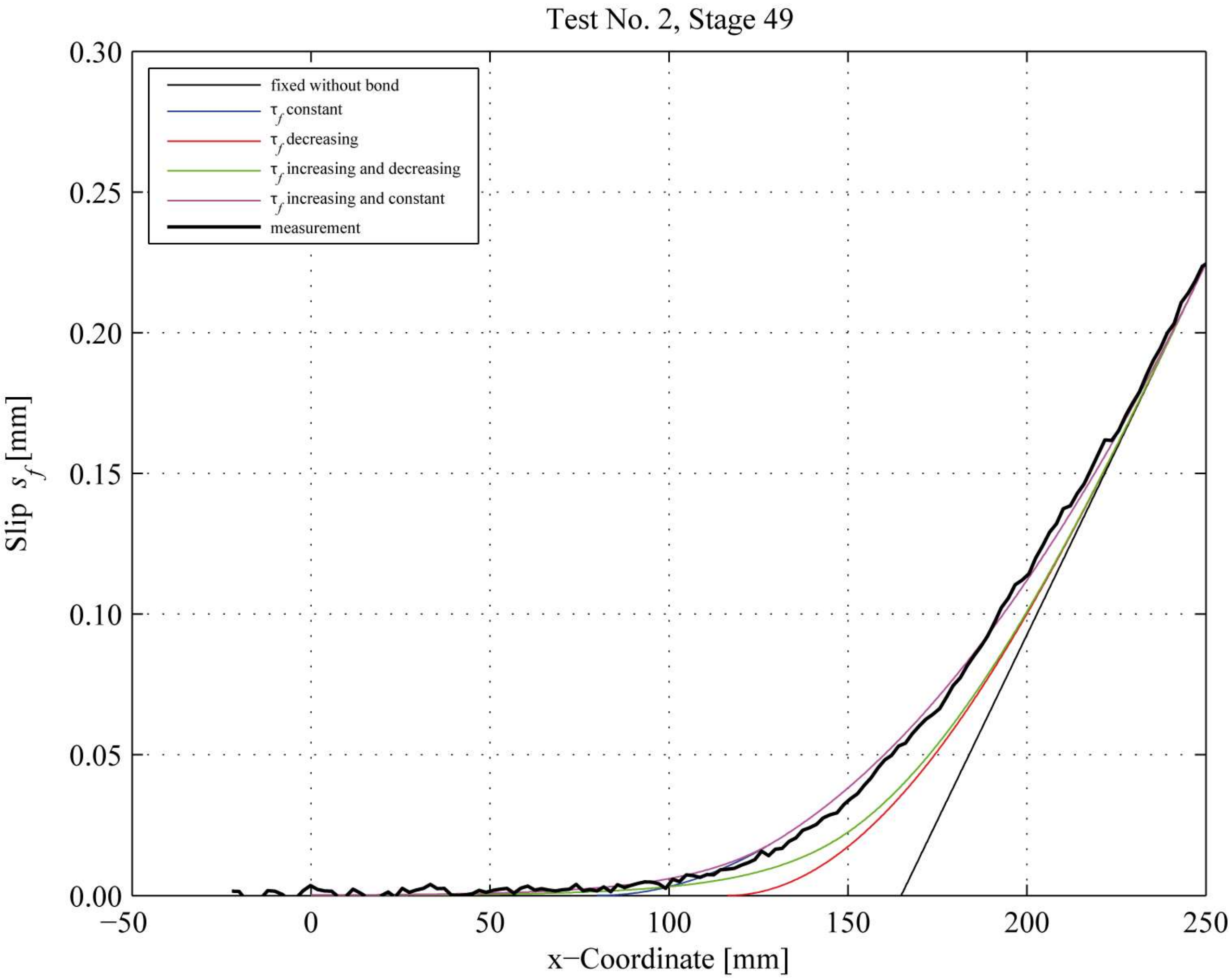

0.25
0.20
0.15
0.10
0.05
0.00
Slip s_f [mm]
0
50
100
150
200
250
x−Coordinate [mm]
fixed without bond
τ_f constant
τ_f
$\tau_{f,mean}$
G_f
$s_{f,max}$
Parabolic



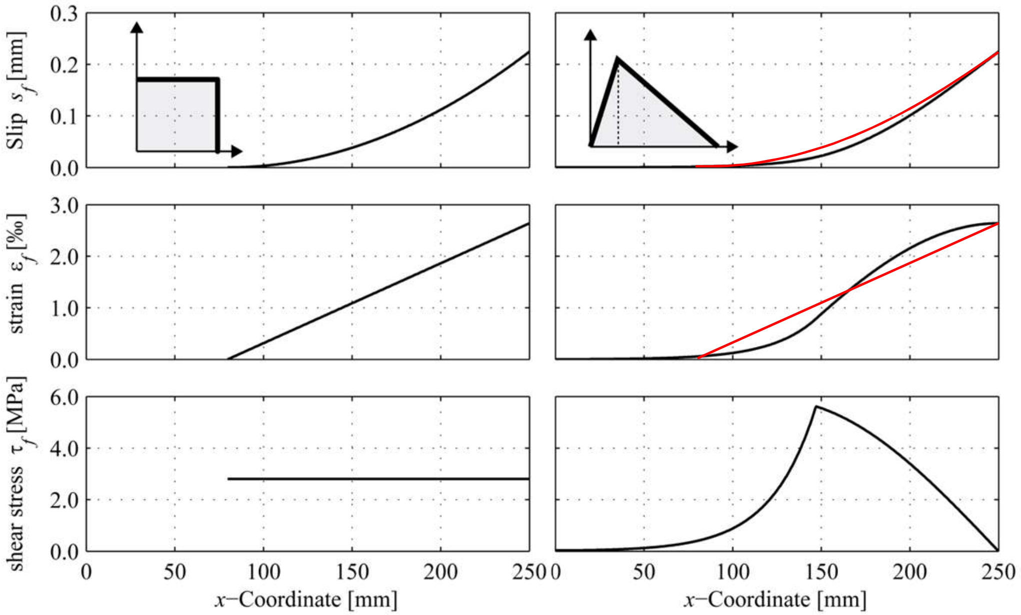
Trigonometric


0.25
0.20
0.15
0.10
0.05
0.00
Slip s_f [mm]
0
50
100
150
200
250
x−Coordinate [mm]
fixed without bond
τ_f constant
τ_f decreasing
τ_f increasing and decreasing
τ_f
$\tau_{f,max}$
G_f
$s_{f,el}$
$s_{f,max}$
Trigonometric
Hyperbolic

Comparison Modelling - Measurement

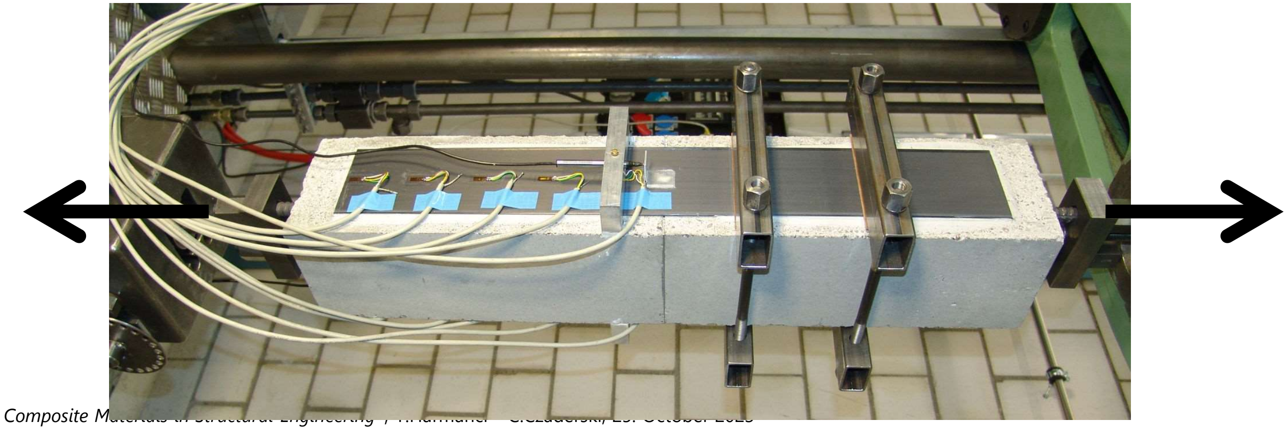


Lap-shear test with strain gauges (SG)

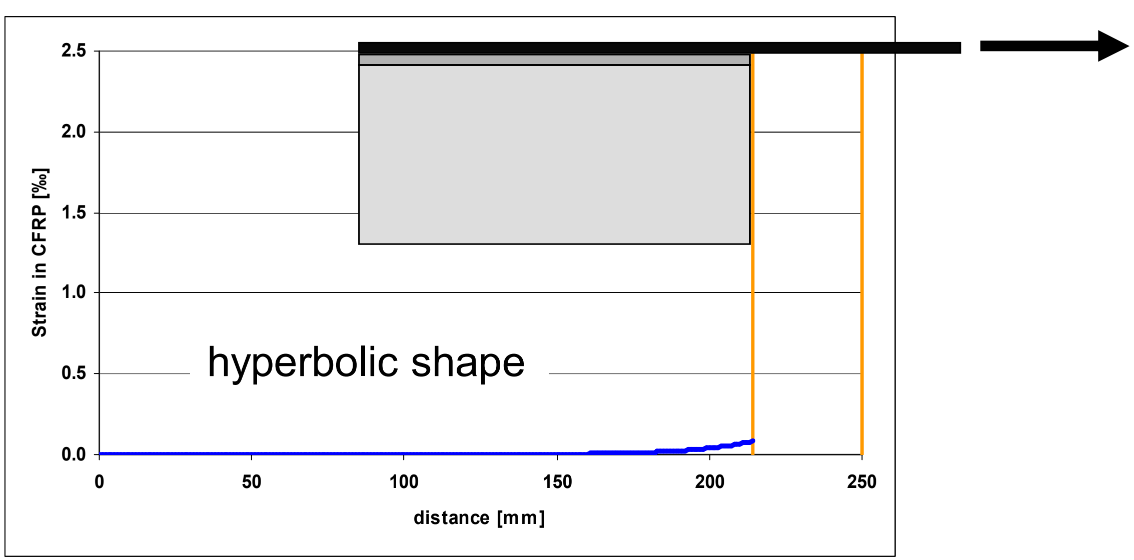


ICS measures slip

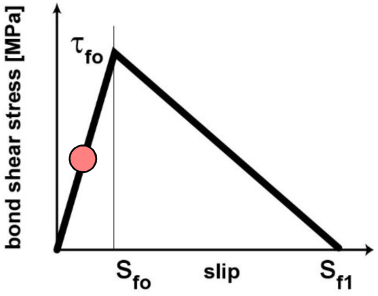
SG's measure strain

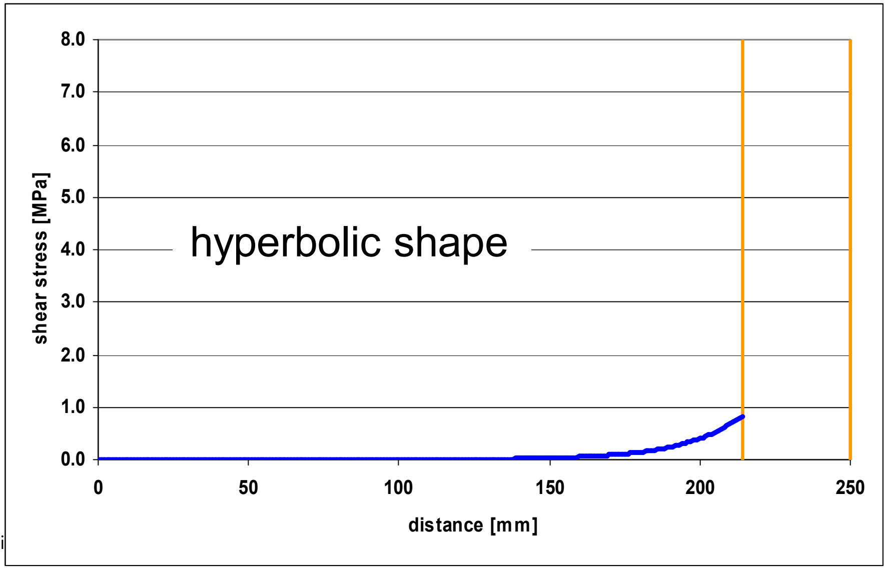


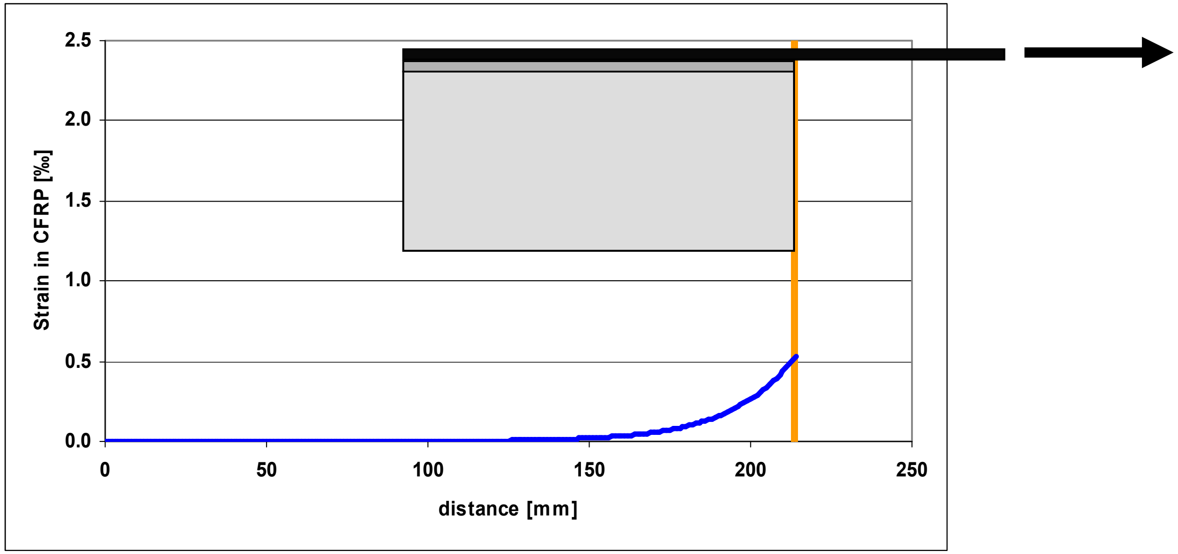
Longitudinal strain

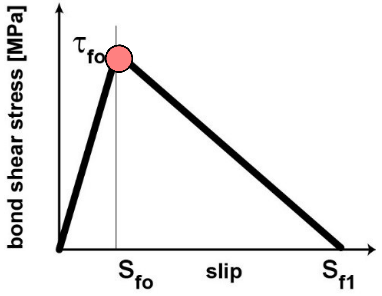


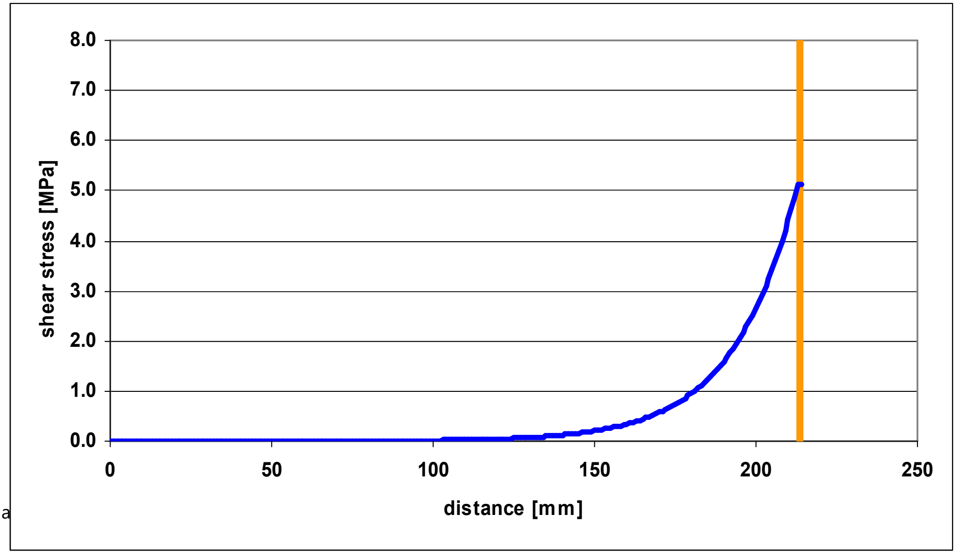
Shear stress between strip and concrete

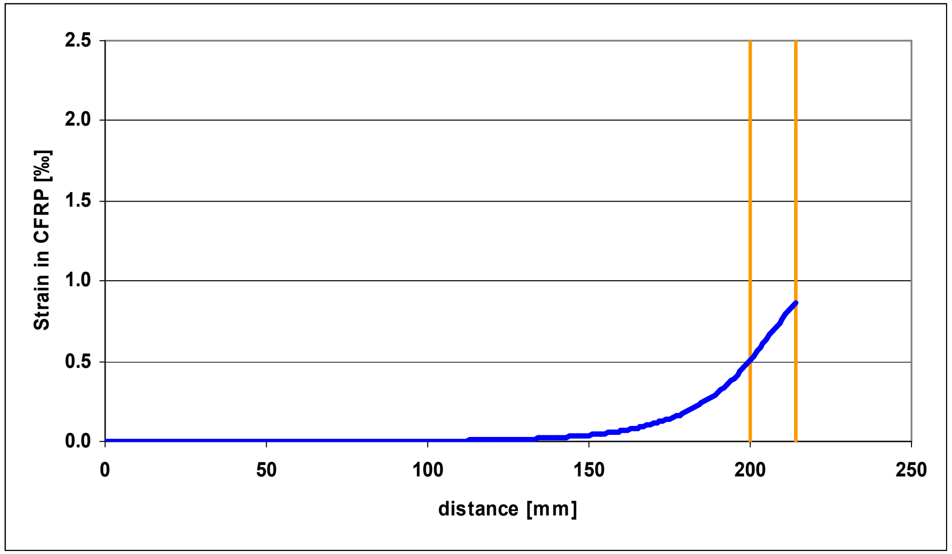


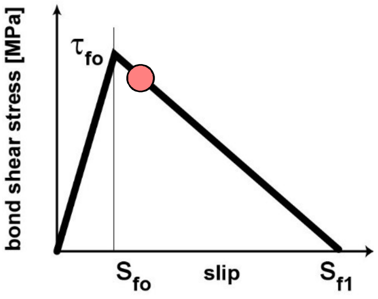


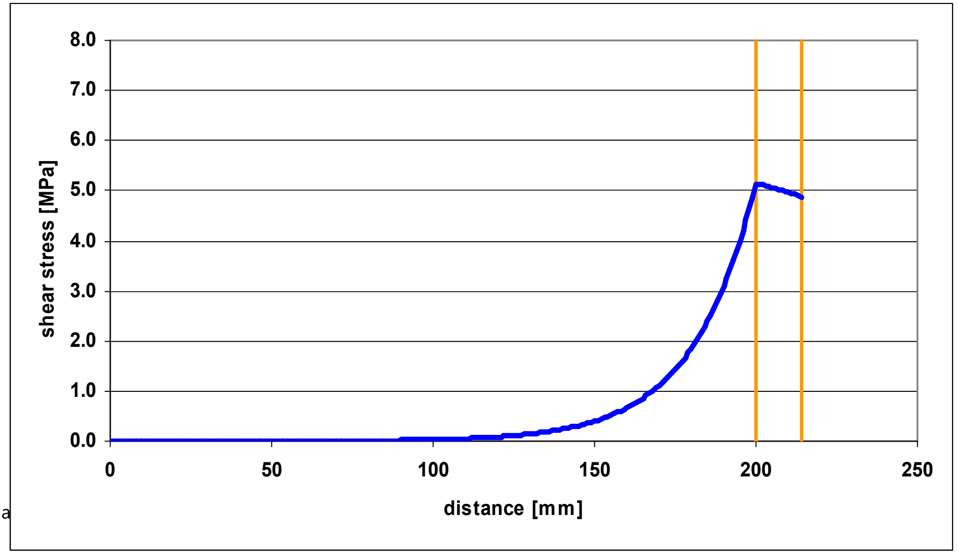

Strain in CFRP [‰]
2.5
2.0
1.5
1.0
0.5
0.0
0
50
100
150
200
250
distance [mm]

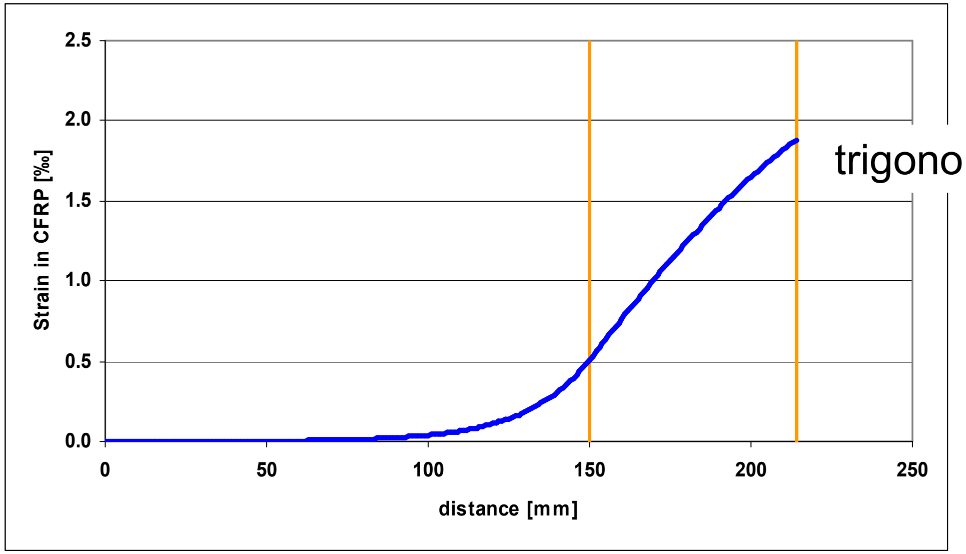

bond shear stress [MPa]
τ_{fo}
S_{fo}
slip
S_{f1}


shear stress [MPa]
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0
50
100
150
200
250
distance [mm]

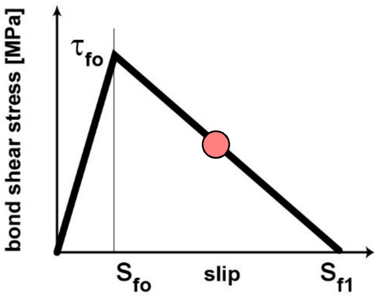

Strain in CFRP [‰]
2.5
2.0
1.5
1.0
0.5
0.0
0
50
100
150
200
250
distance [mm]

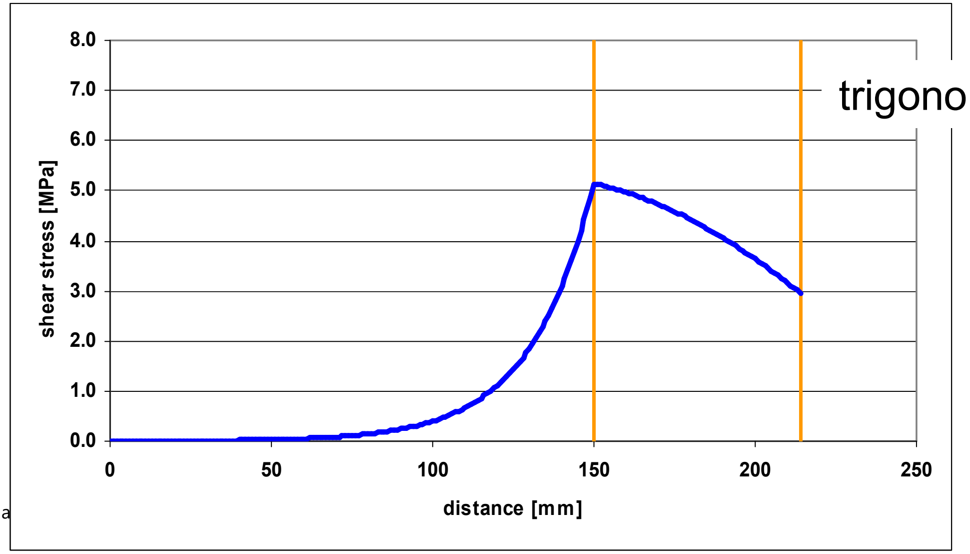

bond shear stress [MPa]
τ_{fo}
S_{fo}
slip
S_{f1}


shear stress [MPa]
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0
50
100
150
200
250
distance [mm]

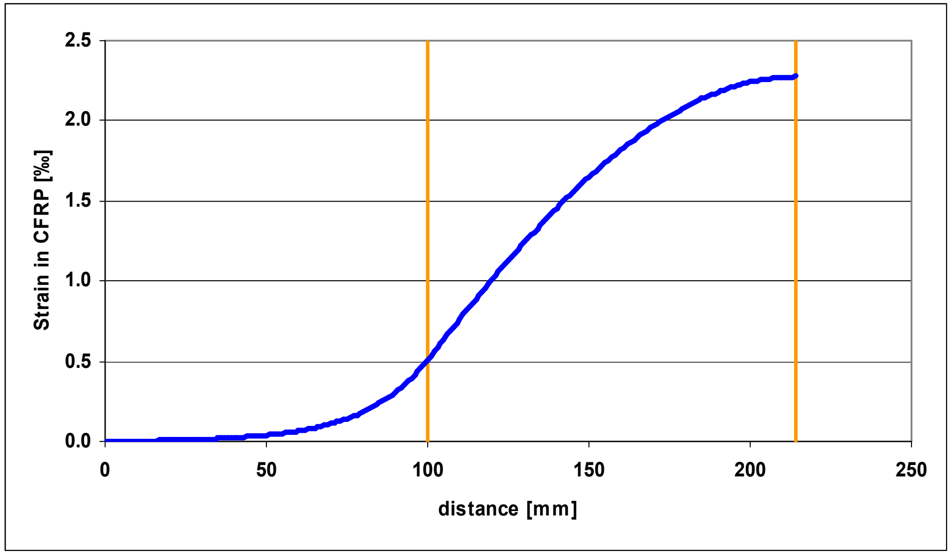


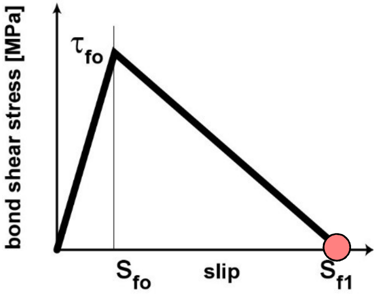
trigonometric shape

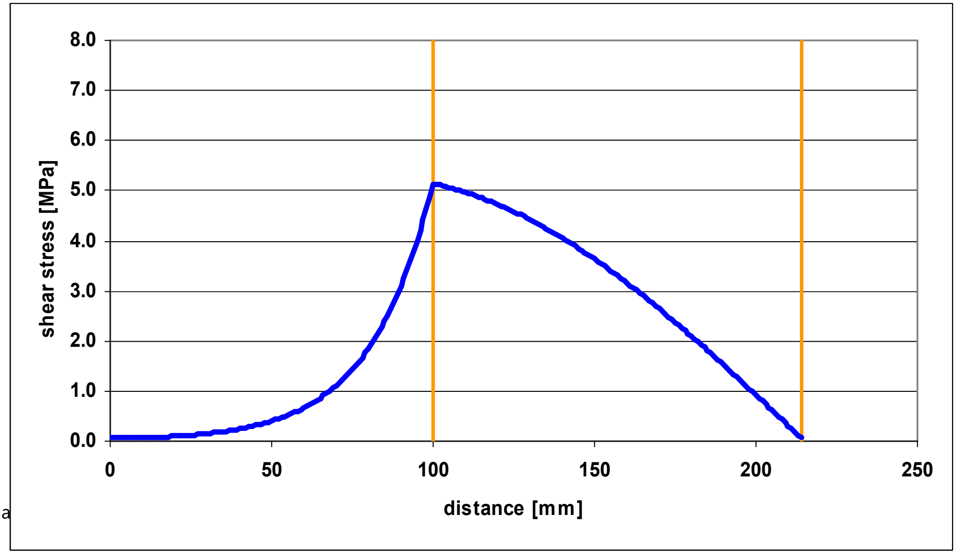




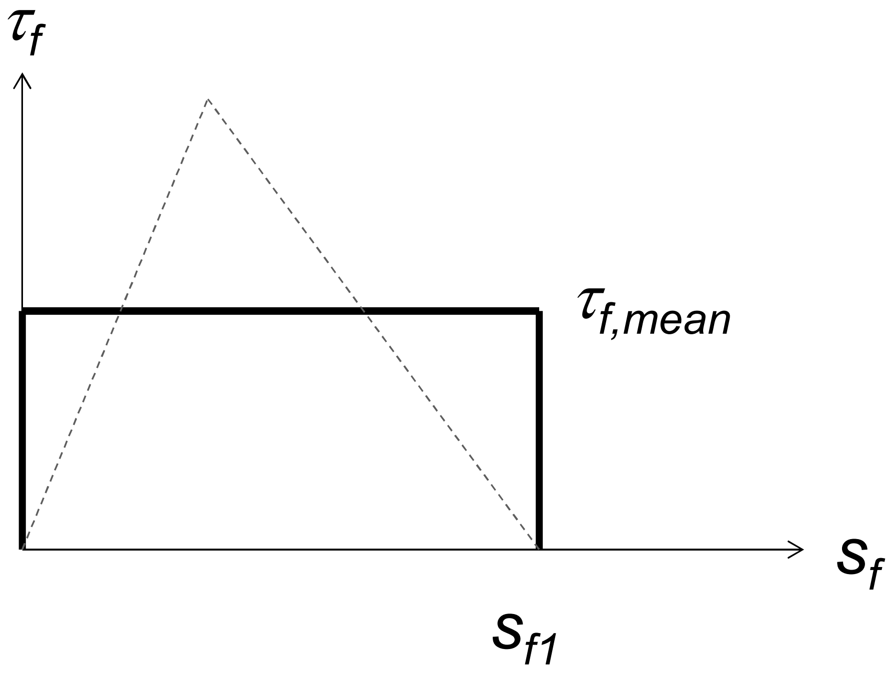
trigonometric shape


Strain in CFRP [‰]
2.5
2.0
1.5
1.0
0.5
0.0
0
50
100
150
200
250
distance [mm]


bond shear stress [MPa]
τfo
Sfo
slip
Sf1

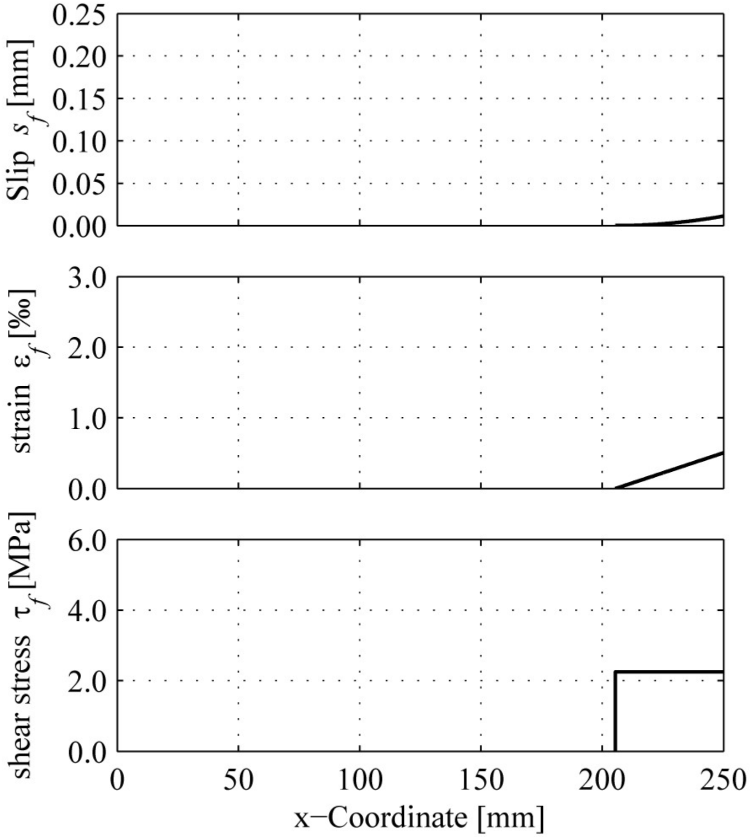

shear stress [MPa]
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0
50
100
150
200
250
distance [mm]

Simplified bond shear stress-slip relation



$$G_{Fb} = \tau_{f,mean} s_{f1}$$

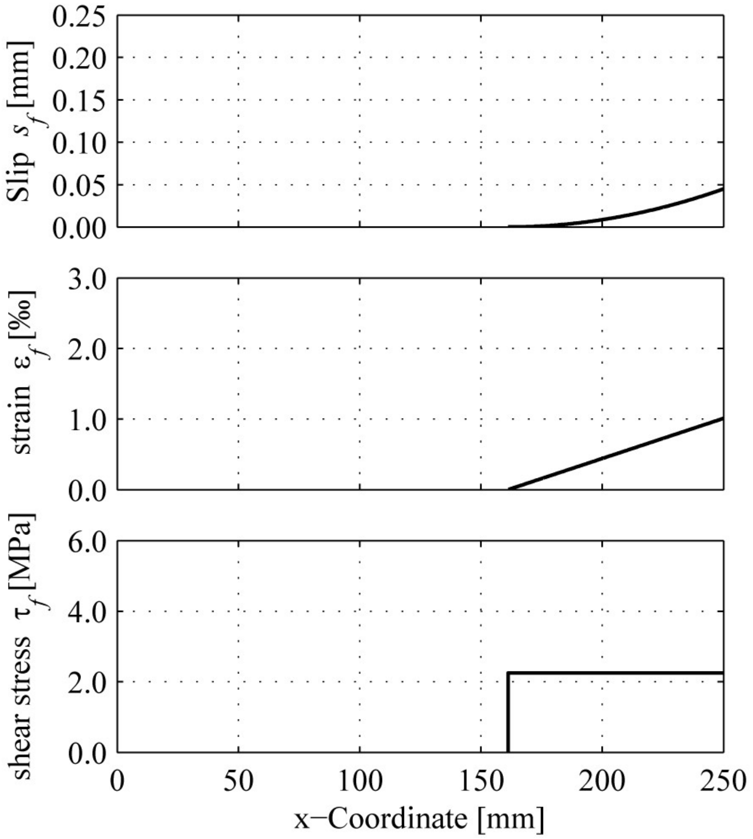
F = 5 kN



parabolic shape of slip

$E_f = 165$ GPa, $b_f = 50$mm, $t_f = 1.2$ mm, $\tau_{f,mean} = 2.25$ MPa

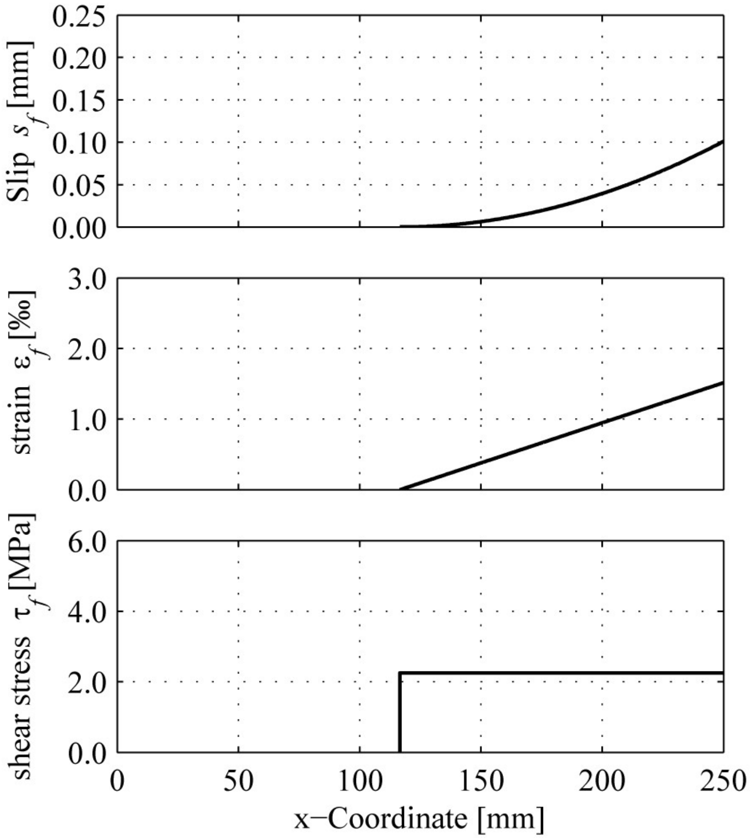
F = 10 kN



parabolic shape of slip

$E_f = 165$ GPa, $b_f = 50$mm, $t_f = 1.2$ mm, $\tau_{f,mean} = 2.25$ MPa

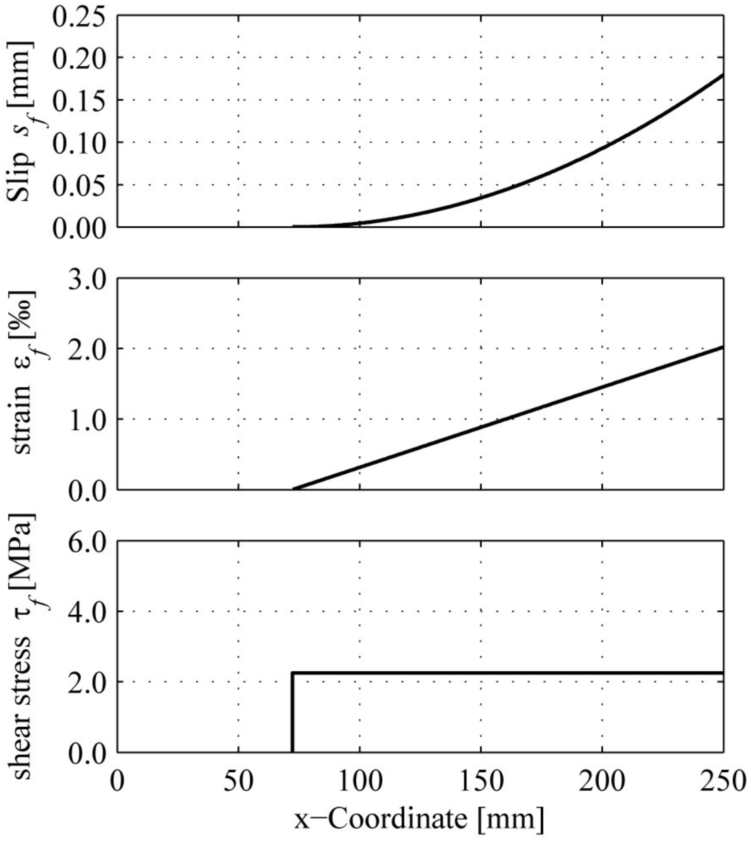
F = 15 kN



parabolic shape of slip

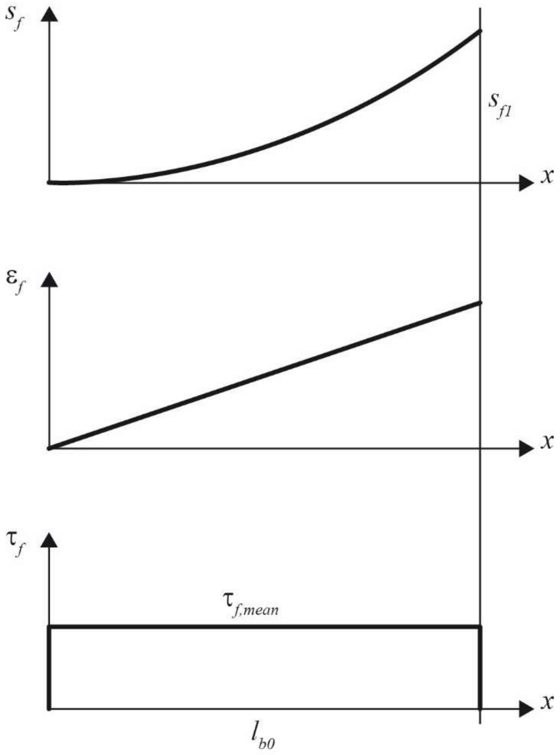
$E_f = 165$ GPa, $b_f = 50$mm, $t_f = 1.2$ mm, $\tau_{f,mean} = 2.25$ MPa

F = 20 kN



parabolic shape of slip

$E_f = 165$ GPa, $b_f = 50$mm, $t_f = 1.2$ mm, $\tau_{f,mean} = 2.25$ MPa



$$s_f(x)=\frac{\tau_f}{E_f t_f}\frac{x^2}{2} \qquad (1)$$

$$\varepsilon_f(x)=\frac{\tau_f}{E_f t_f}x \qquad (2)$$

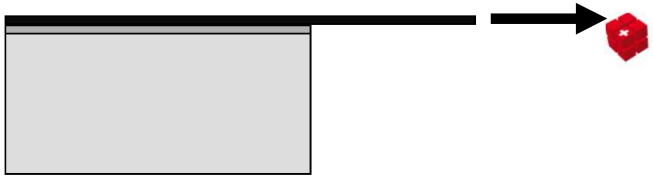
$$\tau_f(x)=\tau_{f,mean} \qquad (3)$$

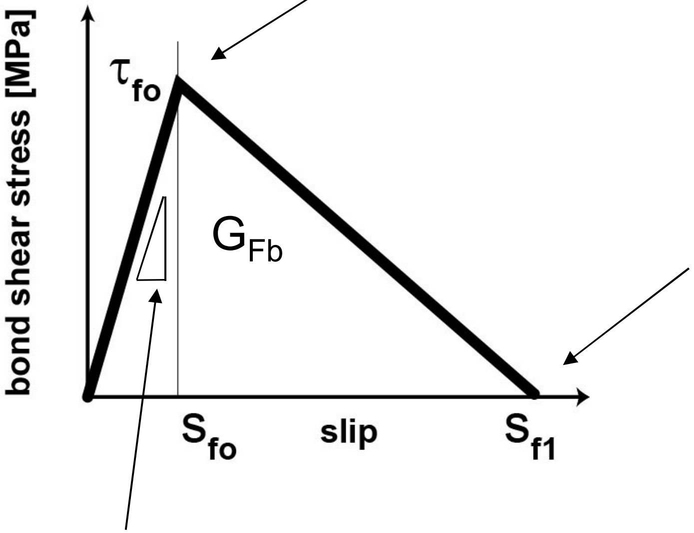
with $l_{b0}=\dfrac{F_{b0,R}}{\tau_{f,mean}\cdot b_f}$ and $s_f(x=l_{b0})=s_{f1}$ we get from Eq. (1) and (3): $s_{f1}=\dfrac{F_{b0,R}^2}{2E_f t_f b_f^2 \tau_{f,mean}}$

and with $G_{Fb}=s_{f1}\tau_{f,mean}$ we get $F_{b0,R}=b_f\sqrt{2G_{Fb}E_f t_f}$

(slide 41)

Bond behavior depends on





Bond damage (Entfestigungsbereich, Verbundschädigung)

- concrete quality

maximum slip

- constant (~0.2 mm)

Elastic deformation

- shear modulus of adhesive and concrete
- thickness of adhesive plus a layer of concrete

Additionally, the stiffness ($E_f t_f$) of the strip influences also the bond behaviour.

Comparison with reinforcement

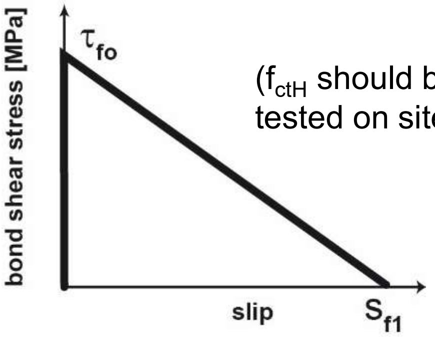
- Internal steel reinforcement is surrounded
  - Deflection perpendicular to longitudinal direction is not possible
  - Normal (confinement) stresses due to interlocking and friction

  → the longer the anchor length, the higher the anchor force up to yielding of the steel reinforcement

- Externally applied strip is free on one side
  - Deflection perpendicular to longitudinal direction is possible

  → maximum anchorage force (anchorage resistance) with corresponding length (active bond length)

CFRP strip according to SIA 166 (2004)



(f_{ctH} should better be tested on site)

$$\tau_{f0} = \frac{4}{3} f_{ctH} \quad G_{Fb} = \frac{1}{8} f_{ctH}$$

f_{ctH} : **mean** value of bond strength (Haftfestigkeit) determined according to DIN 1048 part 2

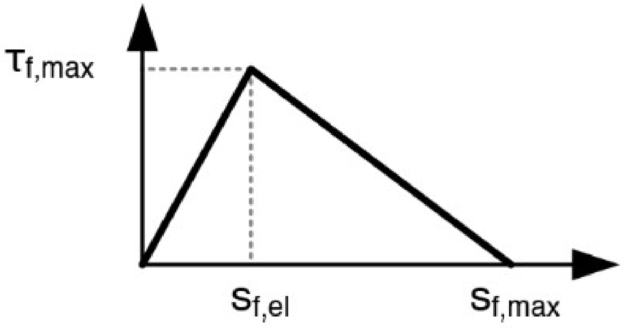
for C30/37: f_{ctm} = 2.9MPa

with $f_{ctm} = f_{ctH}$ (only as an example, the correlation is not correct!)

τ_{f0} = 3.9 MPa

G_{Fb} = 0.36 N/mm

CFRP strip according to prSIA 166 (2023)



$$\tau_{fmax} = \frac{4}{3} f_{hk} \qquad G_{Fb} = \frac{1}{8} f_{hk}$$

f_{hk} : **characteristic** value of bond strength (Haftfestigkeit) determined according to SN EN 1542

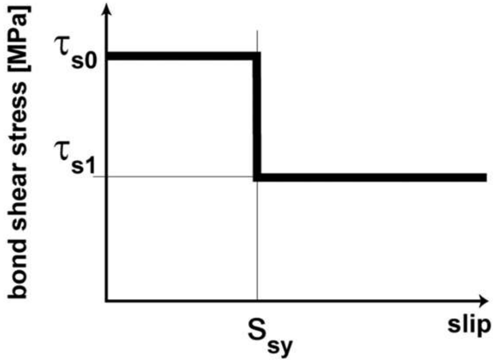
for C30/37: f_{ctm} = 2.9MPa, $f_{ctk0.05}$ = 2.0MPa

with $f_{ctk0.05} = f_{ctH}$ (only as an example, the correlation is not correct!)

$\tau_{f,max}$ = 2.7 MPa

G_{Fb} = 0.25 N/mm

Internal steel reinforcement according to Sigrist and Marti: (Skript Stahlbeton Marti 2009)



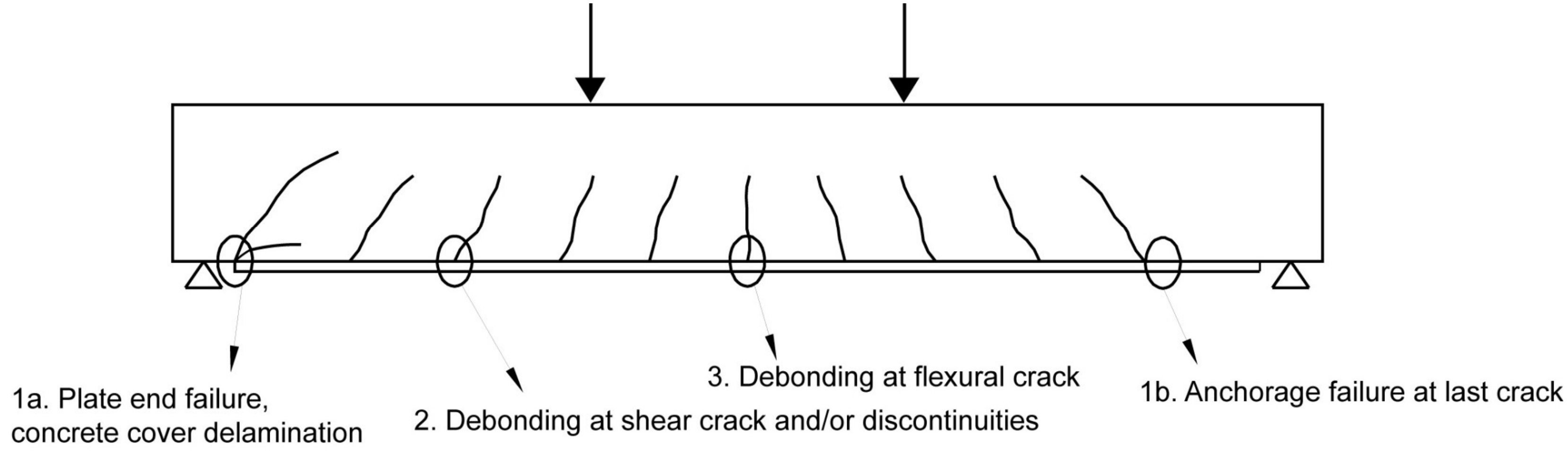
$$\tau_{s0} = 2f_{ctm} \qquad \tau_{s1} = f_{ctm}$$

In SIA262 (2013): $f_b = 1.4f_{ctm}$

τ_{s0} = 5.8 MPa **τ_{s1} = 2.9 MPa**

General comment: for design values, divide with respective γ-values!

Debonding failure modes according to SIA 166



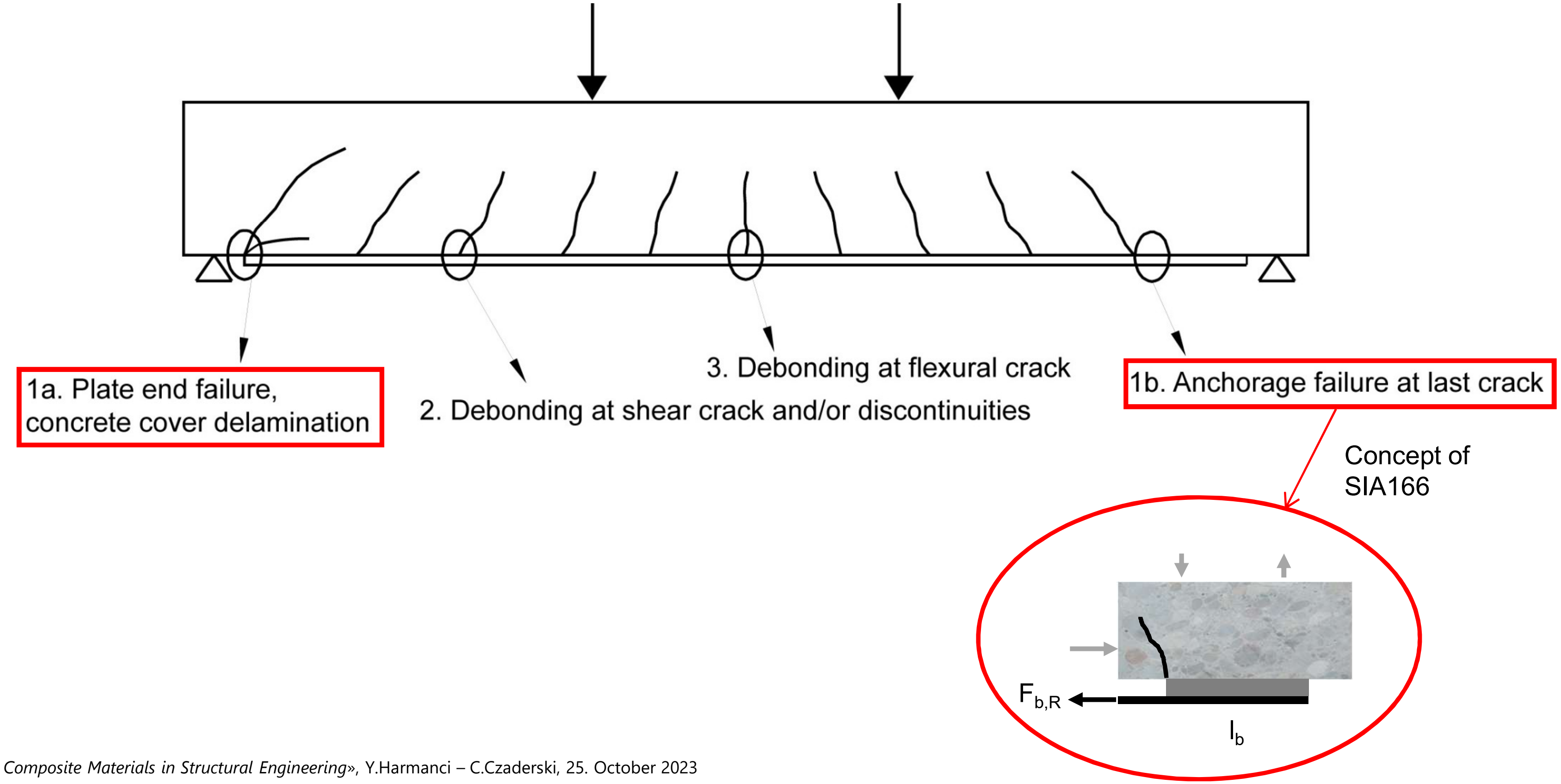
3. Debonding at flexural crack

1a. Plate end failure, concrete cover delamination

2. Debonding at shear crack and/or discontinuities

1b. Anchorage failure at last crack

Debonding failure modes according to SIA 166



1a. Plate end failure,
concrete cover delamination

2. Debonding at shear crack and/or discontinuities

3. Debonding at flexural crack

1b. Anchorage failure at last crack

Concept of
SIA166

Equations for maximum anchorage resistance

please note: without safety factors!! See example on slide 63 for the safety concept!!

SIA 166 $F_{b0,R} = b_f \sqrt{2 \cdot G_{Fb} \cdot E_f \cdot t_f} = b_f \sqrt{2 \frac{f_{ctH}}{8} E_f t_f} = 0.5 b_f \sqrt{E_f t_f f_{ctH}}$

***fib* Bulletin 14** $N_{fa,max} = \alpha \cdot c_1 \cdot k_c \cdot k_b \cdot b_f \cdot \sqrt{E_f \cdot t_f \cdot f_{ctm}}$

$$k_b = 1.06 \sqrt{\frac{2 - \frac{b_f}{b}}{1 + \frac{b_f}{400}}} \geq 1$$

TR55 $T_{k,max} = 0.5 \cdot k_b \cdot b_f \cdot \sqrt{E_{fd} \cdot t_f \cdot f_{ctk}}$

Italian Code $F_{fd} = b_f \sqrt{2 \cdot E_f \cdot t_f \cdot \Gamma_{Fk}}$; $\Gamma_{Fk} = 0.03 \cdot k_b \cdot \sqrt{f_{ck} \cdot f_{ctm}}$

Take care to symbols. They depend on the reference.

Active bond length

(the length which is actively involved in the force transfer from the strip to the concrete)

If we assume a constant bond shear stress:

$$l_{b0} \approx \frac{F_{b0,R}}{b_f \cdot \tau_{f,mean}}$$

with

$$F_{b0,R} = b_f \sqrt{2 \cdot G_{Fb} \cdot E_f \cdot t_f}$$

$$\longrightarrow \quad l_{b0} \approx \frac{\sqrt{2 \cdot G_{Fb} \cdot E_f \cdot t_f}}{\tau_{f,mean}} \quad \longrightarrow \text{proportional to } \sqrt{\frac{G_{Fb} \cdot E_f \cdot t_f}{\tau_{f,mean}^2}}$$

Equations for active bond length

(minimum necessary length for maximum anchor resistance $F_{b0,R}$)

SIA 166	$l_{b0} = \frac{\pi}{2}\cdot\sqrt{2\cdot\frac{G_{Fb}\cdot E_f\cdot t_f}{\tau_{f0}^2}};\ \tau_{f0} = \frac{4}{3}f_{ctH};\ l_{b0} = \frac{3\pi}{16}\cdot\sqrt{\frac{E_f\cdot t_f}{f_{ctH}}}$
***fib* Bulletin 14**	$l_{b,max} = \sqrt{\frac{E_f\cdot t_f}{c_2\cdot f_{ctm}}}$
TR55	$l_{t,max} = 0.7\cdot\sqrt{\frac{E_{fd}\cdot t_f}{f_{ctk}}}$
Italian Code	$l_e = \sqrt{\frac{E_f\cdot t_f}{2\cdot f_{ctm}}}$

please note: without safety factors!!

Units in equations in SIA 166

Equations for maximum anchorage resistance and active bond length

Please note, that the units are not conform!

$$F_{b0,R} = b_f \cdot \sqrt{2 \cdot G_{Fb} \cdot E_f \cdot t_f} = b_f \cdot \sqrt{2 \cdot \frac{f_{ctH}}{8} \cdot E_f \cdot t_f} = 0.5 \cdot b_f \cdot \sqrt{E_f \cdot t_f \cdot f_{ctH}}$$

$$G_{Fb}\left[\frac{N}{mm}\right] = \frac{f_{ctH}\left[\frac{N}{mm^2}\right]}{8\left[\frac{1}{mm}\right]}$$

$$l_{b0} = \frac{\pi}{2} \cdot \sqrt{2 \cdot \frac{G_{Fb} \cdot E_f \cdot t_f}{\tau_{f0}^2}} \; ; \; \tau_{f0} = \frac{4}{3} f_{ctH} \; ; \; l_{b0} = \frac{3\pi}{16} \cdot \sqrt{\frac{E_f \cdot t_f}{f_{ctH}}}$$

Example

Concrete C30/37

Sika CarboDur S512 tensile strength > 2800 MPa

tensile strain > 17‰

$$G_{Fb} = \frac{f_{ctH}}{8}\left[\frac{N}{mm}\right]$$

$$\tau_{f0} = \frac{4}{3}\cdot f_{ctH}\,[MPa]$$

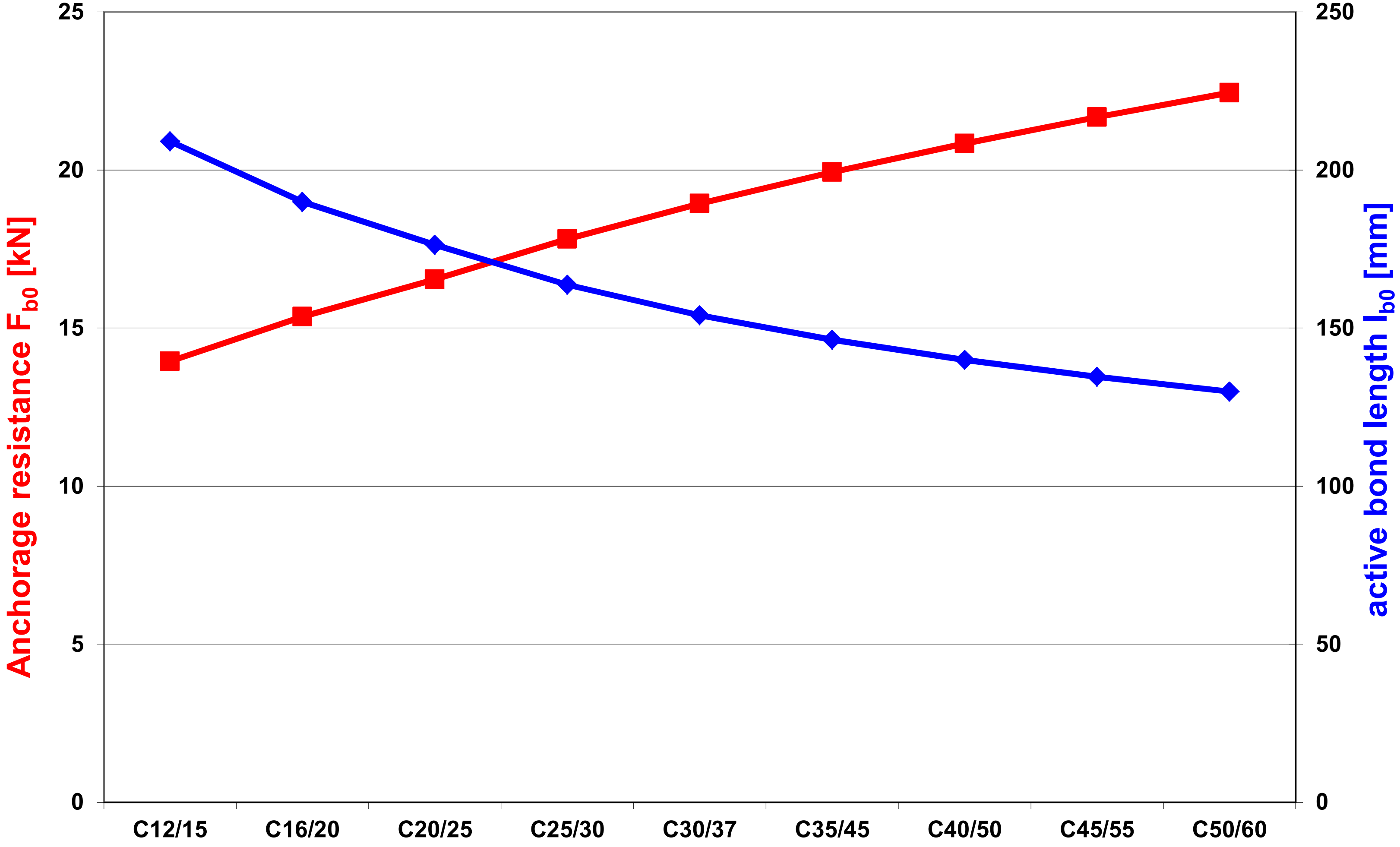
for concrete
C 20/25 to C 50/60

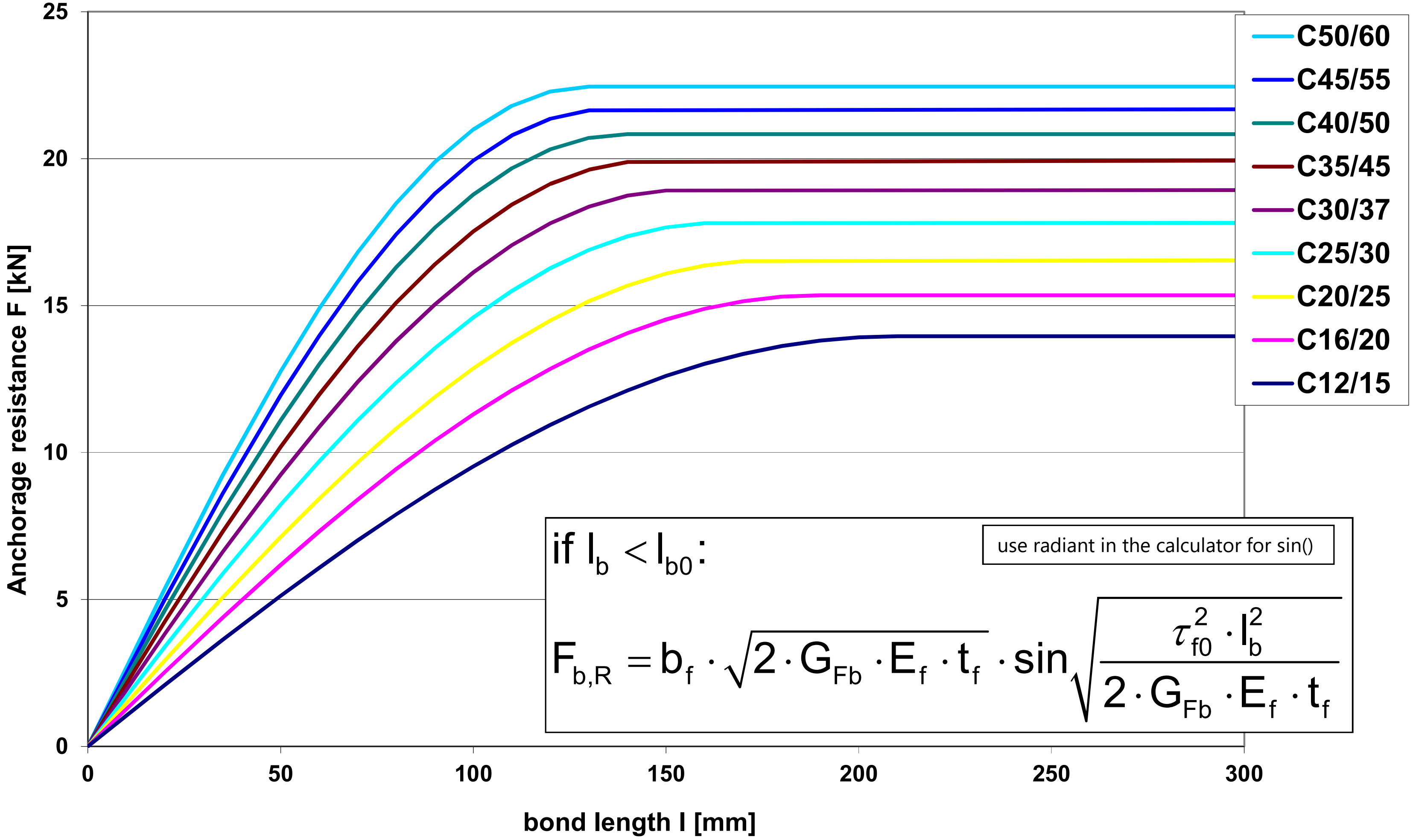
$$G_{Fb} = \frac{2.9}{8} = 0.36\frac{N}{mm}$$

$$\tau_{f0} = \frac{4}{3}\cdot 2.9 = 3.9MPa$$

$$F_{b0,R} = b_f \cdot \sqrt{2\cdot G_{Fb}\cdot E_f\cdot t_f} = 50\cdot\sqrt{2\cdot 0.36\cdot 165'000\cdot 1.2} = 18.9kN$$

$$\sigma_{b0,R} = \frac{F_{b0,R}}{A_f} = 315MPa \qquad \varepsilon_{b0,R} = \frac{\sigma_{b0,R}}{E_f} = 1.91‰$$


Anchorage resistance F_{b0} [kN]
active bond length l_{b0} [mm]
0
5
10
15
20
25
0
50
100
150
200
250
C12/15
C16/20
C20/25
C25/30
C30/37
C35/45
C40/50
C45/55
C50/60



use radiant in the calculator for sin()

if $l_b < l_{b0}$:

$$F_{b,R} = b_f \cdot \sqrt{2 \cdot G_{Fb} \cdot E_f \cdot t_f} \cdot \sin\sqrt{\frac{\tau_{f0}^2 \cdot l_b^2}{2 \cdot G_{Fb} \cdot E_f \cdot t_f}}$$

End anchorage in prSIA 166 (2023)

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$$F_{b,Rd} = F_{b0,Rd} = b_f\sqrt{2G_{Fcd}E_{fd}t_f}$$ wenn $l_{bd} \geq l_{b0d}$

$$F_{b,Rd} = F_{b0,Rd}\frac{l_{bd}}{l_{b0d}}\left(2-\frac{l_{bd}}{l_{b0d}}\right)$$ wenn $l_{bd} < l_{b0d}$

$$l_{b0d} = 2.5\sqrt{\frac{G_{Fcd}E_{fd}t_f}{\tau^2_{c,max,d}}}$$

$$G_{Fcd} = \eta_u \cdot \eta_l \cdot \frac{f_{hd}}{8} = \frac{1}{8} \cdot \frac{\eta_u \cdot \eta_l \cdot f_{hk}}{\gamma_h}$$ G_{Fcd} in N/mm f_{hk} in N/mm²

$$\tau_{c,max,d} = \eta_u \cdot \eta_l \cdot \frac{4}{3} \cdot f_{hd}$$

The reduction values η_u and η_l are tabulated in prSIA 166(2023), they consider environment and loading conditions.

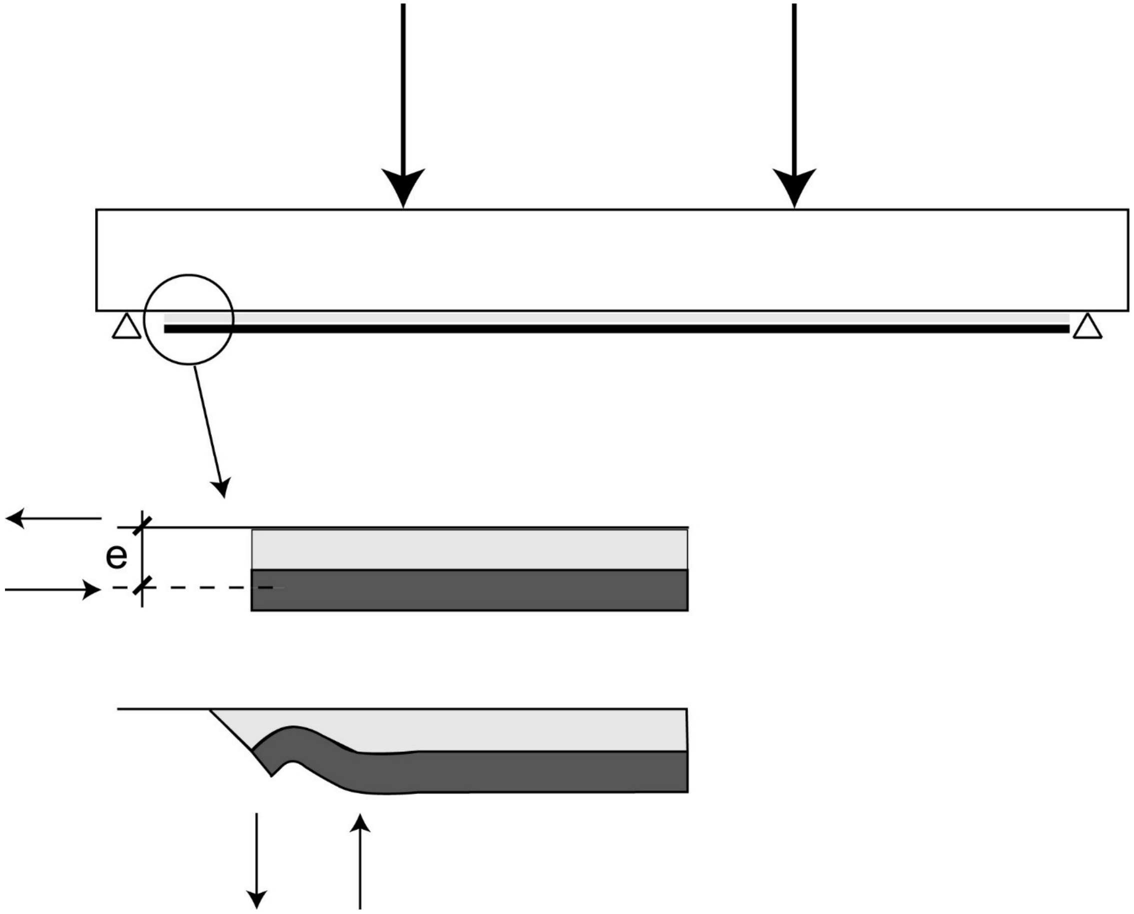
Design philosophy of preventing end anchorage failure

- SIA166: anchorage in the uncracked zone
- ACI:
  - If V_u > 0.67 V_c at strip end, then transverse reinforcement is necessary (they give a design equation for U-wrap reinforcement $A_{f,anchor}$)

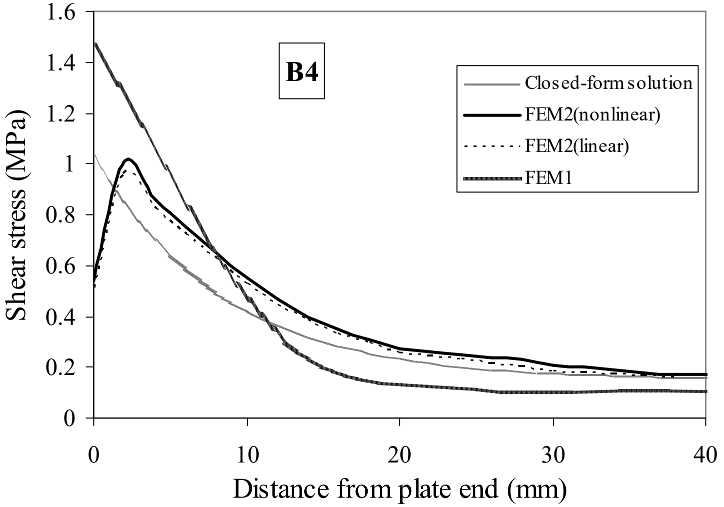
  or (instead of detailed analysis)

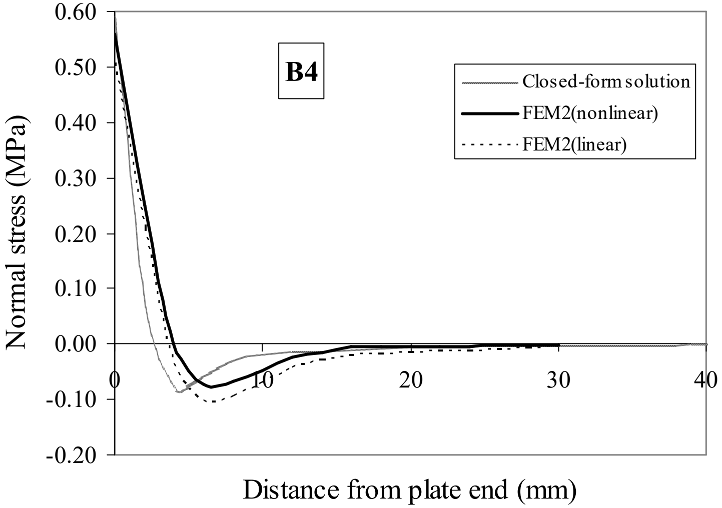
  - for simply supported beams: length l_{df} after last crack
- Elastic solutions for calculating shear and normal stresses at strip end

End anchorage, normal stresses at strip end



Shear and normal stresses at strip end





see:

Aram, M.R., C. Czaderski, and M. Motavalli, Debonding failure modes of flexural FRP-strengthened RC beams. Composites Part B: Engineering, 2008. 39(5): pp. 826-841.

Safety concept of the new prSIA 166 (2023)

Example: concrete tensile strength at the concrete surface

Mean value from tests f_{hm} = 4.0 GPa
Characteristic value (5% fractile) f_{hk} = 3.2 GPa

Design value:

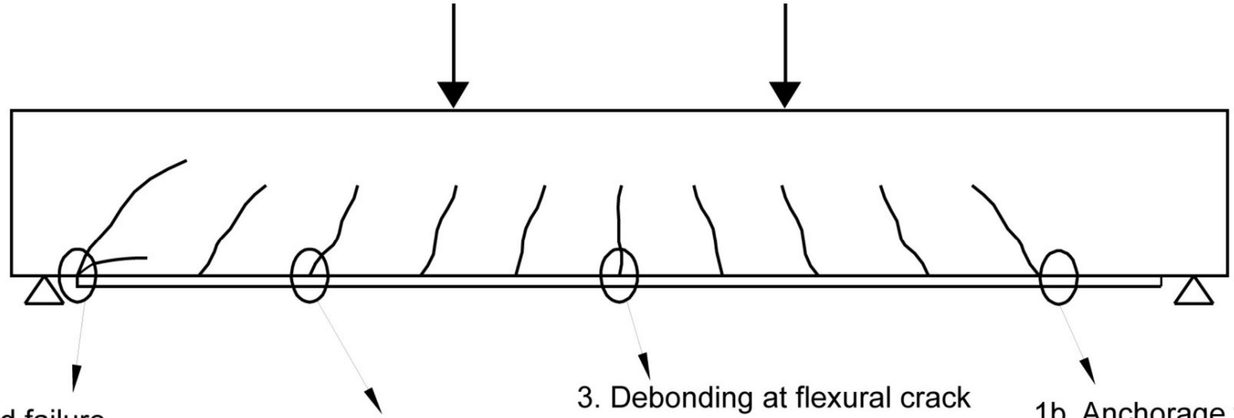
$$f_{hd} = \frac{f_{hk}}{\gamma_h} = \frac{3.2}{1.5} = 2.1MPa$$

γ_h = 1.5, from Tabelle 5 in prSIA166(2023), see slide 110

Design value of fracture energy:

$$G_{fcd} = \frac{f_{hd}}{8} = \frac{2.1}{8} = 0.26N/mm$$

Debonding failure modes



1a. Plate end failure,
concrete cover delamination

2. Debonding at shear crack and/or discontinuities

3. Debonding at flexural crack

1b. Anchorage failure at last crack

Example

Concrete C30/37

Sika CarboDur S512 tensile strength > 2800 MPa
tensile strain > 17‰

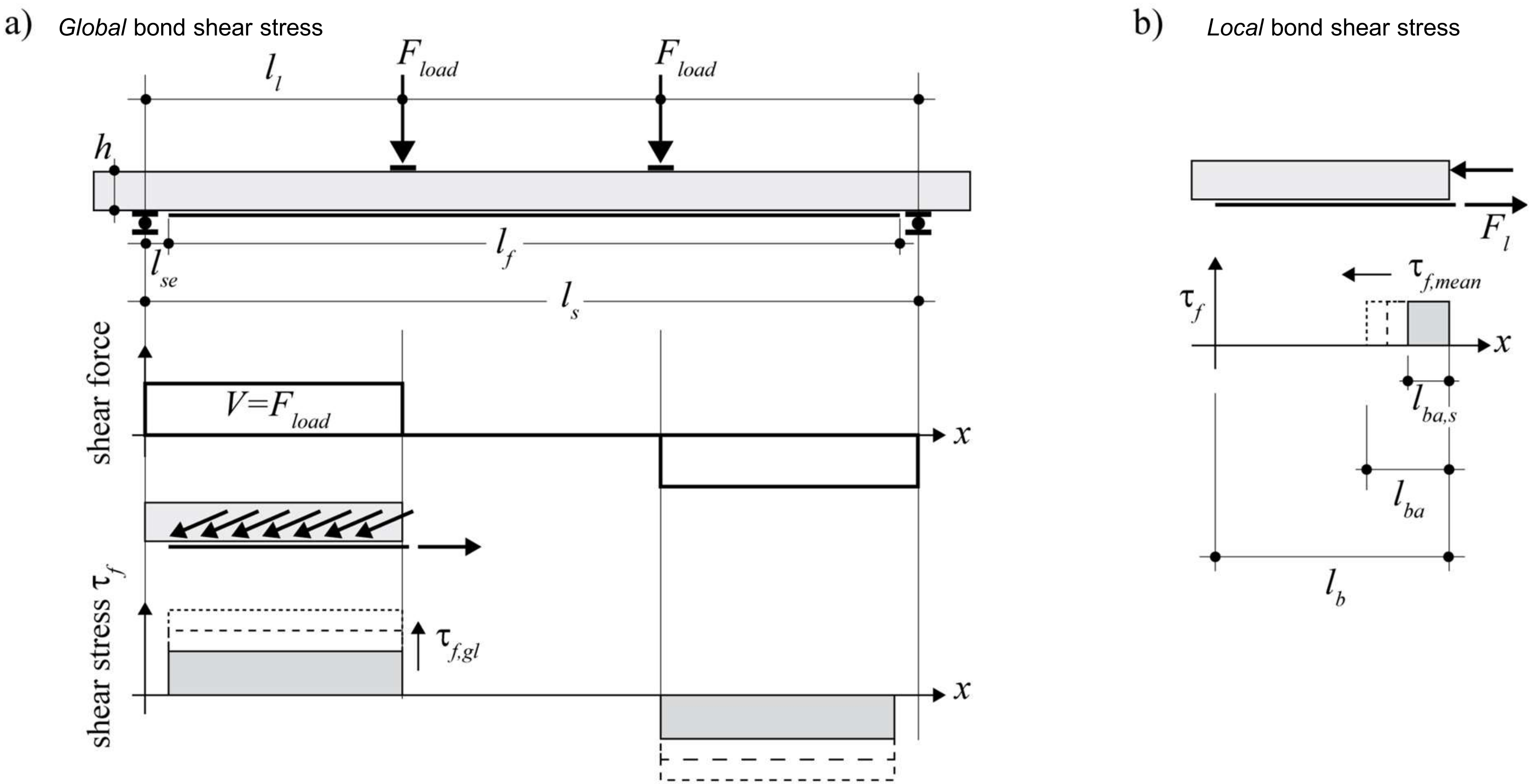
$$F_{b0,R} = b_f \cdot \sqrt{2 \cdot G_{Fb} \cdot E_f \cdot t_f} = 50 \cdot \sqrt{2 \cdot 0.36 \cdot 165'000 \cdot 1.2} = 18.9\text{kN}$$

$$\sigma_{b0,R} = \frac{F_{b0,R}}{A_f} = 315\text{MPa} \qquad \varepsilon_{b0,R} = \frac{\sigma_{b0,R}}{E_f} = 1.91\text{‰}$$

??

According to SIA166,

the maximum allowed strain is 8‰!

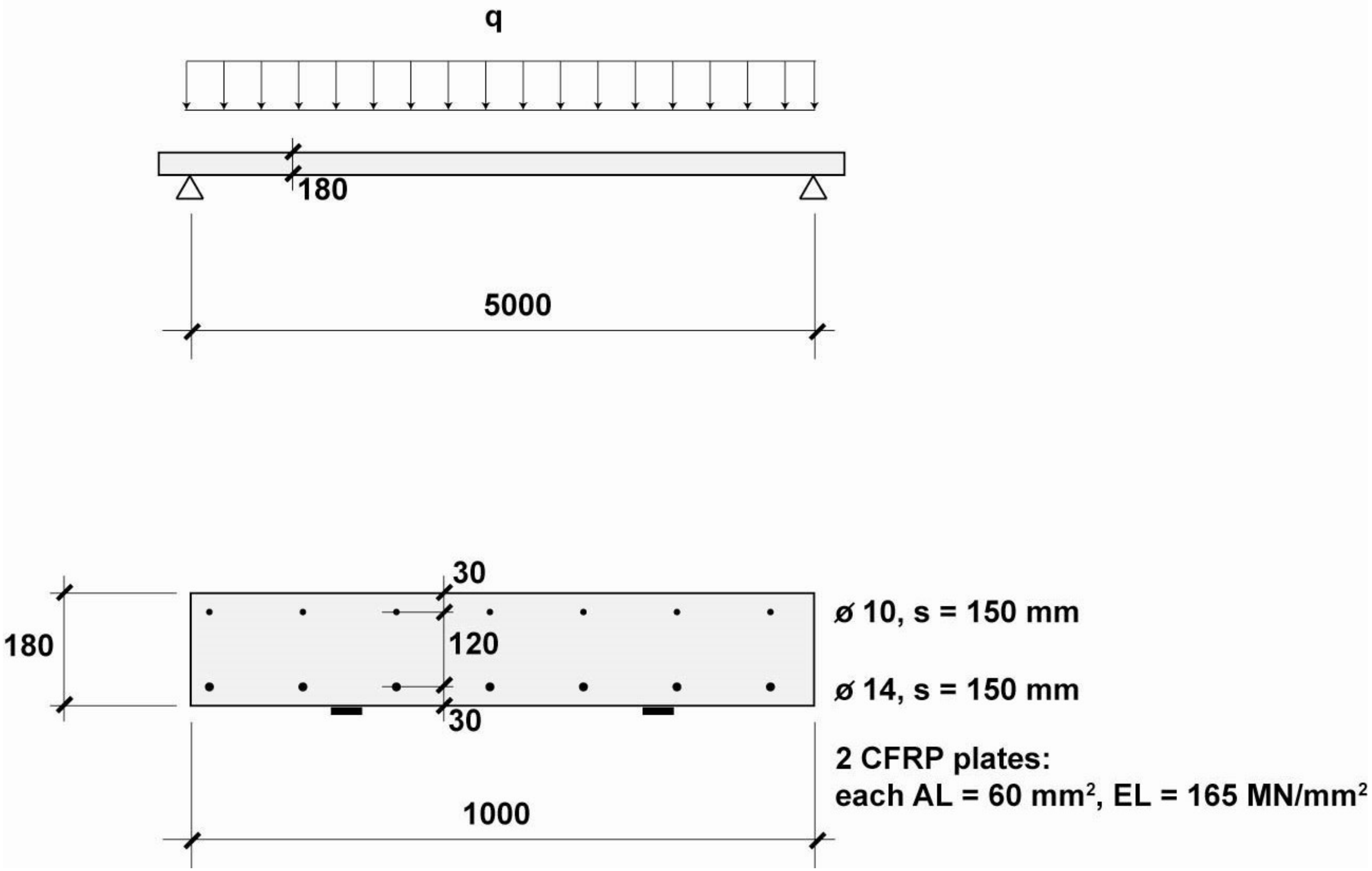


→ bond length constant,
bond shear stress increases

→ bond length increases,
bond shear stress constant

Figure 38 of PhD Thesis of C.Czaderski (Diss ETH No. 20504):

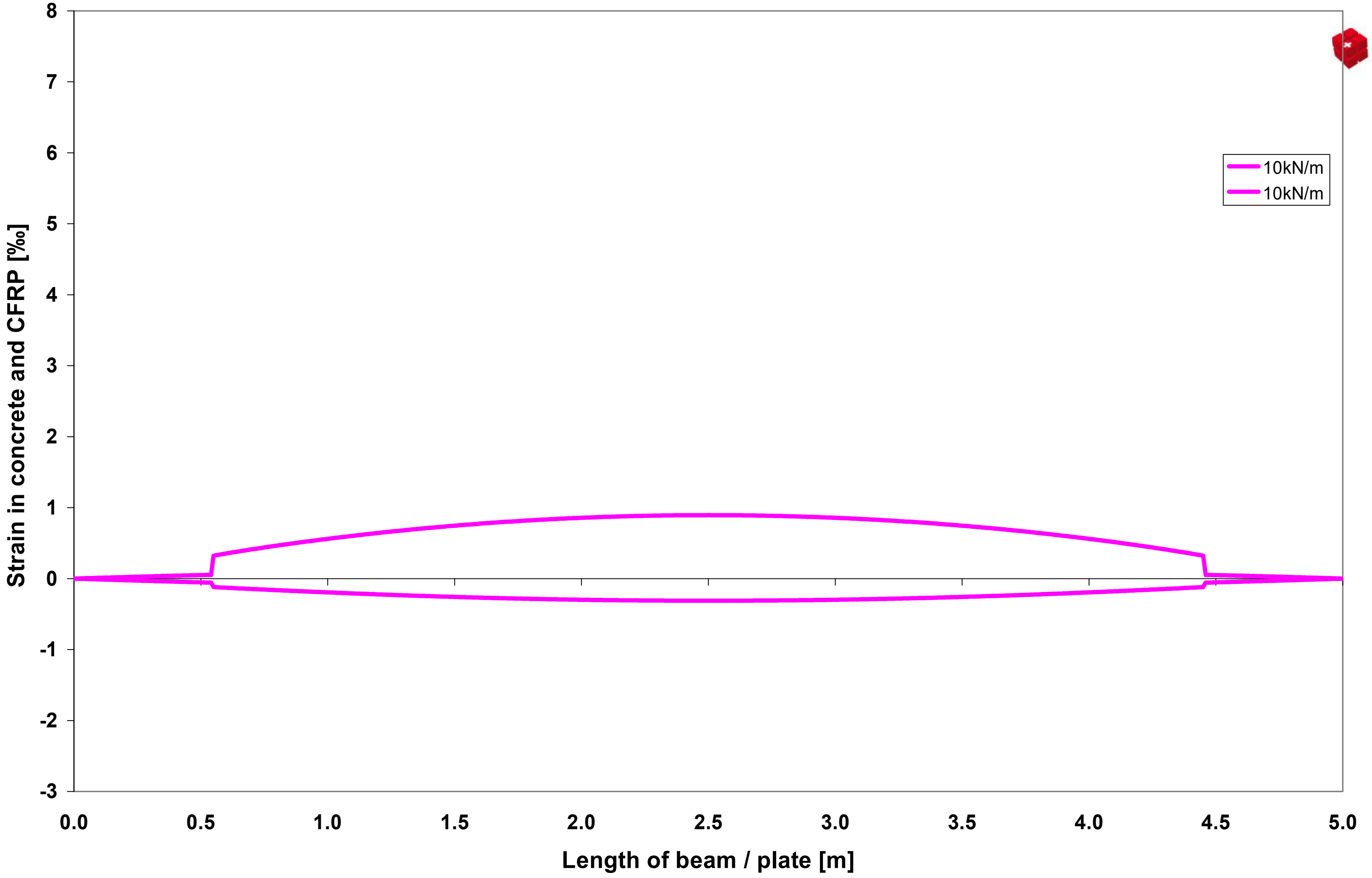
Simple supported plate

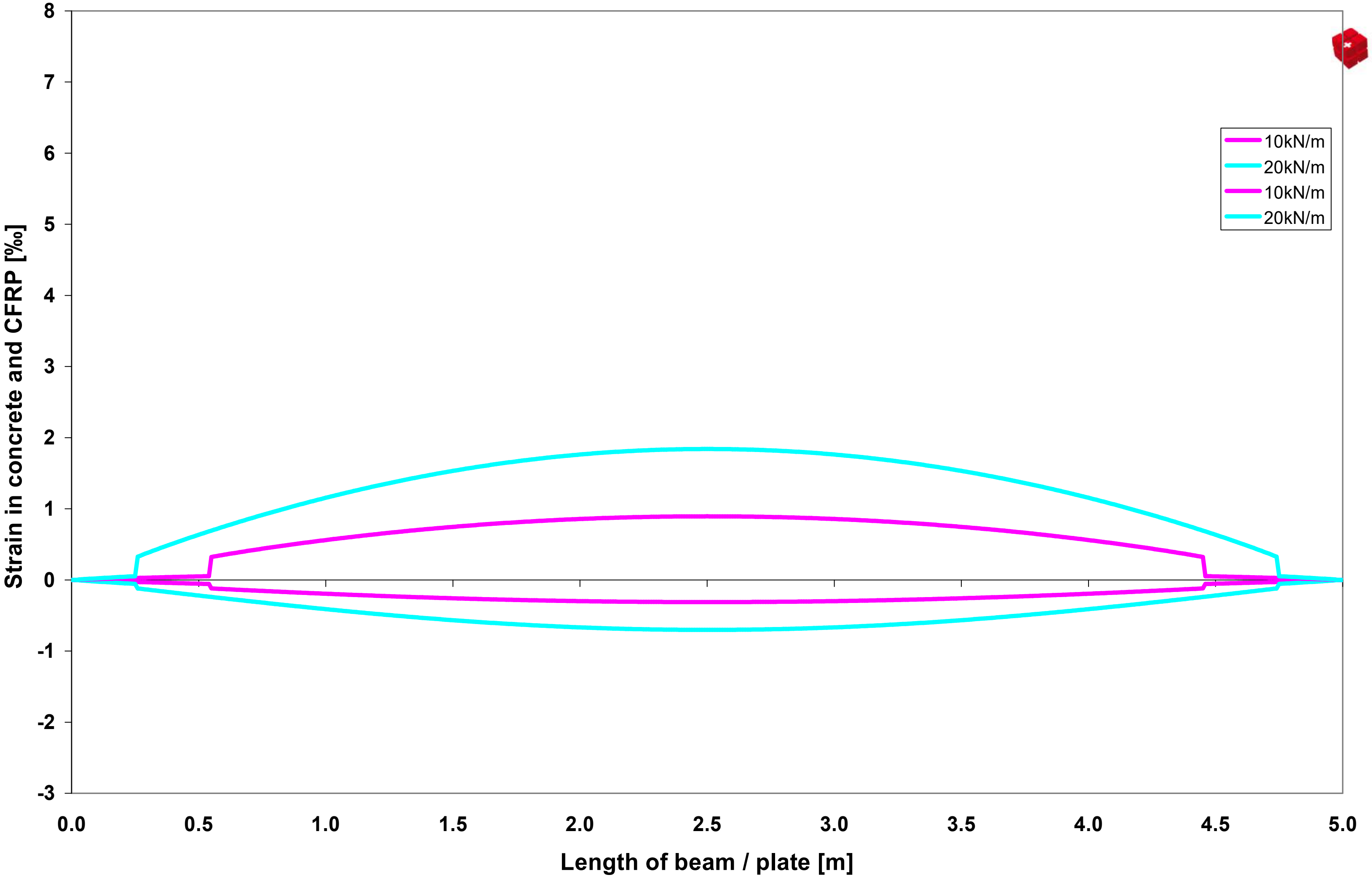


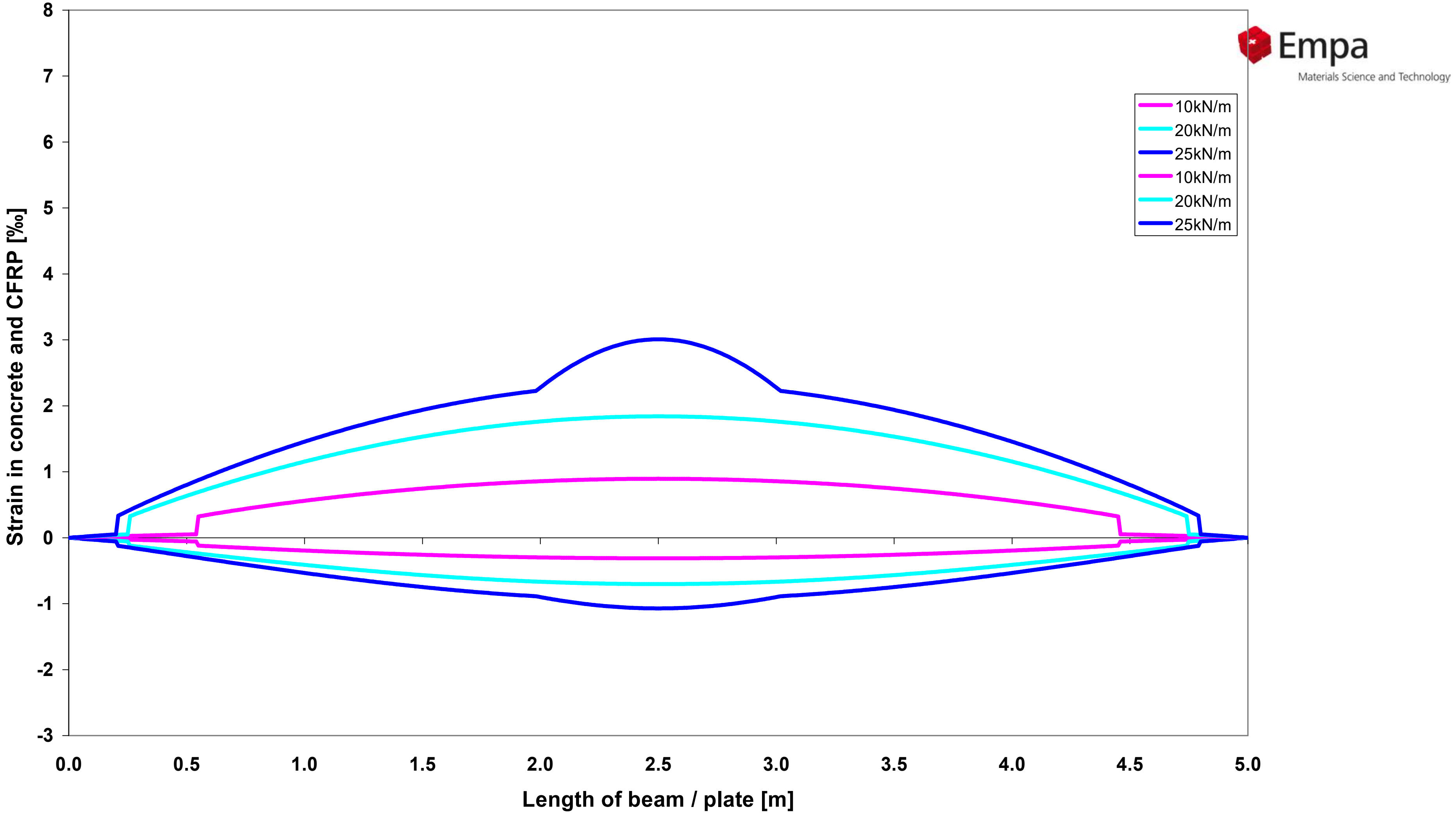
ø 10, s = 150 mm

ø 14, s = 150 mm

2 CFRP plates:
each AL = 60 mm², EL = 165 MN/mm²


8
7
6
5
4
3
2
1
0
-1
-2
-3
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Strain in concrete and CFRP [‰]
Length of beam / plate [m]
10kN/m
10kN/m


Strain in concrete and CFRP [‰]
8
7
6
5
4
3
2
1
0
-1
-2
-3
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Length of beam / plate [m]
10kN/m
20kN/m
10kN/m
20kN/m


Empa
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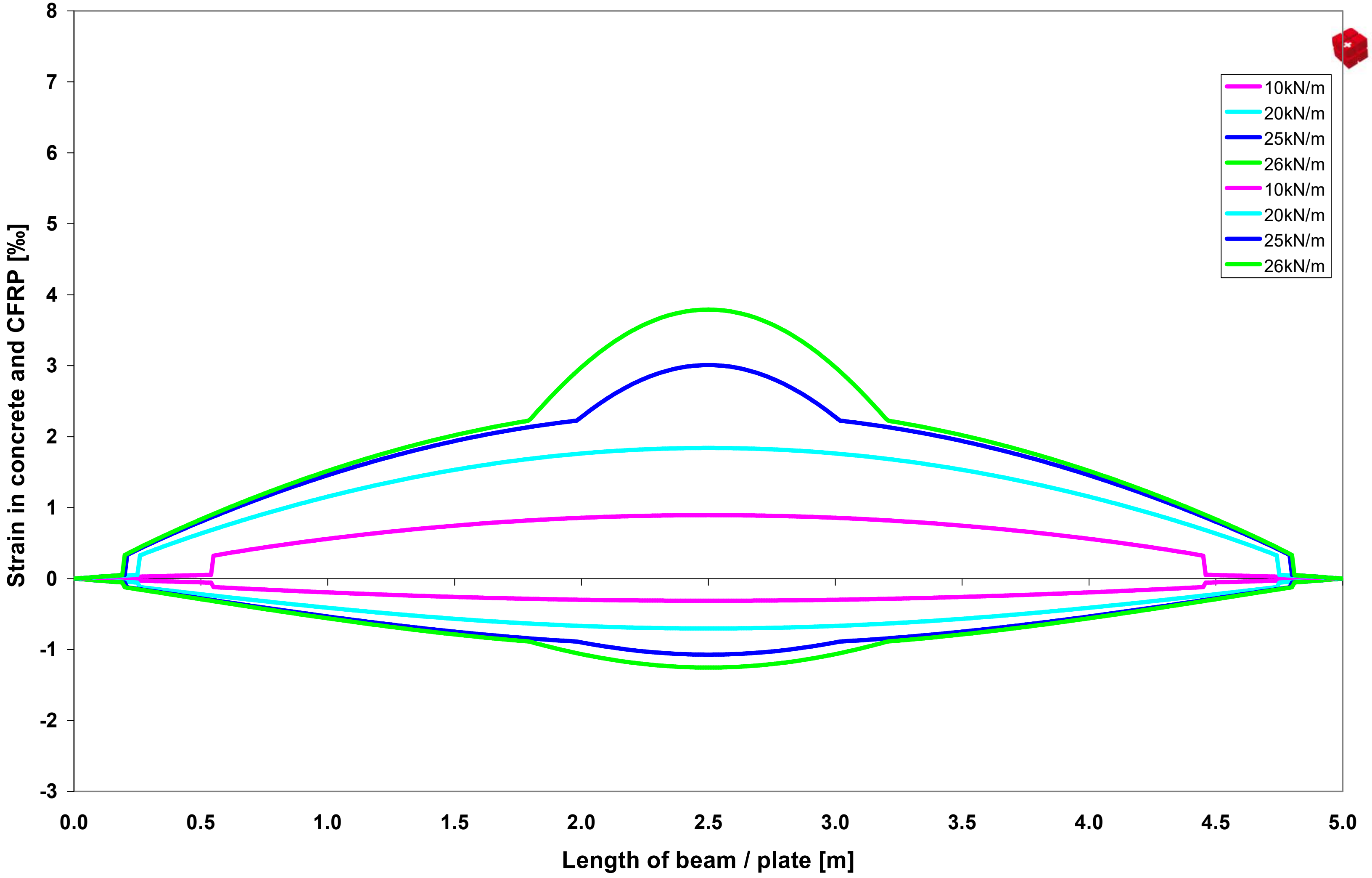
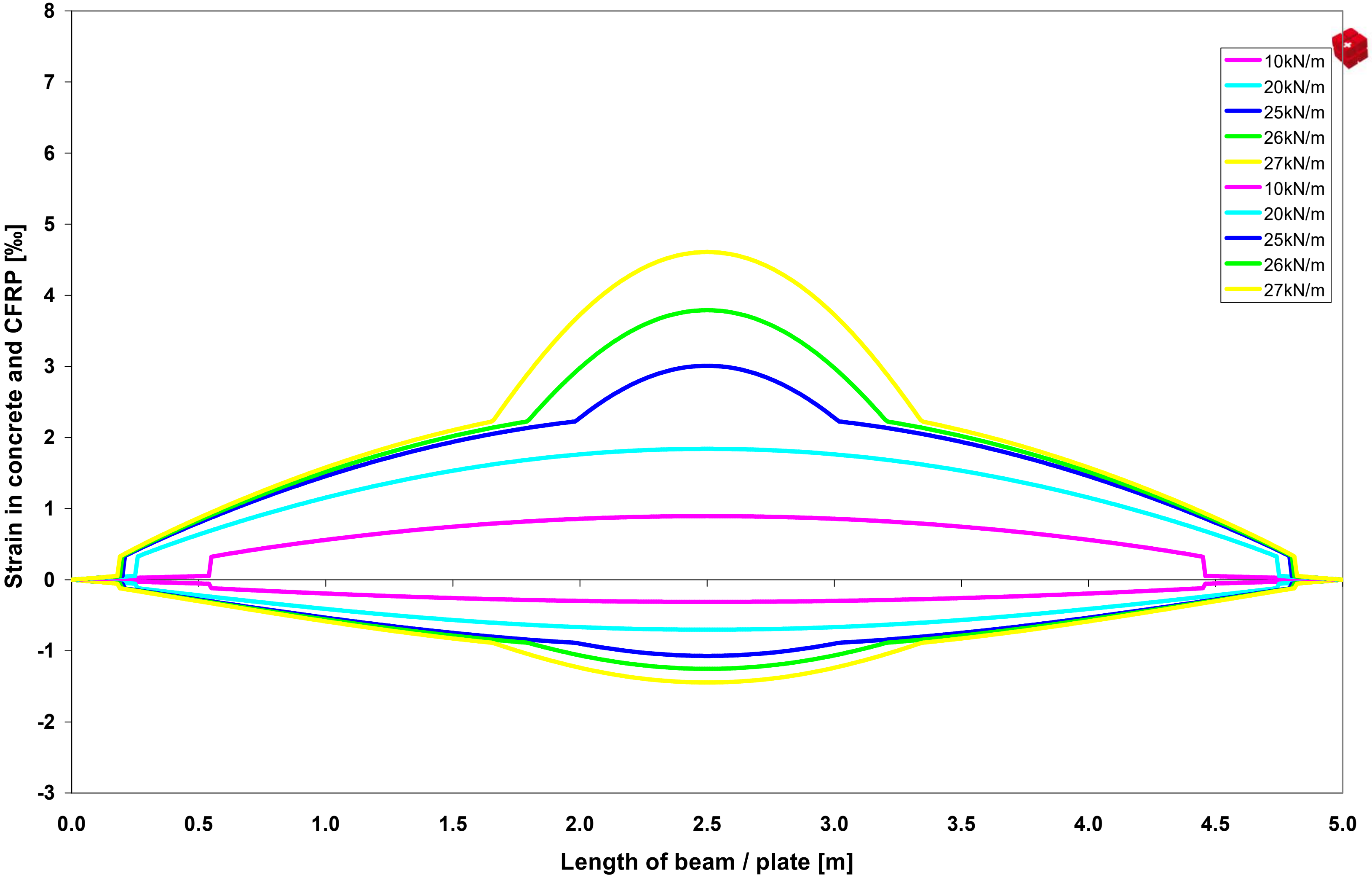
Strain in concrete and CFRP [‰]
8
7
6
5
4
3
2
1
0
-1
-2
-3
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Length of beam / plate [m]
10kN/m
20kN/m
25kN/m
10kN/m
20kN/m
25kN/m



8
7
6
5
4
3
2
1
0
-1
-2
-3
Strain in concrete and CFRP [‰]
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Length of beam / plate [m]
10kN/m
20kN/m
25kN/m
26kN/m
10kN/m
20kN/m
25kN/m
26kN/m



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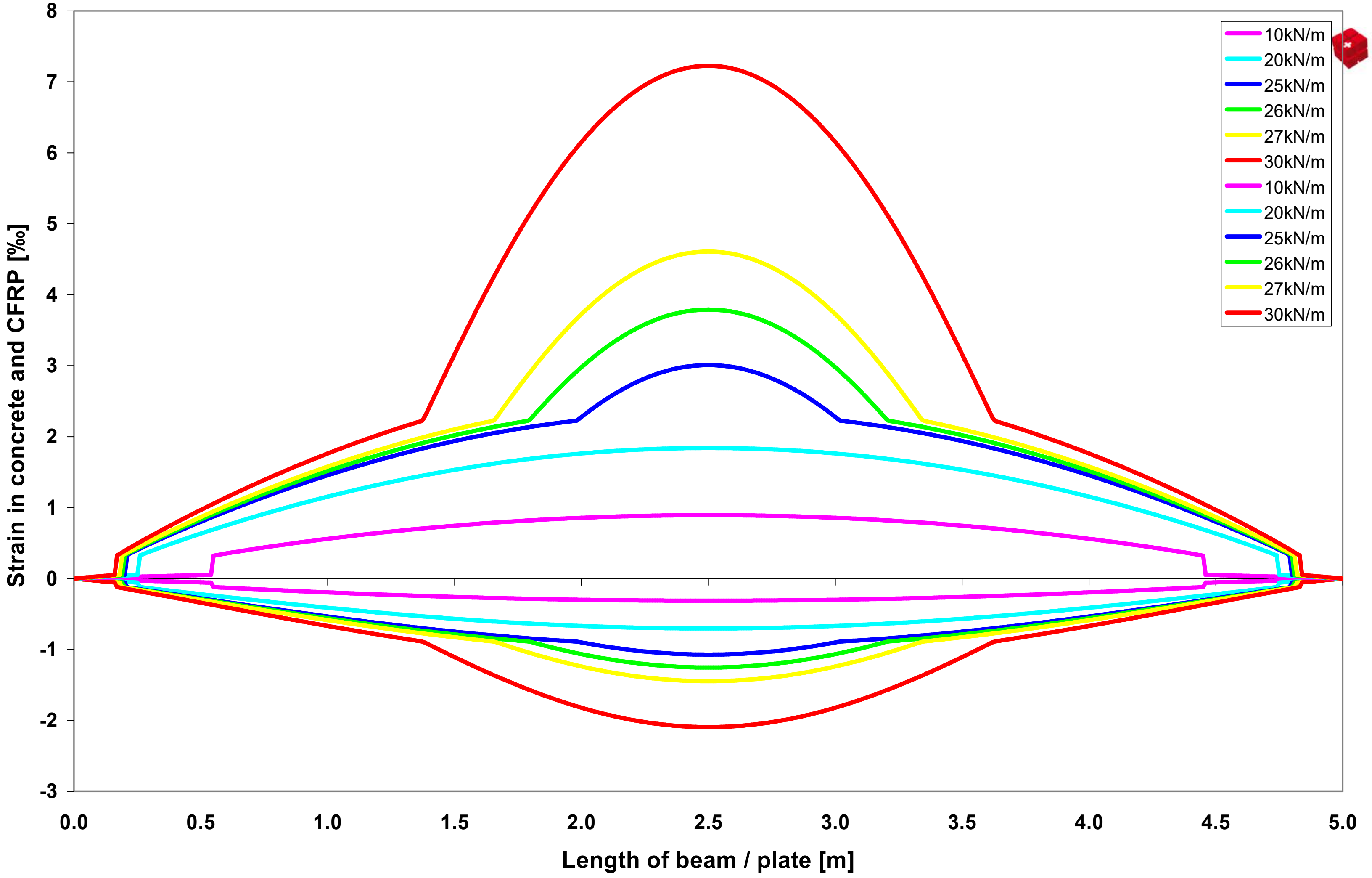
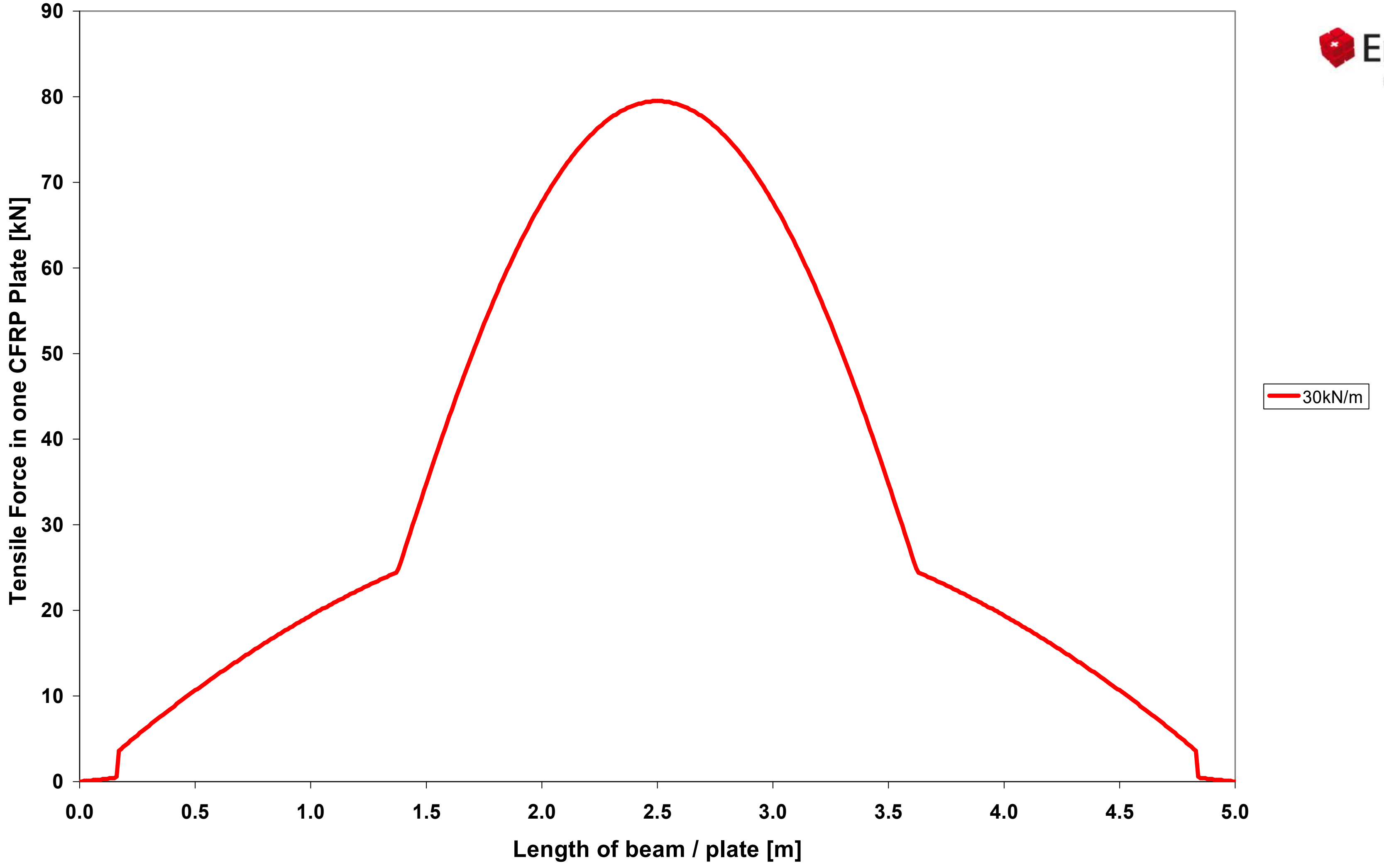
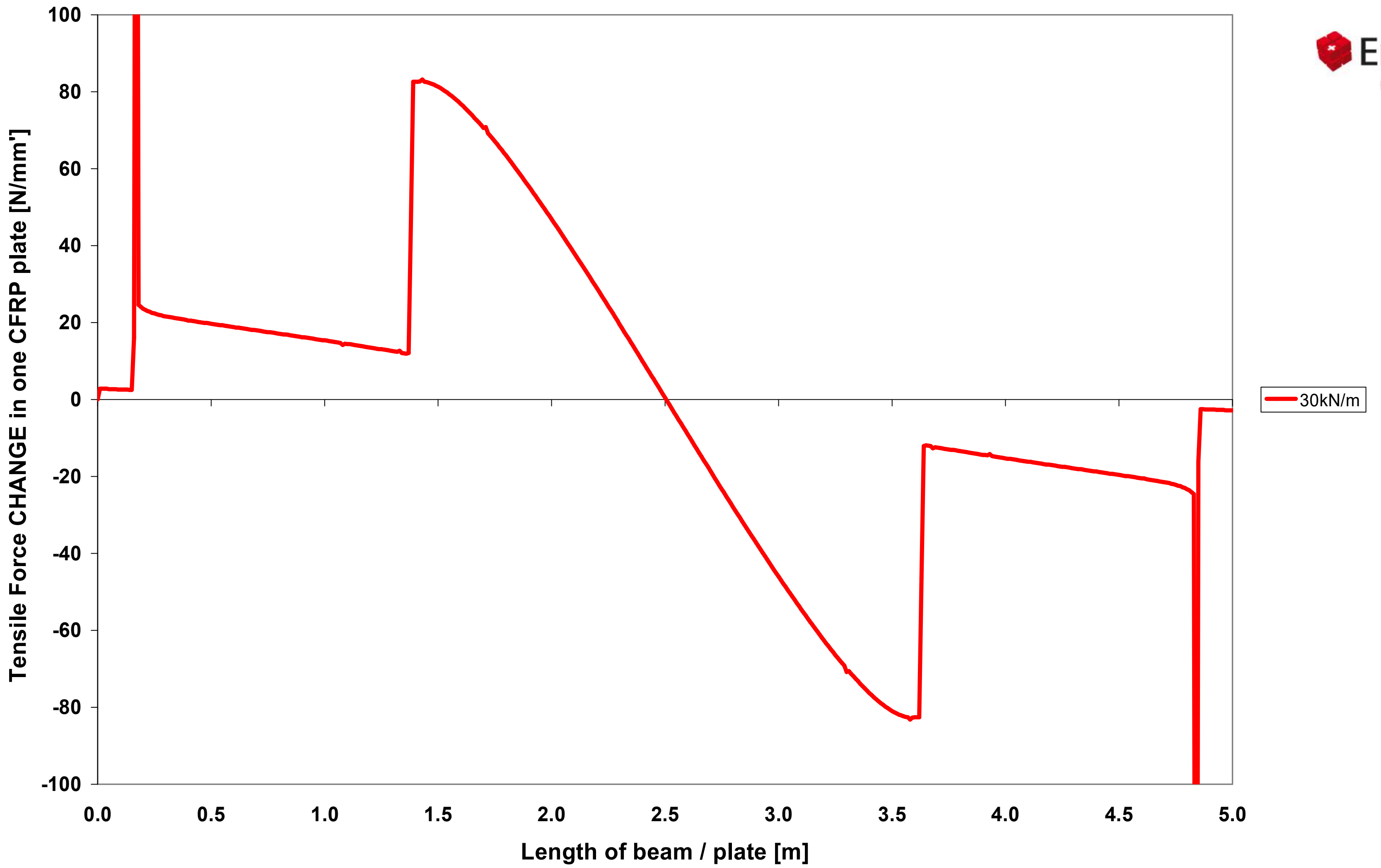


Empa
Materials Science and Technology


Tensile Force in one CFRP Plate [kN]
0
10
20
30
40
50
60
70
80
90
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Length of beam / plate [m]
30kN/m



100
80
60
40
20
0
-20
-40
-60
-80
-100
Tensile Force CHANGE in one CFRP plate [N/mm']
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Length of beam / plate [m]
30kN/m

Maximum Tensile Force CHANGE according to SIA 166

$$\left(\frac{\Delta F_f}{\Delta x}\right)_R = \tau_{f,\lim} \cdot b_l$$

$$\tau_{f,\lim} = 2.5 \cdot \tau_c = 2.5 \cdot 0.3 \cdot \sqrt{f_{ck}}$$ (τ_c from SIA 262)

Example

Concrete C30/37

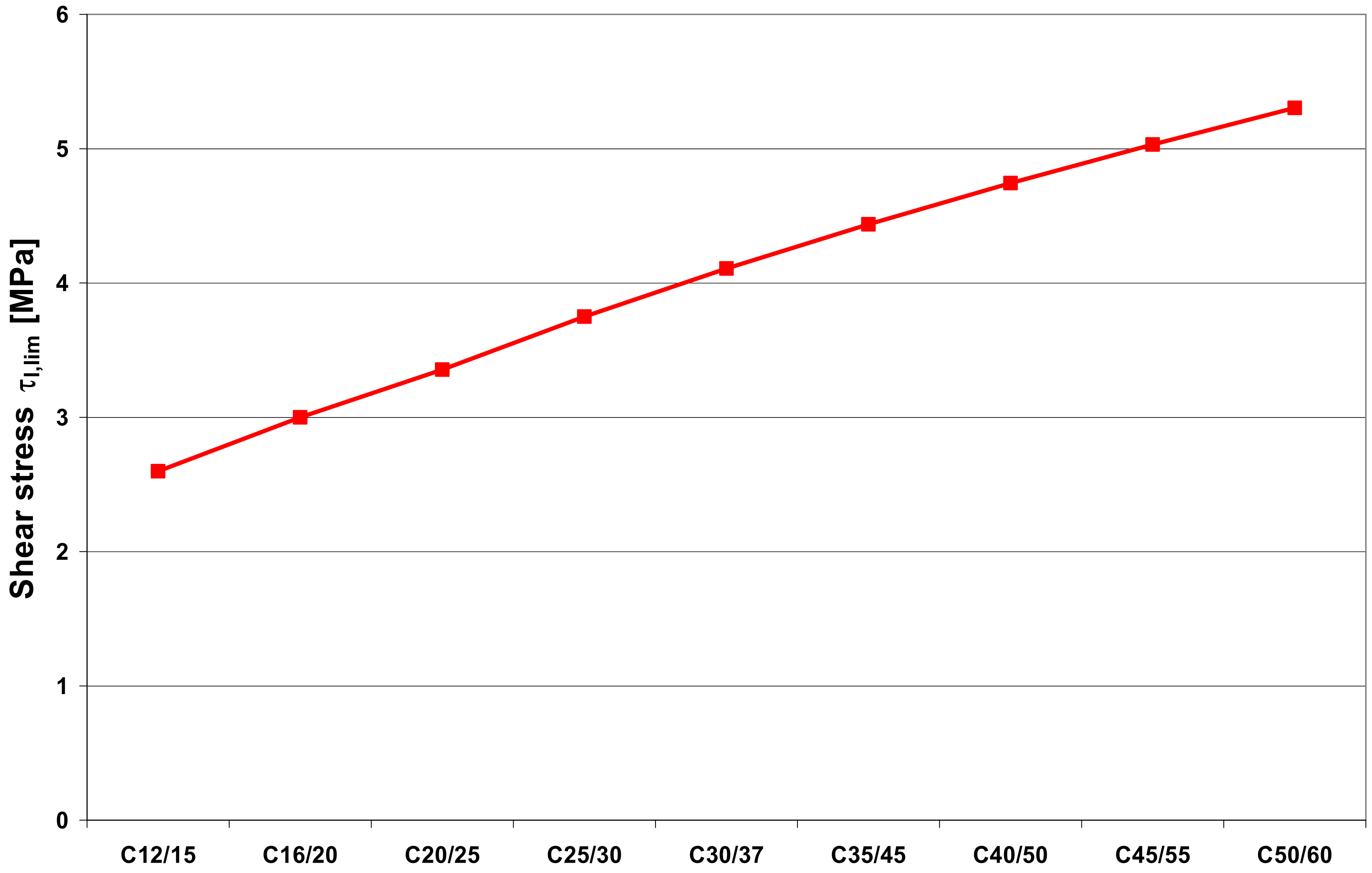
Sika CarboDur S512

$$\left(\frac{\Delta F_f}{\Delta x}\right)_R = \tau_{f,\lim} \cdot b_f = 205 N / mm$$

$$\tau_{f,\lim} = 2.5 \cdot \tau_c = 2.5 \cdot 0.3 \cdot \sqrt{30} = 4.1 MPa$$

In comparison, the local maximum bond shear stress (Slide 56): $\tau_{f0} = \frac{4}{3} \cdot 2.9 = 3.9 MPa$

please note: without safety factors!!


Shear stress $\tau_{l,lim}$ [MPa]
6
5
4
3
2
1
0
C12/15
C16/20
C20/25
C25/30
C30/37
C35/45
C40/50
C45/55
C50/60

Shear stress limitations, other guidelines

fib **Bulletin 14 (approach 3)**

$$\tau_b \leq f_{cb}$$

$$f_{cb} = 1.8 \cdot f_{ctk} = 1.8 \cdot 0.7 \cdot 2.9 = 3.7\text{MPa}$$

---

fib **Bulletin 14 (approach 2)**

$\Delta\sigma_f$ **(between cracks …)**

TR55

$$\tau = V_{add}\alpha_f A_f \frac{h\text{-}x}{I_{cs} b_a}$$

$$\tau_{lim,c} = 0.8 f_{ctk}$$ **outside yield zone**

$$\underbrace{\tau_t = t_f \left[\frac{\sigma_{fmax} - \sigma_{fy}}{\Delta x}\right]}_{\tau_m = \text{ mean shear stress}} + \underbrace{7.8\left[1.1\text{-}\frac{M_y}{M_{ed}}\right] f_{ctk}}_{\tau_{sc} = \text{additional shear stress due to stress concentration at flexural cracks}}$$

τ_m= mean shear stress

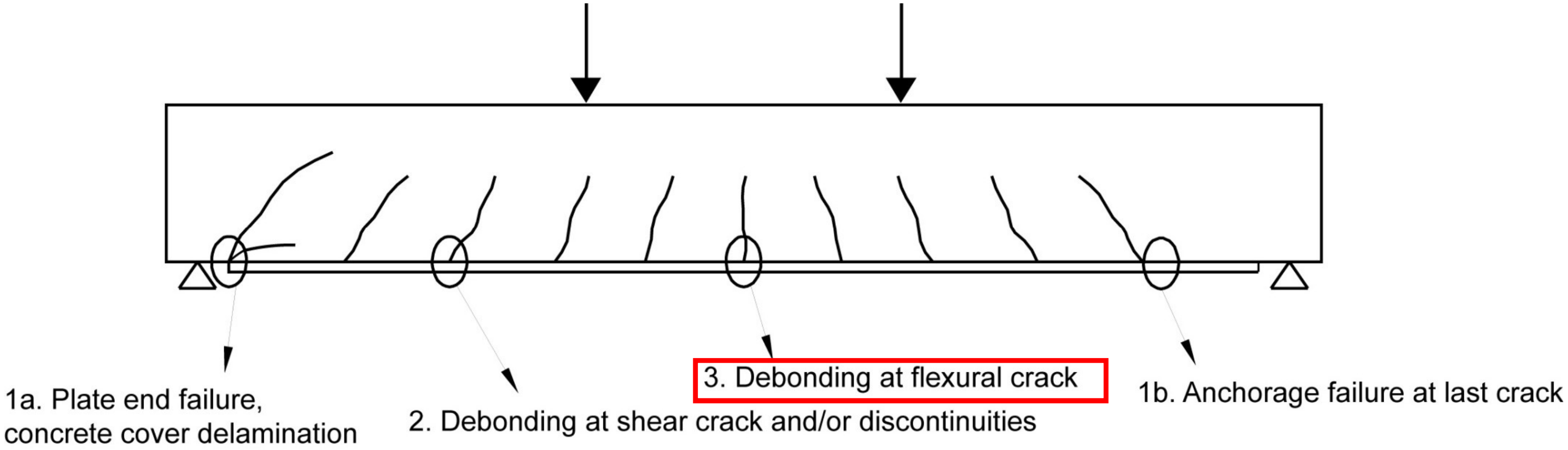
τ_{sc}=additional shear stress due to stress concentration at flexural cracks

$$\tau_{lim,y} = 4.5 f_{ctk}$$ **in the yield zone**

---

please note: without safety factors!!

Debonding failure modes



Maximum strain in the strip according to SIA 166

- Local debonding due to compatibility problems between strip and concrete at flexural cracks

$$F_{f,R} = A_f \cdot E_f \cdot \varepsilon_{f,lim} \leq A_f \cdot E_f \cdot \varepsilon_{fu}$$

$\varepsilon_{f,lim,d} = 8‰$ design value!

$\varepsilon_{fu} =$ supplier of the material

Maximum strain in strip, other guidelines

***fib* Bulletin 14 (approach 1)**

$$\varepsilon_{f,lim} = 6.5 - 8.5‰$$

ACI

$$\varepsilon_{fd} = 0.41\sqrt{\frac{f_c^{'}}{n \cdot E_f \cdot t_f}}$$

in SI units

(equation for DFM 2 and 3)

Italian Code

$$\varepsilon_{fdd} = k_{cr}\sqrt{\frac{0.06k_b\sqrt{f_{ck}f_{ctm}}}{E_f \cdot t_f}}$$

With k_{cr} = 3.0

(equation for DFM 2 and 3)

TR55

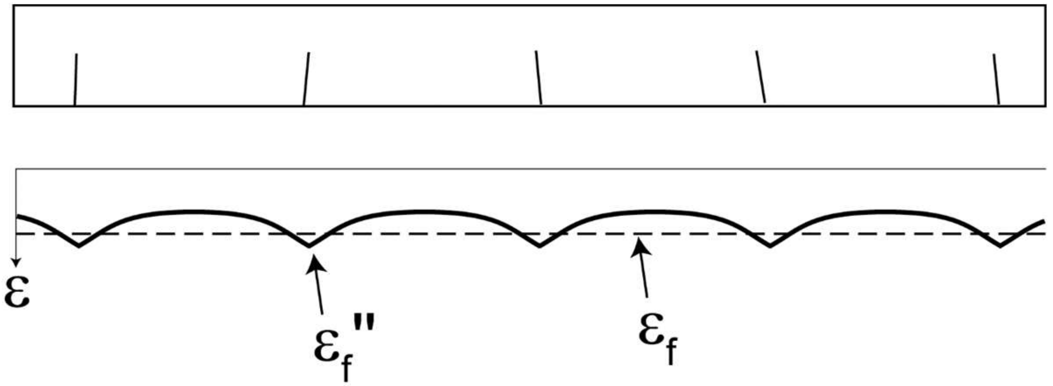
$$\varepsilon_{fmax} + \underbrace{0.114\frac{\tau_{sc}}{\sqrt{E_{fd}t_f}}}_{\tau_{sc}=\text{localised strain increase at flexural cracks}} \leq \varepsilon_{fd}$$

please note: without safety factors!!

Cross-section analysis

- Cross-section analysis to determine the strains in the CFRP strips so that the debonding failure modes 1, 2 and 3 according to SIA 166 can be verified.
- Bond factor according to SIA166: $\kappa_s=0.7$, $\kappa_f=0.9$

$$\kappa = \frac{\varepsilon}{\varepsilon''}$$



- Equilibrium with ε''
- Compatibility with ε

Equations for cross-section analysis

Equilibrium: $C = T \rightarrow F_c + F_{s'} = F_s^{"} + F_f^{"}$

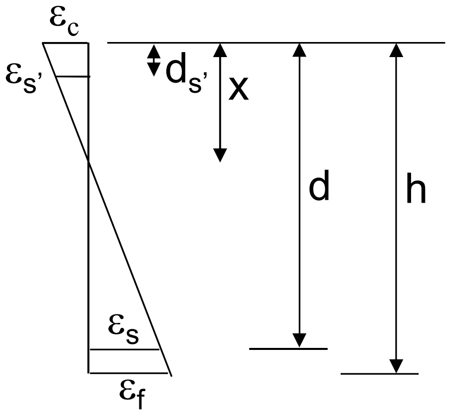
Tensile force: $\left\{ T = F_s^{"} + F_f^{"} = \frac{\varepsilon_s}{\kappa_s} E_s A_s + \frac{\varepsilon_f}{\kappa_f} E_f A_f \right.$

Compression force: $\left\{ \begin{array}{l} F_{s'} = \varepsilon_{s'} E_s A_{s'} \\ F_c = \text{ next slides} \end{array} \right.$

Compatibility:

$$\frac{\varepsilon_c}{x} = \frac{\varepsilon_f}{h-x}$$

$$\frac{\varepsilon_c}{x} = \frac{\varepsilon_s}{d-x}$$



$$\frac{\varepsilon_c}{x} = \frac{\varepsilon_{s'}}{d_{s'}-x}$$

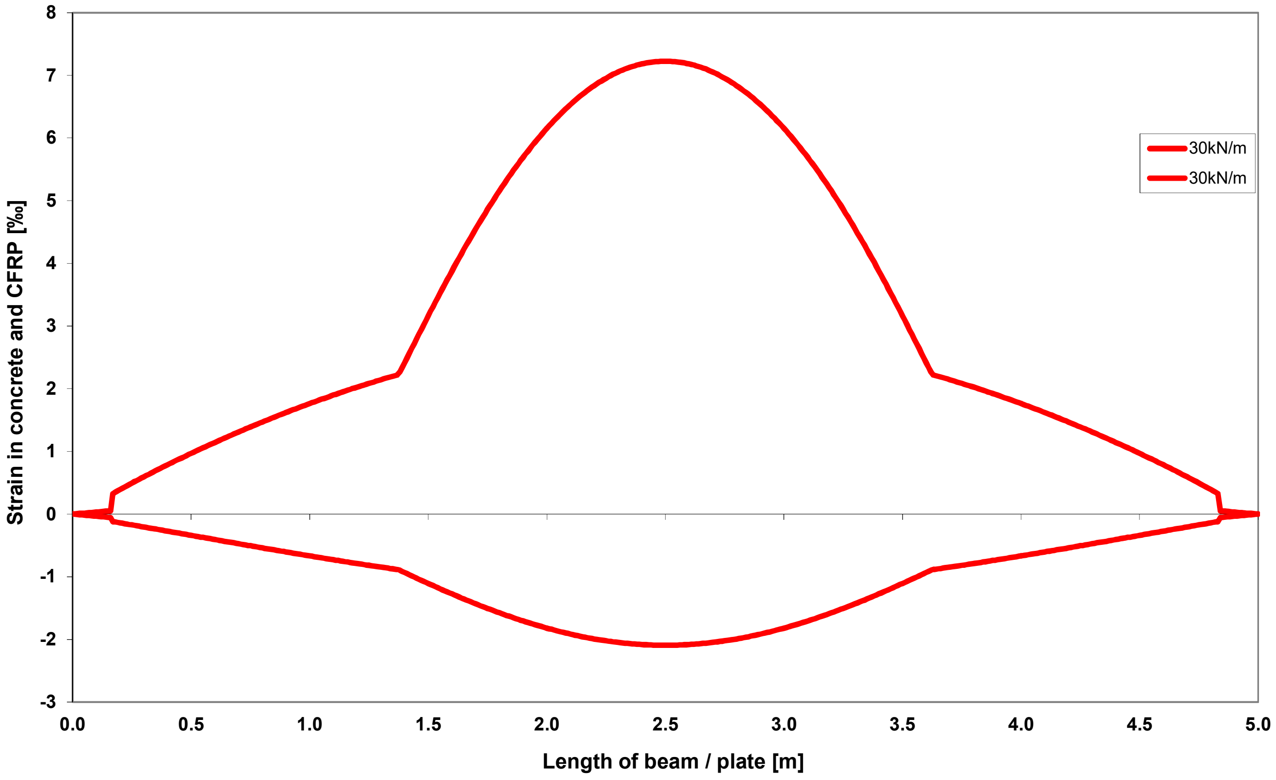
→ one equation with unknowns ε_c and x

Cross-section analysis

- Excel
- Matlab
- Software programs e.g. FAGUS
- …

Summary of calculation procedure: first step

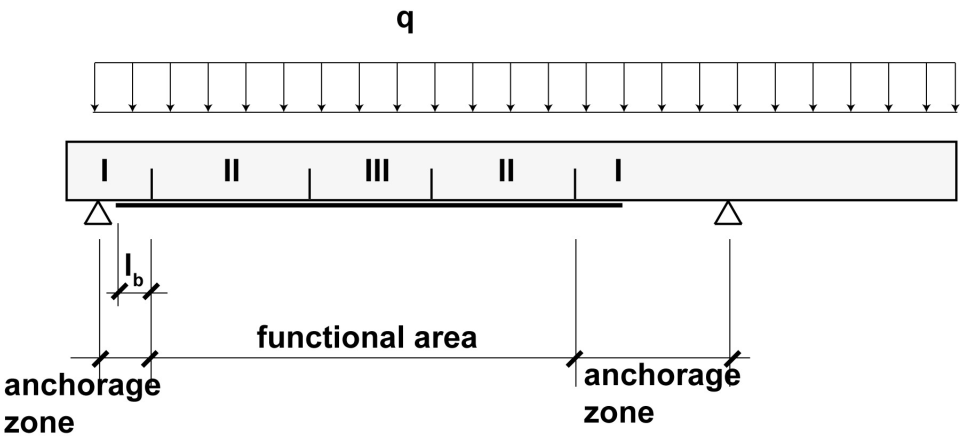
Cross-section analysis to determine the strain and forces along the structure for an assumed maximum load



Check debonding failure mode 1: end strip failure

Determine the location of last crack and define anchorage zone

Compare the existing force in the strip at the last crack with anchorage resistance which can be anchored in the anchorage zone

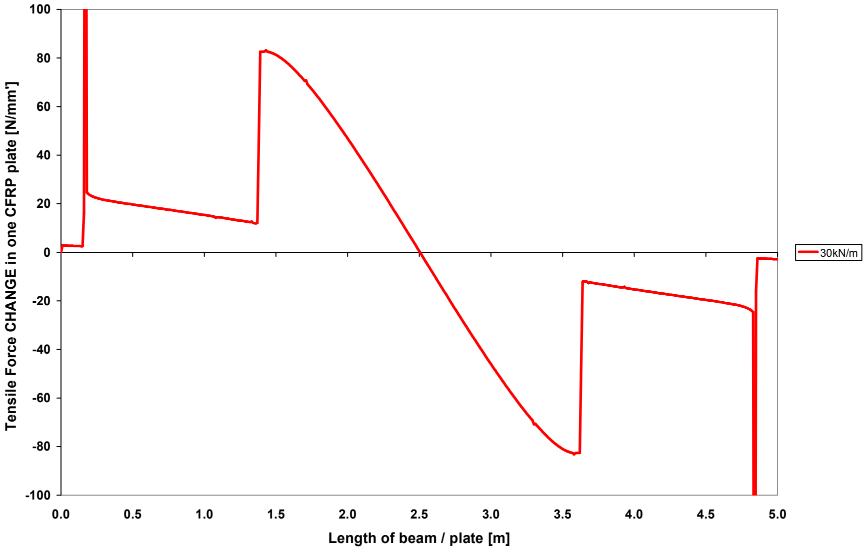


- I: uncracked cross-section
- II: cracked cross-section, internal steel in elastic state
- III: cracked cross-section, internal steel in yielding state

Check debonding failure mode 2:
Debonding at strong strain increase in strip

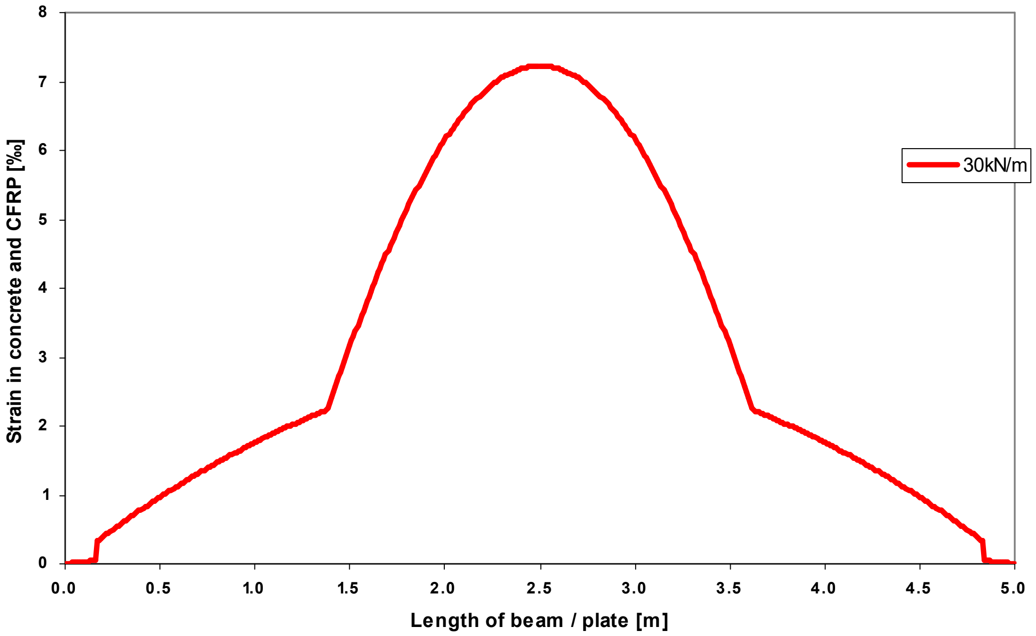
Calculation of shear stress between strip and concrete (force change) and comparison with bond shear strength

$$\tau_f = \frac{\Delta F_f}{\Delta x \cdot b_f}$$



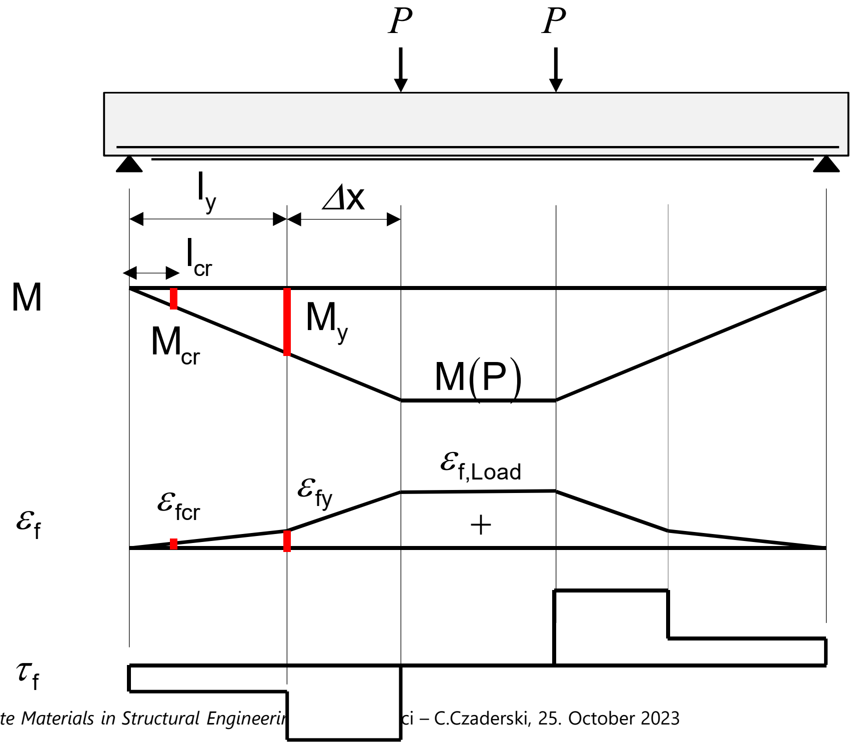
Check debonding failure mode 3: at flexural cracks

Compare maximum existing strain with maximum admissible strain



Calculation procedure for a four point beam

1. Calculation cracking moment M_{cr} and yielding moment M_y with cross-section analysis (CSA)
   - Strip strain at last crack and at the point of steel yielding
2. Assumption of a failure load P_{ult} (larger as P_y), then:
   - with CSA calculation of the maximum strain in the strip
   - calculation of the location of the last crack l_{cr} and the location of M_y (l_y *und* Δx)
   - with this $\Delta\varepsilon_f$ and τ_f
3. The three SIA verifications and assumption of a new failure load P_{ult} etc.



$$\Delta\varepsilon_f = \varepsilon_{f,Load} - \varepsilon_{fy}$$

$$\tau_f = \frac{\Delta\varepsilon \cdot E_f \cdot A_f}{\Delta x \cdot b_f}$$

Summary of the Equations from SIA

$$G_{Fb} = \frac{f_{ctH}}{8}\left[\frac{N}{mm}\right]$$

$$F_{b0,R} = b_f\sqrt{2G_{Fb}E_f t_f}$$

$$\tau_{f0} = \frac{4}{3}\cdot f_{ctH}\,[MPa]$$

$$f_{ctH} \approx f_{ctm} = 0.3\cdot f_{ck}^{2/3}$$

$$l_{b0} = \frac{\pi}{2}\cdot\sqrt{2\cdot\frac{G_{Fb}\cdot E_f\cdot t_f}{\tau_{f0}^2}}$$

if $l_b < l_{b0}$:

use radiant in the calculator for sin()

$$F_{b,R} = b_f\cdot\sqrt{2\cdot G_{Fb}\cdot E_f\cdot t_f}\cdot\sin\sqrt{\frac{\tau_{f0}^2\cdot l_b^2}{2\cdot G_{Fb}\cdot E_f\cdot t_f}}$$

$$\tau_{f,lim} = 2.5\cdot\tau_c = 2.5\cdot 0.3\cdot\sqrt{f_{ck}}$$

$$\left(\frac{\Delta F_f}{\Delta x}\right)_R = \tau_{f,lim}\cdot b_f$$

Summary of the three SIA 166 verifications

1. End strip debonding failure at the last crack

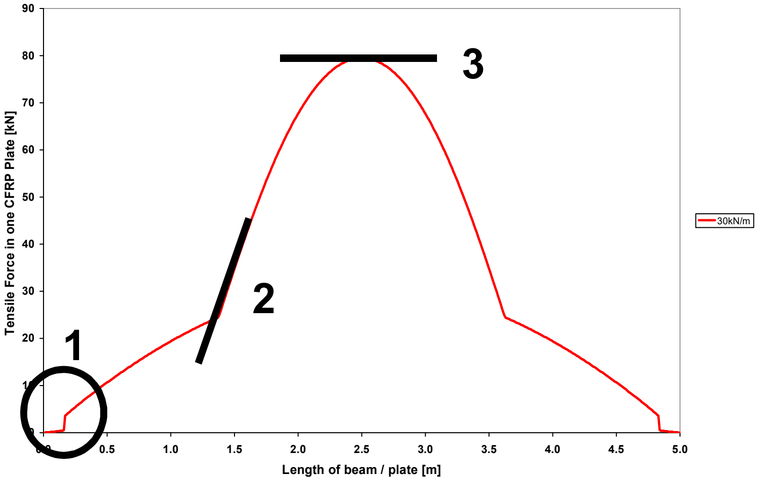
$$\boxed{F_{f\,cr} \leq F_{b,R}}$$

2. Debonding at strong strain increase in strip

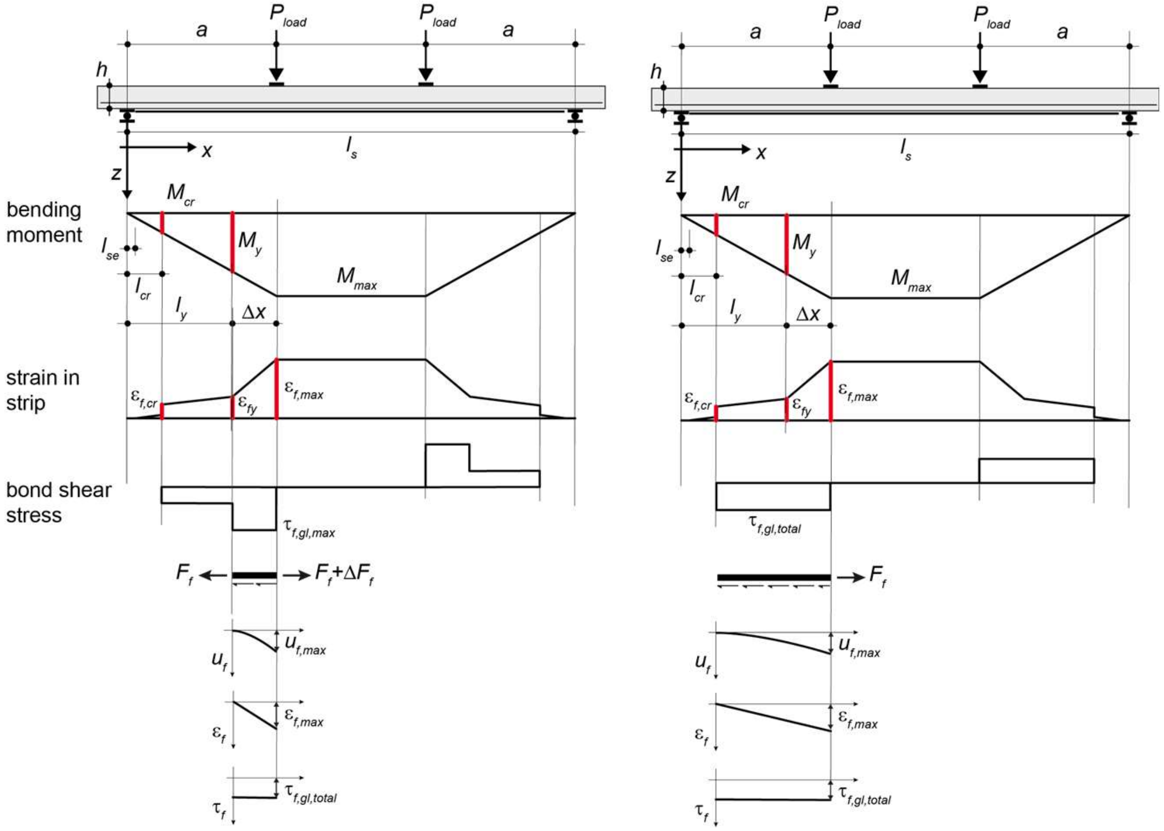
$$\boxed{\left(\frac{\Delta F_f}{\Delta x}\right) \leq \left(\frac{\Delta F_f}{\Delta x}\right)_R = \tau_{f,lim} \cdot b_f}$$

3. Debonding at flexural cracks

$$\boxed{\varepsilon_f \leq \varepsilon_{f,lim,d} = 8‰}$$



Note: SIA166 is under revision, a new approach for debonding mode verifications



$\varepsilon_{f,d} = 8‰$ $\qquad$ $\varepsilon_{f,y} \approx 3.5‰$

$$\tau_{f,gl,max} = \tau_{f,lim} = \frac{\left(\varepsilon_{f,max} - \varepsilon_{f,y}\right) E_f t_f}{a\left(1 - \dfrac{M_y}{M_{max}}\right)}$$

$$\left\{ \begin{array}{l} \boxed{\varepsilon_f \le \varepsilon_{f,lim,d} = 8‰} \\ \boxed{\left(\dfrac{\Delta F_f}{\Delta x}\right) \le \left(\dfrac{\Delta F_f}{\Delta x}\right)_R = \tau_{f,lim} \cdot b_f} \end{array} \right.$$

Combination of debonding failure modes 2 and 3 with one verification:

$$\varepsilon_{f,max} = \sqrt{\frac{2G_{F,total}}{E_f t_f}}$$

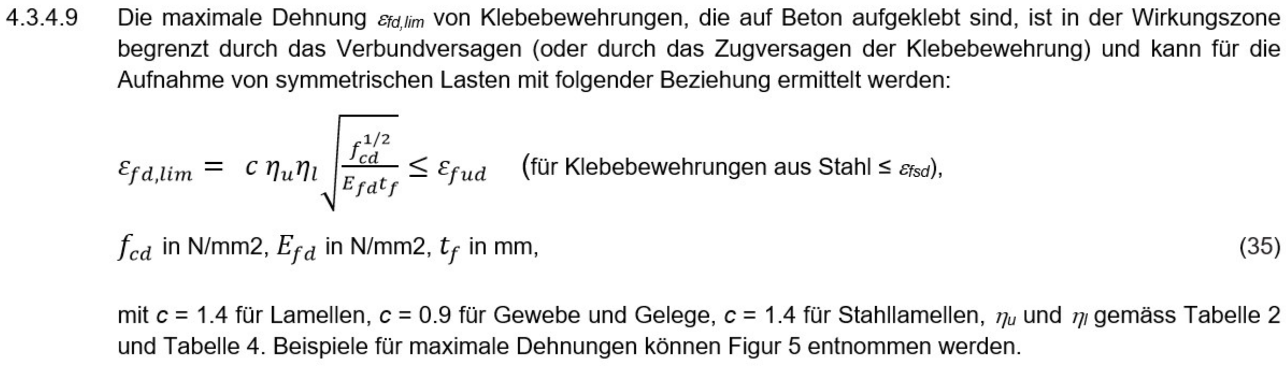
Flexural strengthening

- SIA 166 (2004) 3 Nachweise

1. Endverankerung Versagen
2. Maximale Zugkraftänderung
3. Beschränkung der Dehnung ε=0.8%

- prSIA 166 (2022) 2 Nachweise

1. Endverankerung Versagen Ziffer 4.3.2
2. Beschränkung der Dehnung Ziffer 4.3.4
3. ~~Beschränkung der Dehnung ε=0.8%~~



4.3.4.9 Die maximale Dehnung $\varepsilon_{fd,lim}$ von Klebebewehrungen, die auf Beton aufgeklebt sind, ist in der Wirkungszone begrenzt durch das Verbundversagen (oder durch das Zugversagen der Klebebewehrung) und kann für die Aufnahme von symmetrischen Lasten mit folgender Beziehung ermittelt werden:

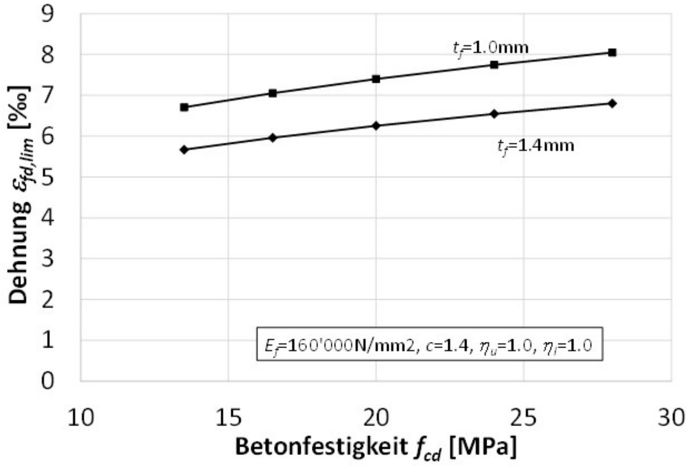
$$\varepsilon_{fd,lim} = c\,\eta_u\eta_l\sqrt{\frac{f_{cd}^{1/2}}{E_{fd}t_f}} \le \varepsilon_{fud}$$

(für Klebebewehrungen aus Stahl ≤ ε_{fsd}),

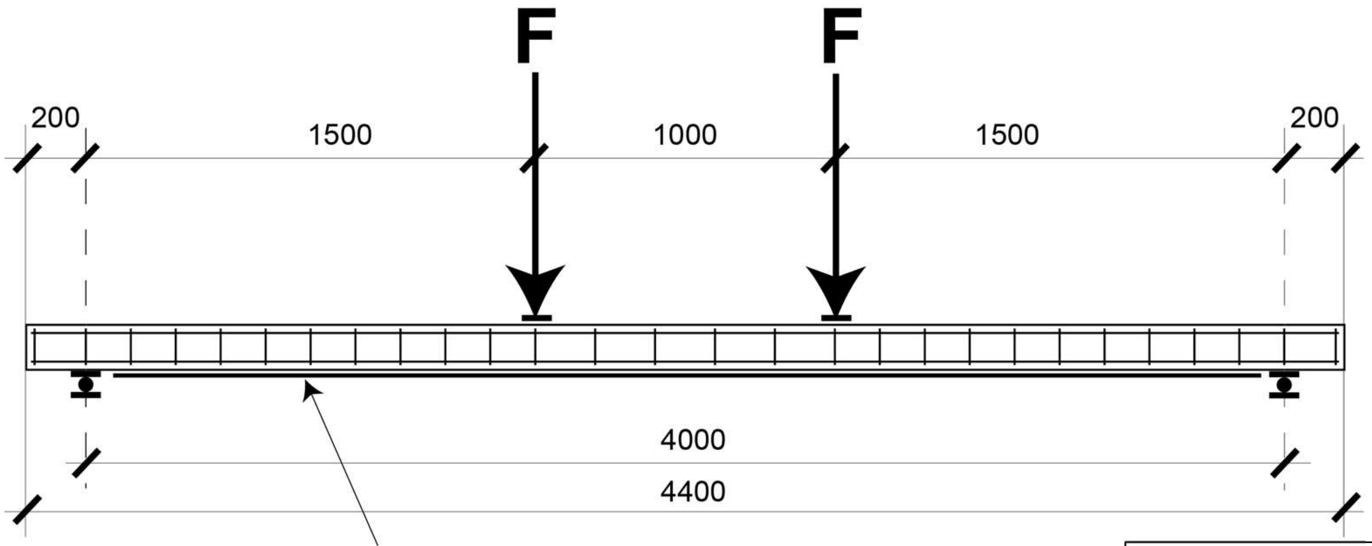
f_{cd} in N/mm2, E_{fd} in N/mm2, t_f in mm, (35)

mit c = 1.4 für Lamellen, c = 0.9 für Gewebe und Gelege, c = 1.4 für Stahllamellen, η_u und η_l gemäss Tabelle 2 und Tabelle 4. Beispiele für maximale Dehnungen können Figur 5 entnommen werden.

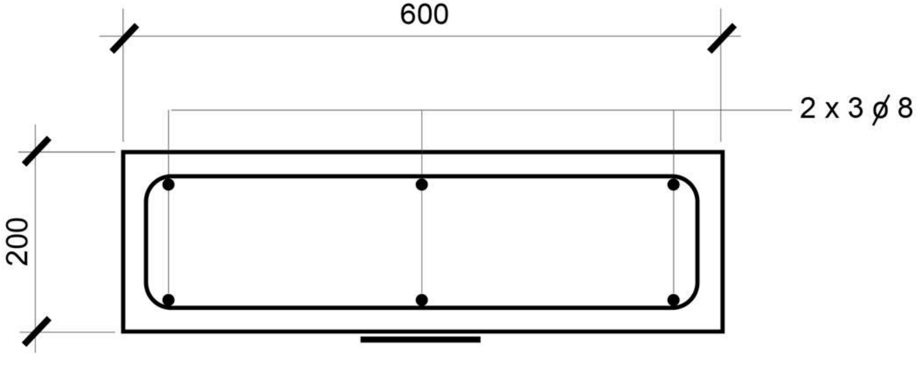
Figur 5: Beispiele für Bemessungswerte für Lamellen für die maximale D
Abhängigkeit der Betonfestigkeit.



Example



CFRP-strip 100x1.2mm, length 3400mm



concrete cover 20 mm

Concrete C30/37

- Cylinder compressive strength f_{ck} = 30 MPa
- Tensile strength f_{ctm}=2.9 MPa

CFRP

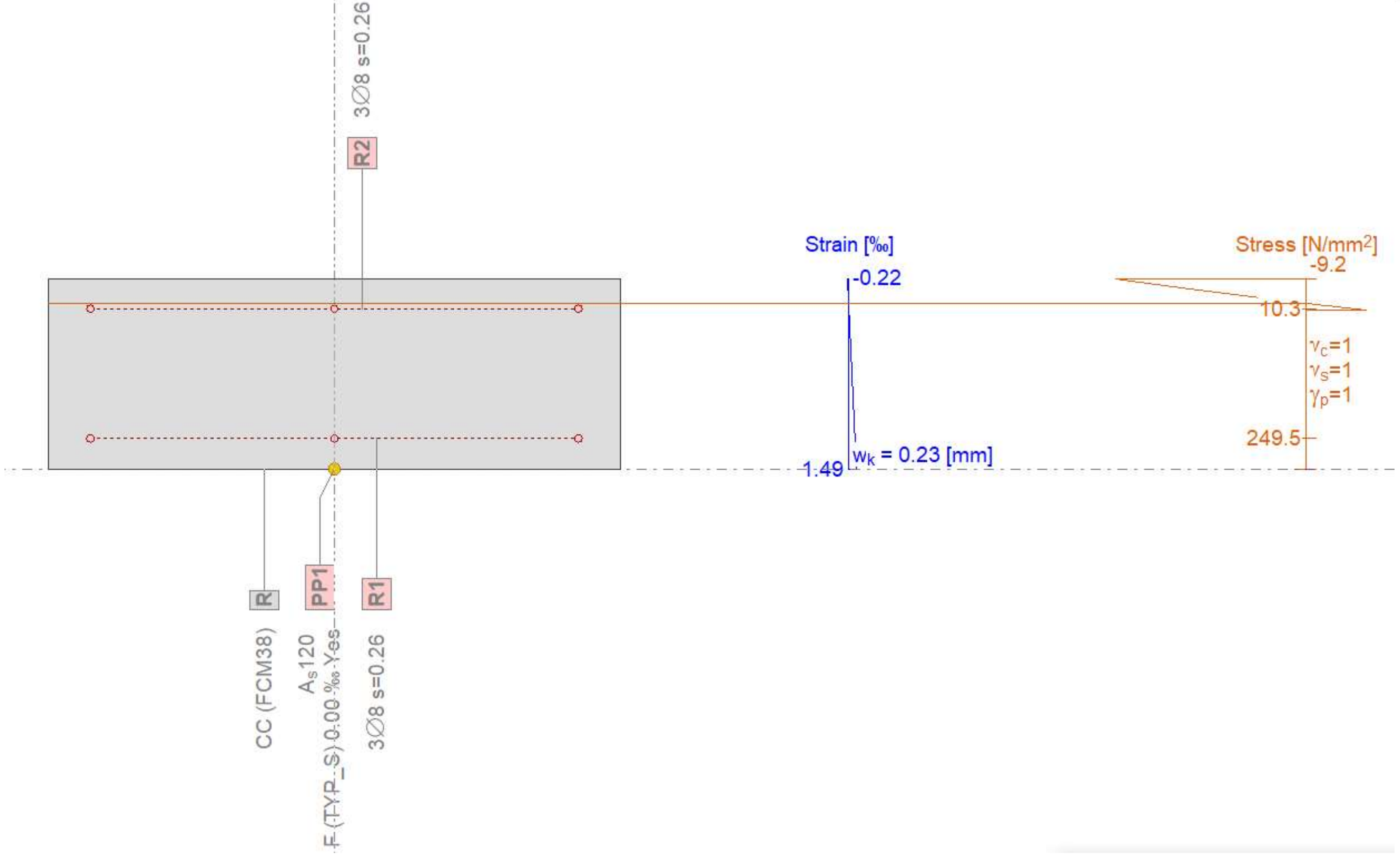
- Elastic modulus E_f= 170 GPa
- Tensile strength f_f = 3000 MPa

Reinforcing steel B500

- f_{sk}=500 MPa
- E_s =205 GPa

Crack Moment

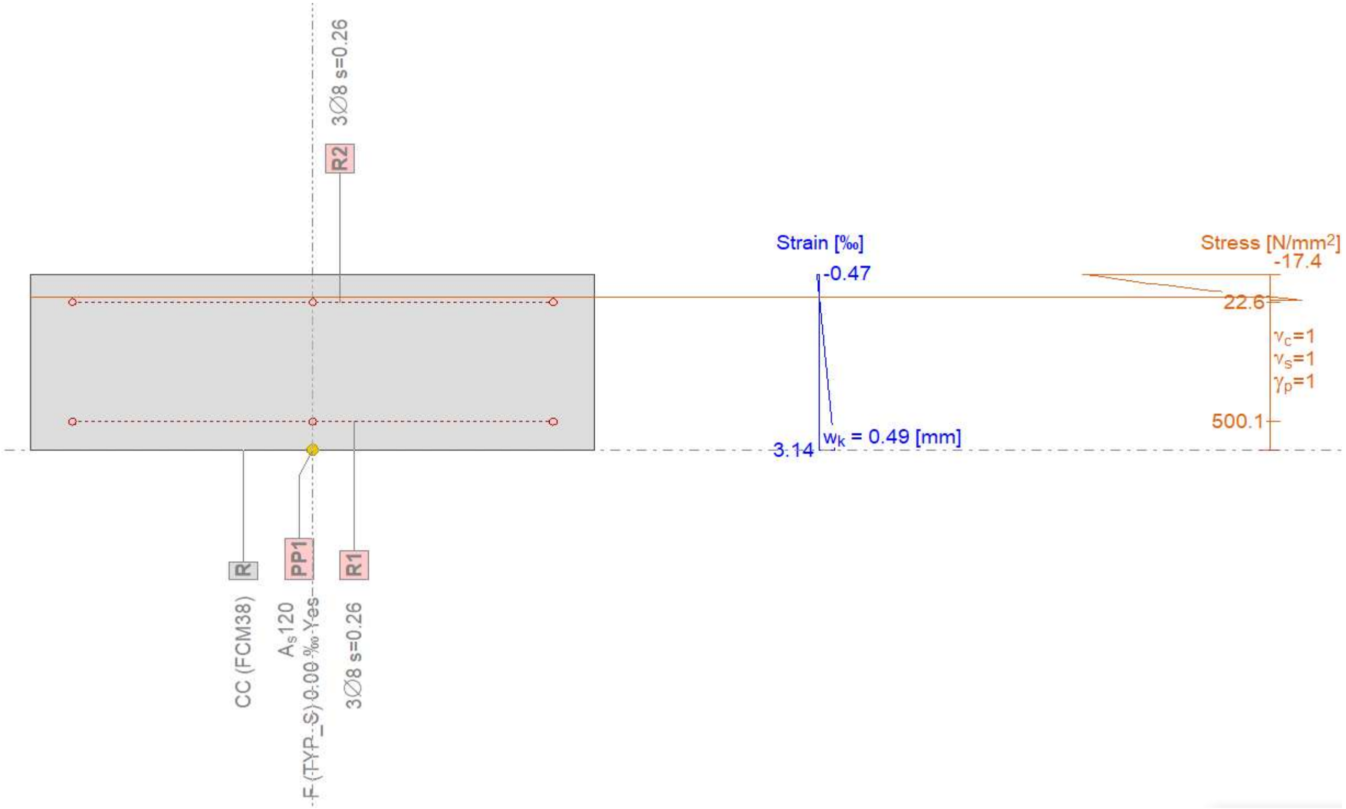
- M_{cr} = 11.8 kNm
- $\varepsilon_{f,cr}$=1.49‰



Calculation with FAGUS from Company Cubus

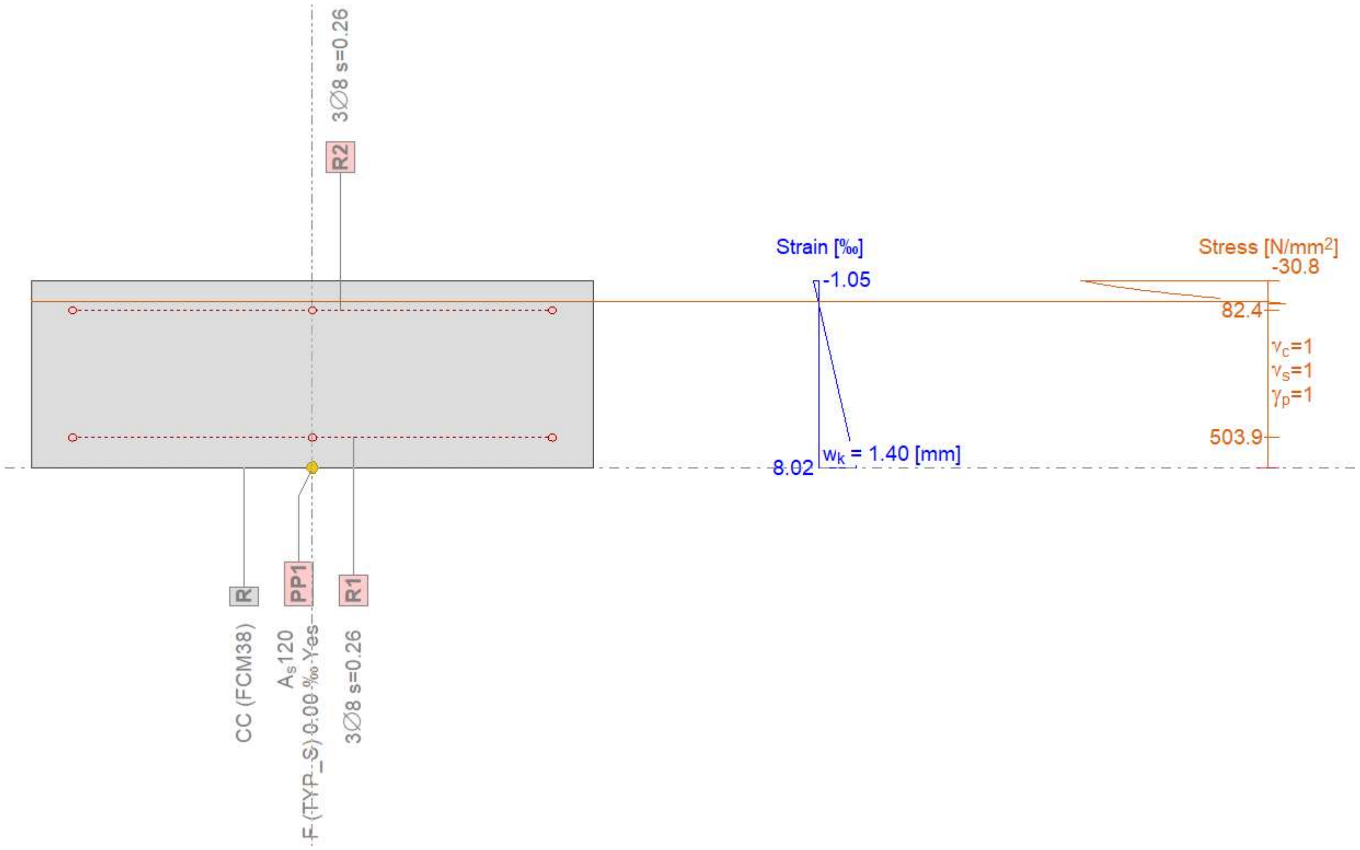
Yielding Moment

- M_y = 24.0 kNm
- $\varepsilon_{f,y}$=3.14‰



Maximum Moment

- M_{max} = 42.9 kNm
- $\varepsilon_{f,max}$=8.00‰



1. Step: evaluation of the end anchorage force capacity with the SIA equations (see slide 90)

Tensile strength f_{ctm}=2.9 MPa

$$G_{Fb} = \frac{f_{ctH}}{8} \left[\frac{N}{mm}\right]$$

$$F_{b0,R} = b_f \sqrt{2 G_{Fb} E_f t_f}$$

$$\tau_{f0} = \frac{4}{3} \cdot f_{ctH} \, [MPa]$$

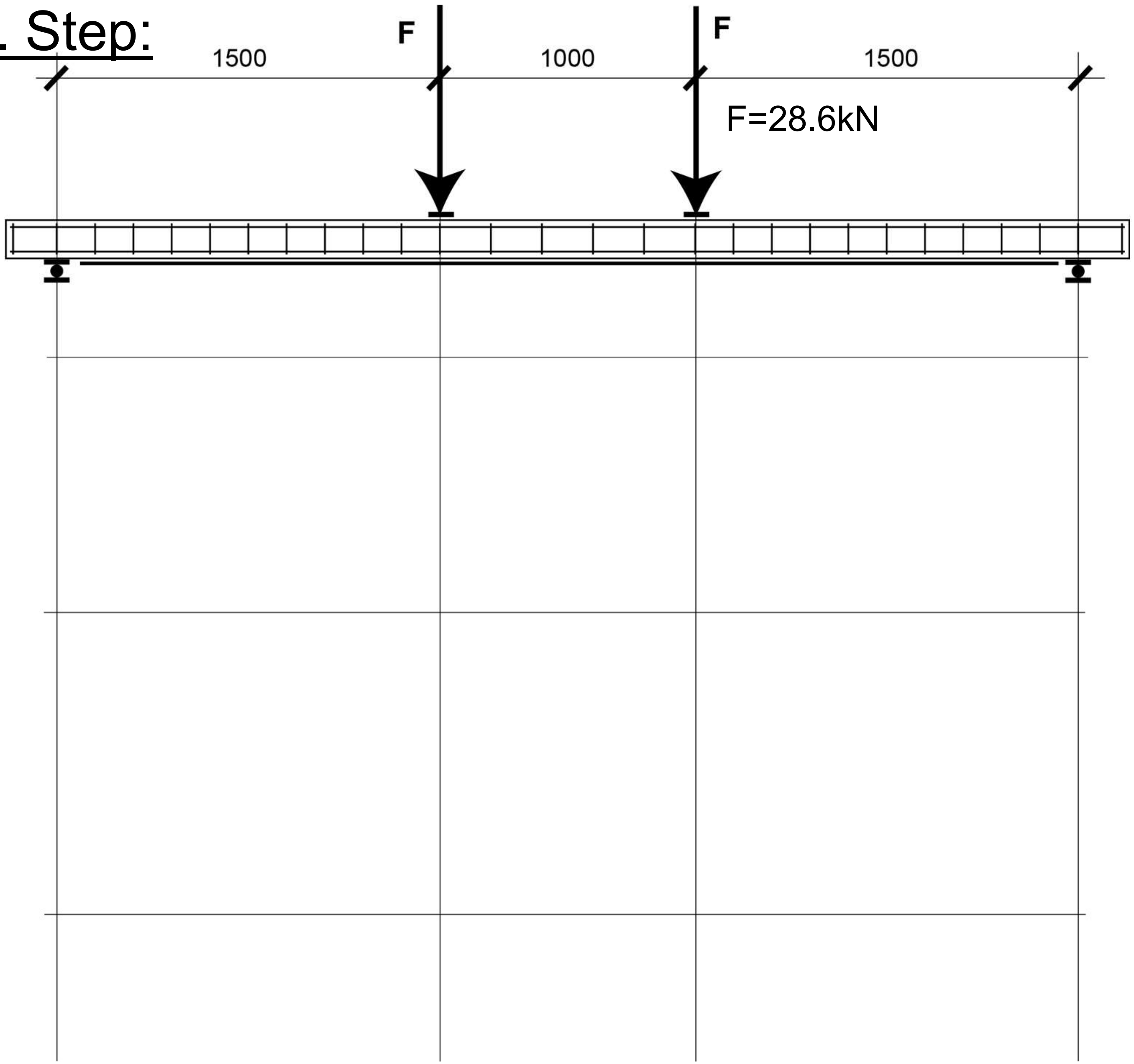
$$l_{b0} = \frac{\pi}{2} \cdot \sqrt{2 \cdot \frac{G_{Fb} \cdot E_f \cdot t_f}{\tau_{f0}^2}}$$

if $l_b < l_{b0}$:

use radiant in the calculator for sin()

$$F_{b,R} = b_f \cdot \sqrt{2 \cdot G_{Fb} \cdot E_f \cdot t_f} \cdot \sin\sqrt{\frac{\tau_{f0}^2 \cdot l_b^2}{2 \cdot G_{Fb} \cdot E_f \cdot t_f}}$$

2. Step:



from QSA:

M_{cr} =11.8 kNm
$\varepsilon_{f,cr}$=1.49‰

M_y = 24.0 kNm
$\varepsilon_{f,y}$=3.14‰

M_{max} =42.9 kNm
(with F=28.6kN)
$\varepsilon_{f,max}$=8.0‰

Notes

Notes

Use this procedure to solve the lab competition!

Several additional topics according to Swiss code pr166 (2023)

- Assessment of existing structures
- Ultimate limit state (ULS)
- Serviceability (SLS)
  - Stresses
  - Deflections
- Check of deformation capacity

Assessment of existing structure

- Consider existing stresses and deflections in the structure before strengthening
- Good assessment: geometry, strength
- Internal reinforcement (we need a minimum value…, otherwise we risk a brittle behavior, risk of premature concrete crushing if steel is neglected in calculations)
- Concrete property (tensile strength good enough?)
- Static system
- …
- Use conversation codes (Erhaltungsnormen) to check if a strengthening is necessary or not!
    - SIA269 Basics
    - SIA269/1 Actions
    - SIA269/2 Reinforced Concrete
- If a strengthening is necessary, use SIA260, 261, 262 and 166

Ultimate limit state

$$E_d \leq R_d$$

Design value of effects of action ≤ Design value of ultimate resistance

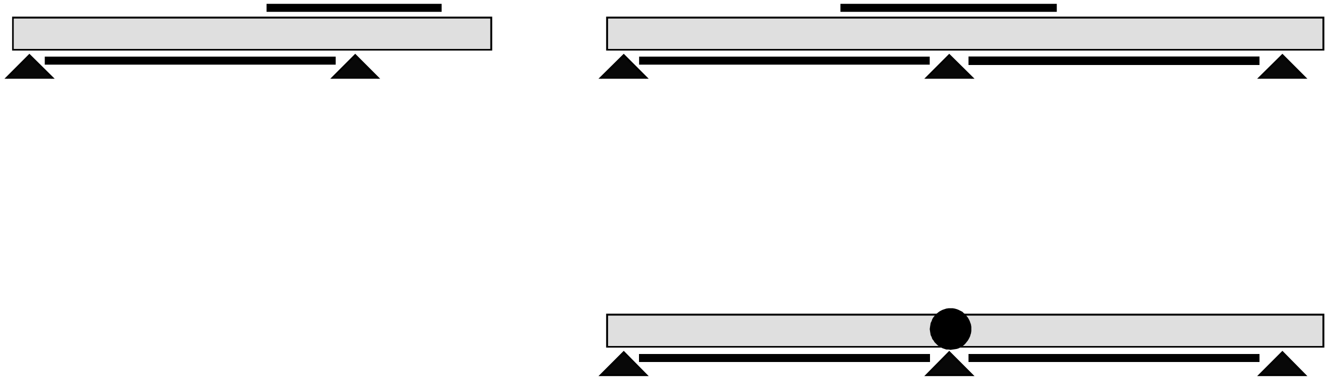
Definition of hazard scenarios (Gefährdungsbilder)

do not forget the hazard scenario “failure of externally bonded reinforcement” (accidential scenario)

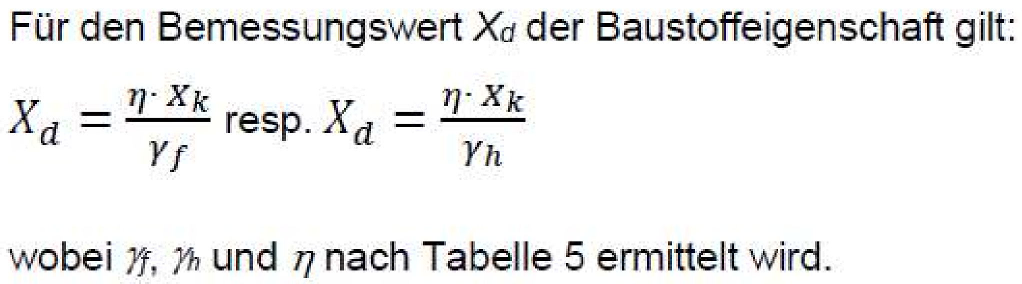
Determination of action effects M and V (Schnittkräfte)

- Plastic rotation capacity is reduced!

→ Elastic determination of action effects if FRP are used, e.g. two span beam



Determination of the design values



Für den Bemessungswert X_d der Baustoffeigenschaft gilt:

$$X_d = \frac{\eta \cdot X_k}{\gamma_f} \text{ resp. } X_d = \frac{\eta \cdot X_k}{\gamma_h}$$

wobei γ_f, γ_h und η nach Tabelle 5 ermittelt wird.

X_k = characteristic value of material property

Depending on failure in strengthening material or bond failure:

η = reduction factor

γ_f or γ_h = resistance factor

→ see Table 5 in prSIA166(2023)

Table 5 in prSIA 166

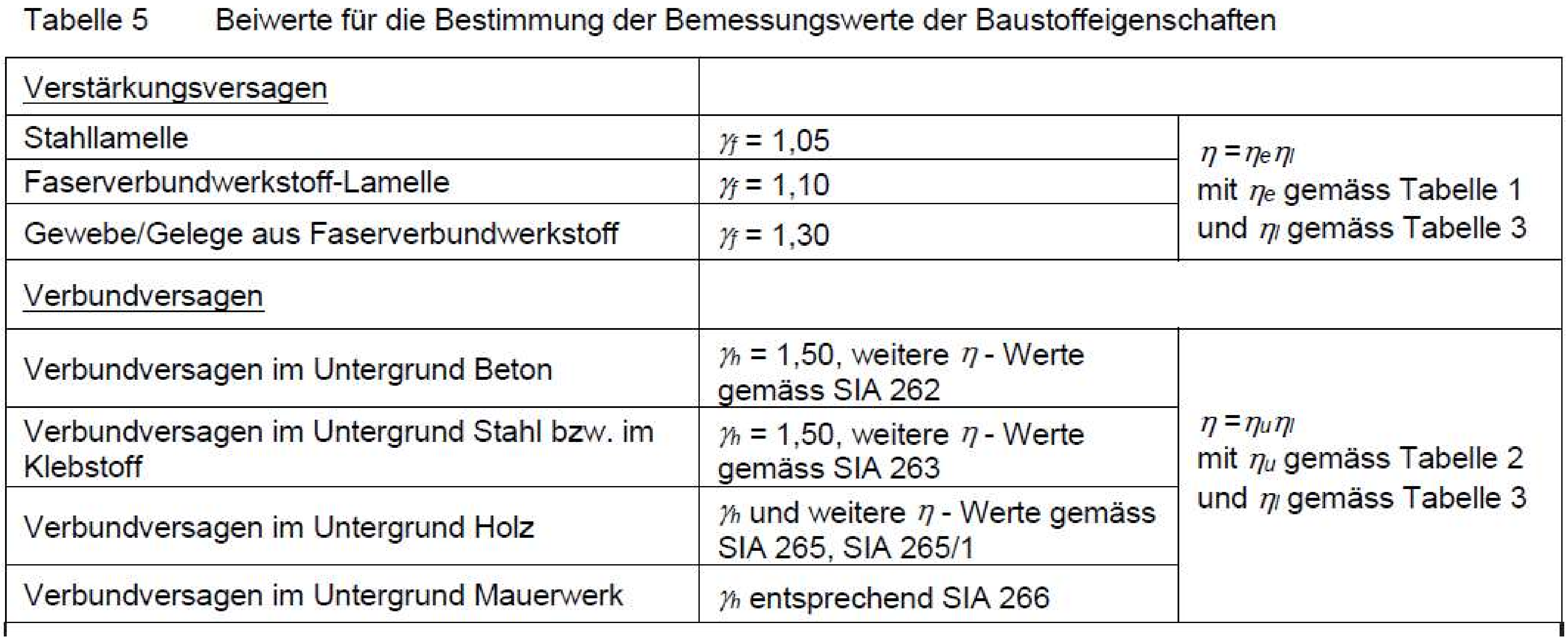


Tabelle 5 Beiwerte für die Bestimmung der Bemessungswerte der Baustoffeigenschaften

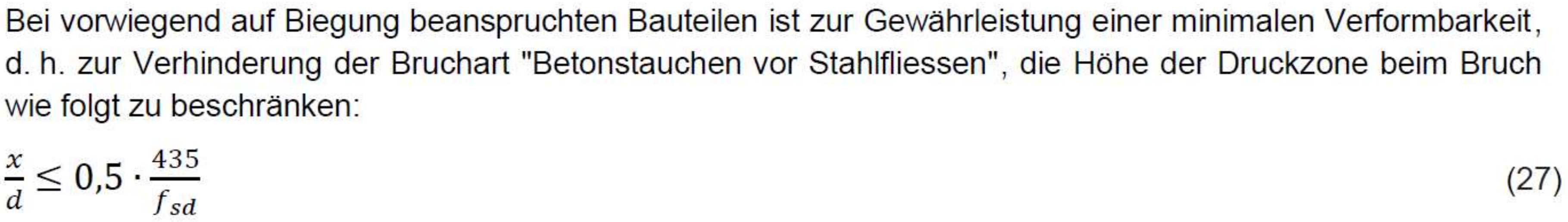
Verstärkungsversagen		
Stahllamelle	$\gamma_f = 1{,}05$	$\eta = \eta_e \eta_l$ mit η_e gemäss Tabelle 1 und η_l gemäss Tabelle 3
Faserverbundwerkstoff-Lamelle	$\gamma_f = 1{,}10$	
Gewebe/Gelege aus Faserverbundwerkstoff	$\gamma_f = 1{,}30$	
Verbundversagen		
Verbundversagen im Untergrund Beton	$\gamma_h = 1{,}50$, weitere η - Werte gemäss SIA 262	$\eta = \eta_u \eta_l$ mit η_u gemäss Tabelle 2 und η_l gemäss Tabelle 3
Verbundversagen im Untergrund Stahl bzw. im Klebstoff	$\gamma_h = 1{,}50$, weitere η - Werte gemäss SIA 263	
Verbundversagen im Untergrund Holz	γ_h und weitere η - Werte gemäss SIA 265, SIA 265/1	
Verbundversagen im Untergrund Mauerwerk	γ_h entsprechend SIA 266	

SLS and evenness

- SIA 166: Serviceability (SLS)
  - Stresses in the internal reinforcement should not exceed the stresses as defined in SIA262 (depending on the requirements: normal, increased, high)
  - Deflections have to be checked

- Evenness of concrete surface
  - for 2 m measurement length (Messlatte): max. 5 mm tolerance
  - for 0.3 m measurement length (Messlatte): max. 1 mm tolerance

Deformation capacity according prEN 166 (2023)



Bei vorwiegend auf Biegung beanspruchten Bauteilen ist zur Gewährleistung einer minimalen Verformbarkeit, d. h. zur Verhinderung der Bruchart "Betonstauchen vor Stahlfliessen", die Höhe der Druckzone beim Bruch wie folgt zu beschränken:

$$\frac{x}{d} \leq 0{,}5 \cdot \frac{435}{f_{sd}} \qquad (27)$$

x: height of bending compression zone
d: static height
f_{sd}: design value of yield strength of internal reinforcement steel

Literature: Codes/Guidelines

- *fib* (2001) Externally bonded FRP reinforcement for RC structures - Bulletin 14. International Federation for Structural Concrete (fib), Switzerland.
- *fib* (2019) Externally applied FRP reinforcement for concrete structures - Bulletin 90. International Federation for Structural Concrete (fib), Switzerland.
- SIA166 (2004) Klebebewehrungen (Externally bonded reinforcement). Schweizerischer Ingenieur- und Architektenverein SIA.
- SIA (2004) D 0209, Dokumentation, Klebebewehrung, Einführung in die Norm SIA 166.
- CNR (2004) Guide for the Design and Construction of Externally Bonded FRP Systems for Strengthening Existing Structures, CNR-DT 200/2004. CNR - Advisory Committee on Technical Recommendations for Construction, Rome, Italy.
- ACI (2008) ACI440.2R-08, Guide for the design and construction of externally bonded FRP systems for strengthening concrete structures. American Concrete Institute.
- TR55 (2012) Design guidance for strengthening concrete structures using fibre composite materials, Third Edition. Technical Report No. 55 of the Concrete Society, UK.

Literature

PhD Theses (can freely be downloaded from www.research-collection.ethz.ch)

- Ulaga T (2003) Dissertation ETH Nr. 15062, Betonbauteile mit Stab- und Lamellenbewehrung: Verbund- und Zuggliedmodellierung, http://dx.doi.org/10.3929/ethz-a-004525392
- Czaderski C (2012) Dissertation ETH No. 20504, Strengthening of reinforced concrete members by prestressed, externally bonded reinforcement with gradient anchorage, http://dx.doi.org/10.3929/ethz-a-007569614

Book chapters

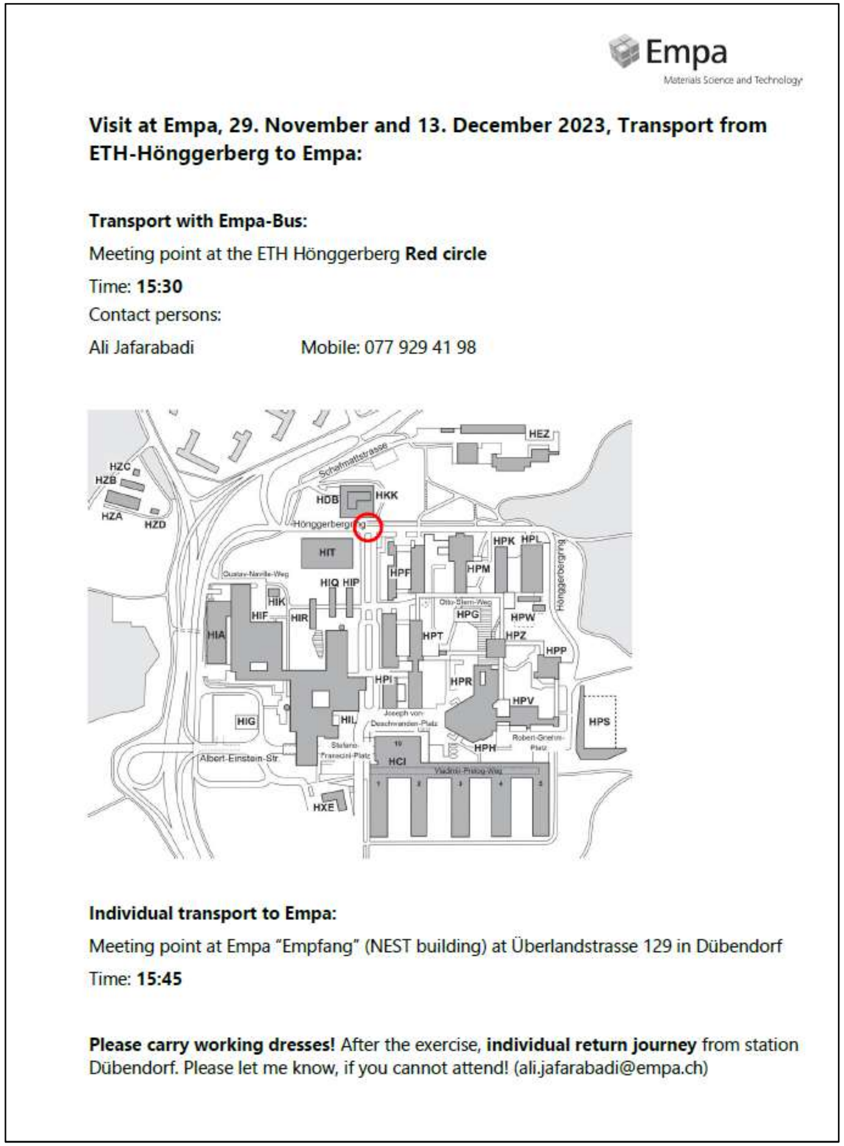
- Motavalli, M., C. Czaderski, A. Schumacher, and D. Gsell, Fibre reinforced polymer composite materials for building and construction, in *Textiles, polymers and composites for buildings*, G. Pohl, Editor. 2010, Woodhead Publishing Limited: Cambridge UK. p. 69-128.
- Czaderski C., Flexural and Shear Strengthening of Reinforced Concrete Structures, in *The International Handbook of FRP Composites in Civil Engineering*, CRC Press, 2013.
  p. 235-252

Program overview of the lectures and laboratory work

- Wednesday 25.10.2023, 15:45-17:30 (ETH Hönggerberg, HIL E7), Lecturer Yunus Harmanci
  - Lecture on Flexural Strengthening
  - Preparations for laboratory competition (Beam) and second written intermediate exam

- Wednesday 29.11.2023, 15:45-17:30 (Empa Dübendorf) Responsible Ali Jafarabadi
  - Meeting point at Busstation ETH Hönggerberg ETH Link 15:30!!
  - Application of Externally Bonded FRP Reinforcement (Confinement) for laboratory competition
  - Video of the beam failure test
  - Empa structural laboratory tour (if time available)

- Wednesday 13.12.2023, 15:45 – ca.18:00 (Empa Dübendorf) Responsible Ali Jafarabadi
  - Meeting point at Busstation ETH Hönggerberg ETH Link 15:30!!
  - Laboratory experiments and awarding of lab competition
  - Second written interim exam



Visit at Empa, 29. November and 13. December 2023, Transport from ETH-Hönggerberg to Empa:

Transport with Empa-Bus:

Meeting point at the ETH Hönggerberg **Red circle**

Time: **15:30**

Contact persons:

Ali Jafarabadi Mobile: 077 929 41 98

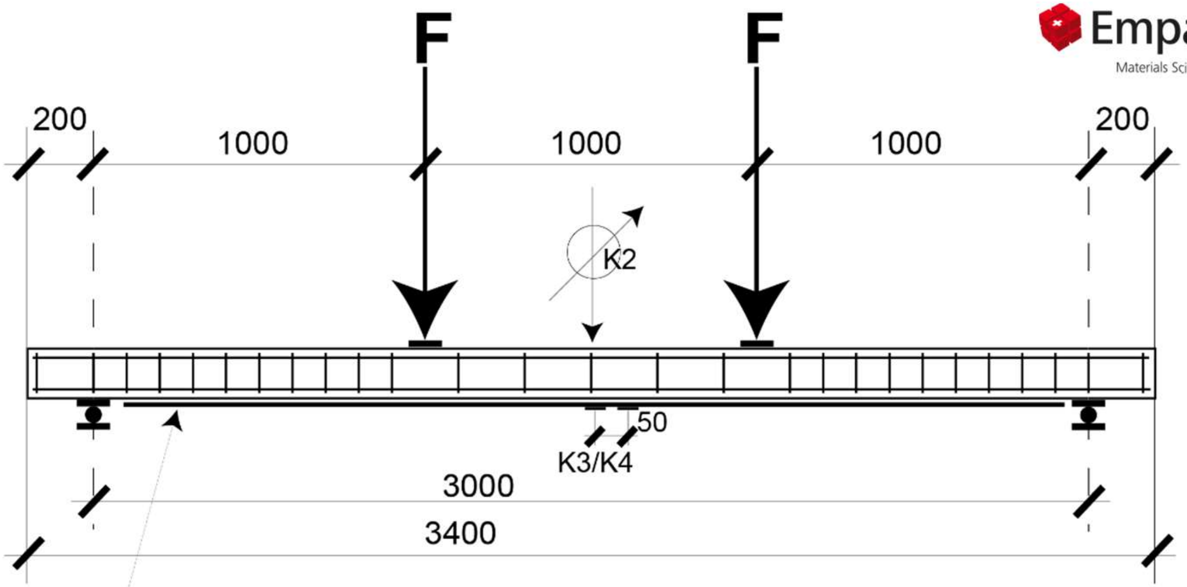
Individual transport to Empa:

Meeting point at Empa "Empfang" (NEST building) at Überlandstrasse 129 in Dübendorf

Time: **15:45**

Please carry working dresses! After the exercise, **individual return journey** from station Dübendorf. Please let me know, if you cannot attend! (ali.jafarabadi@empa.ch)

First part of the laboratory competition: prediction of the failure load of this beam

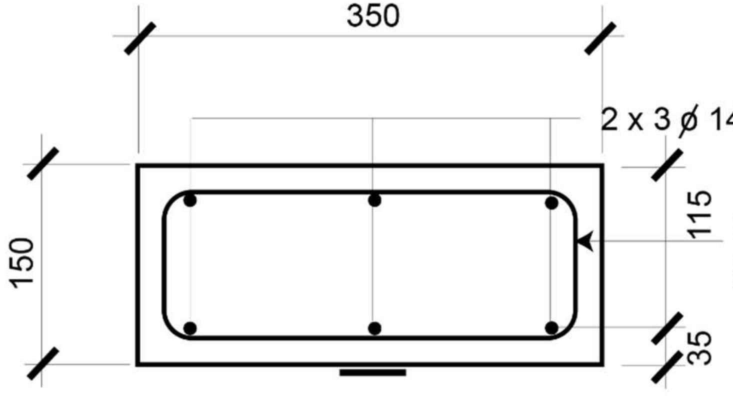


CFRP-strip S&P type CFK 150 / 2000
100x0.9mm, length2850mm

Concrete
C35/45

Steel
f_s = 487 N/mm^2
f_t = 566 N/mm^2

CFRP
E_f = 150’000 N/mm^2



2 x 3 ø 14

27 stirrups
ø 8 s = 100 and 200 mm

concrete cover 20 mm

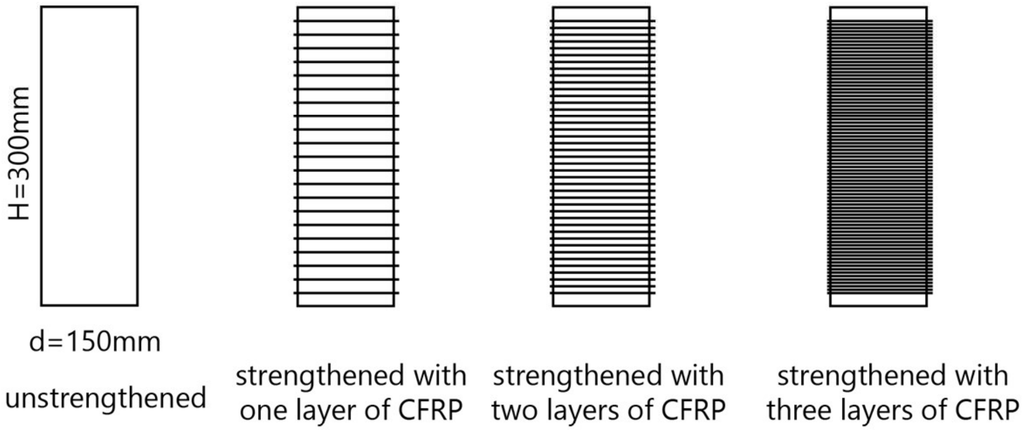
Video of the application of the CFRP strip

- $f_{c,cube}$ 68 days = 47.2 MPa (f_c = 0.8*$f_{c,cube}$ = 37.8 MPa)
- Assumption for calculations: concrete C35/45 (f_{ctm} = 3.2 MPa, f_{ck} = 35 MPa)

Cracking moment	**Yielding** moment	**Max** moment
M_{cr} =	M_y =	M_{max} =
F_{cr} =	F_y =	F_{max} =
$\varepsilon_{f,cr}$ =	$\varepsilon_{f,y}$ =	$\varepsilon_{f,max}$ =

Compression tests on four concrete cylinders

Concrete cylinder 150x300mm



CFRP: S&P C-Sheet 240 200g/m2

Second part of the laboratory competition: prediction of the failure load of four cylinders

$f_{c,cube,28}$ = 27.4 MPa

Time schedule:

Casting on 22.09.2023
20.10.2023 after 28 Tagen: Testing of concrete cube compressive strength
29.11.2023: Application of CFRP sheet on the cylinders
13.12.2022: Failure tests

Lab competition

- Video of the experiment on the beam will be presented on 29.11.2023
- Lap experiments on the cyclinders will be performed on 13.12.2023

- Who makes the best prediction? The best predictions are awarded with a price.
- Predictions (in kN):
  Failure load of Beam (by 28.11.2023)
  Failure loads of Cylinders 1 to 4 (by 12.12.2023)

- →Submission of the numbers by email to:
  ali.jafarabadi@empa.ch