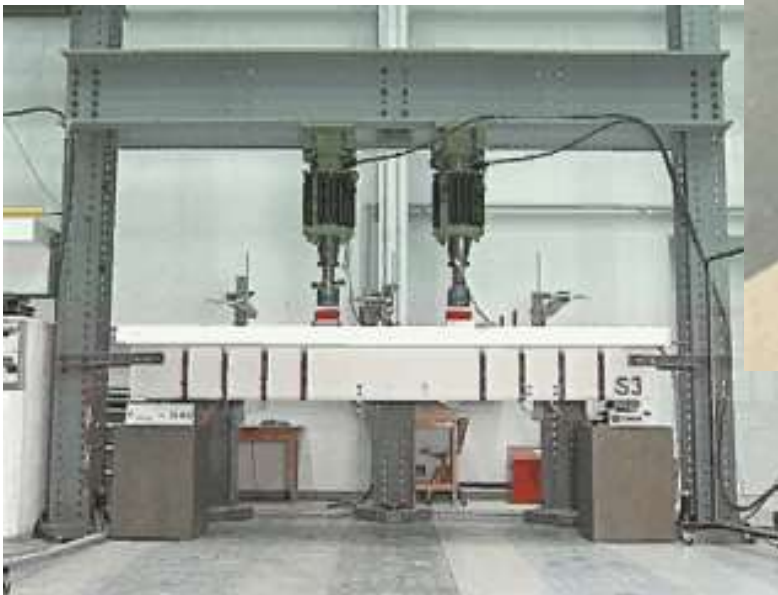


Table of content:

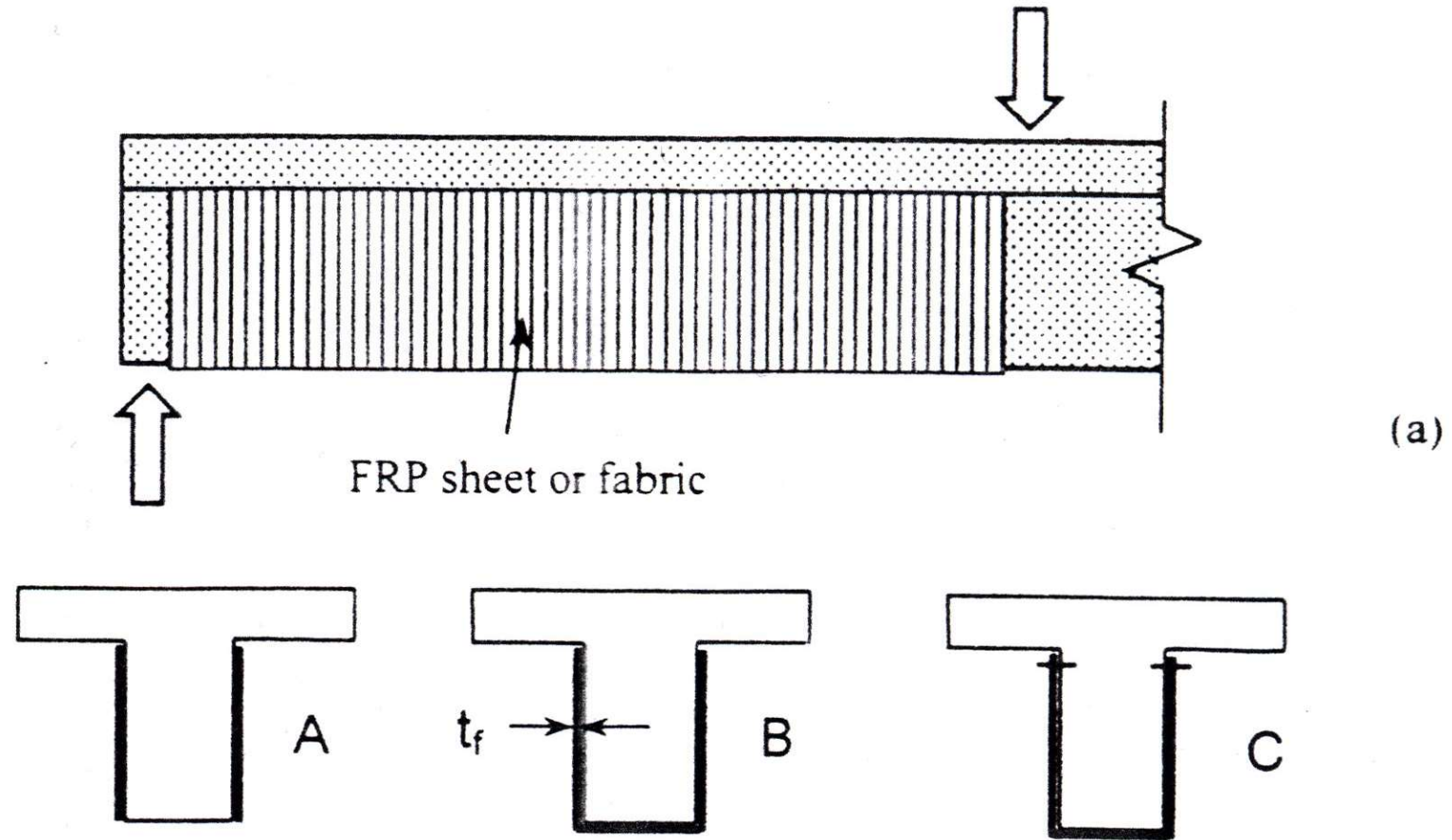
- ✓ Introduction
- ✓ Materials and Properties of Polymer Matrix Composites
- ✓ Mechanics of a Lamina
- ✓ Laminate Theory
- ✓ Ply by Ply Failure Analysis
- ✓ Externally Bonded FRP Reinforcement for RC Structures: Overview
- ✓ Flexural Strengthening
 - **Strengthening in Shear**
 - Column Confinement
 - CFRP Strengthening of Metallic Structures
 - FRP Strengthening of Timber Structures
 - Design of FRP Profiles and all FRP Structures
 - An Introduction to FRP Reinforced Concrete
 - Structural Monitoring with Wireless Sensor Networks
 - Composite Manufacturing
 - Testing Methods

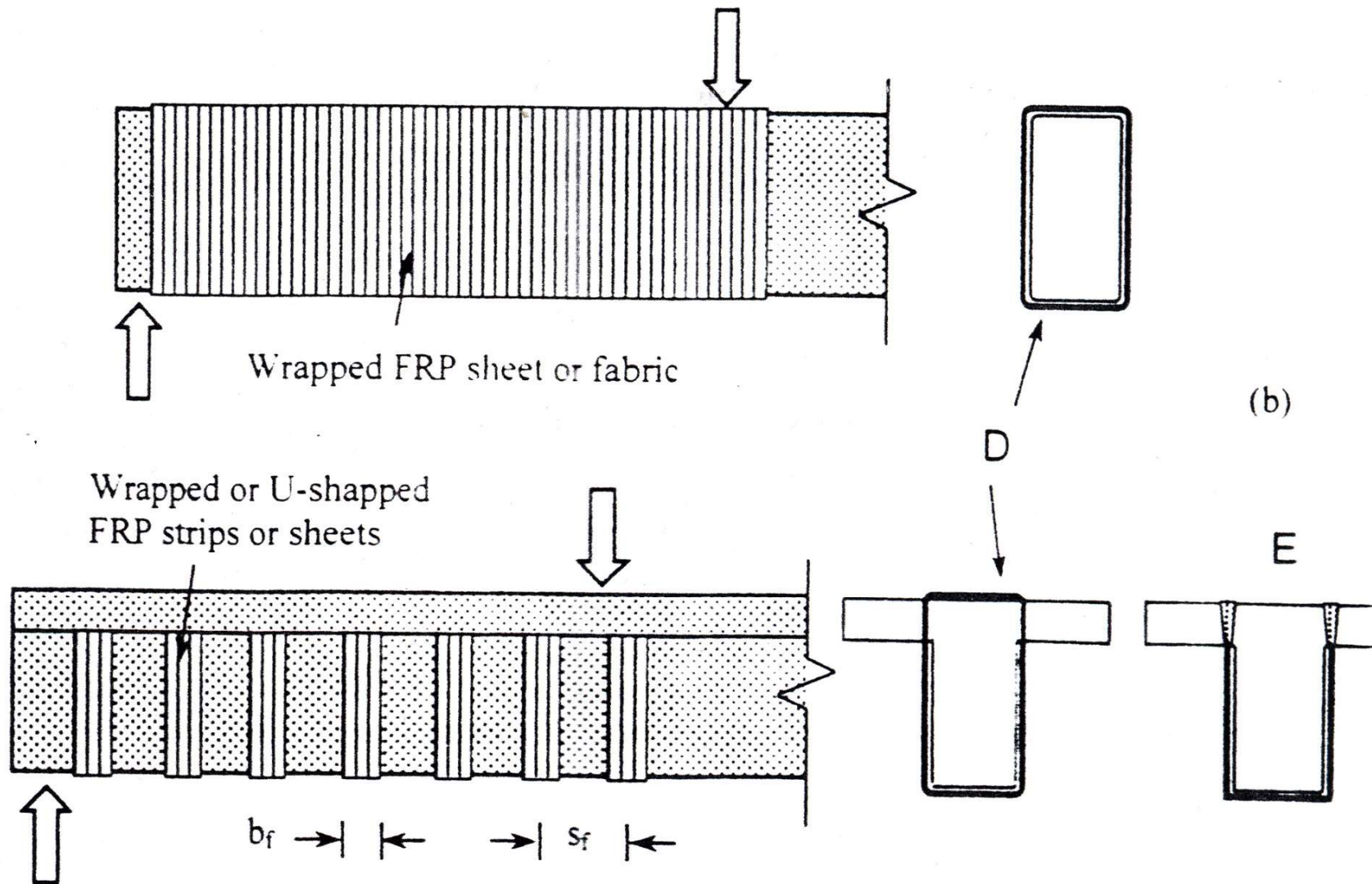
Strengthening in shear

Book Composite for Construction, L. C. Bank, Chapter 10



General

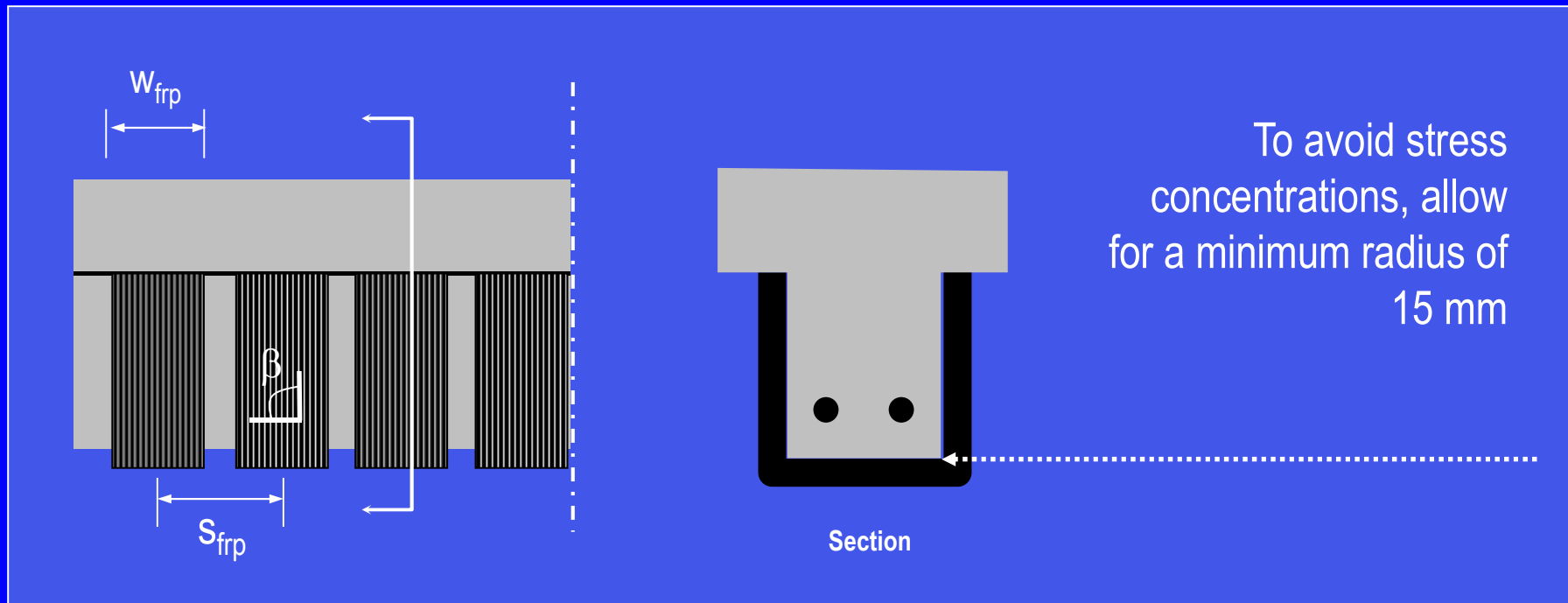




Beam/One-Way Slab Strengthening

Shear Strengthening

Assumptions



Beam/One-Way Slab Strengthening

Shear Strengthening

Design Principles

External strengthening with FRPs:

Flexural failure



Generally fairly ductile

Shear failure

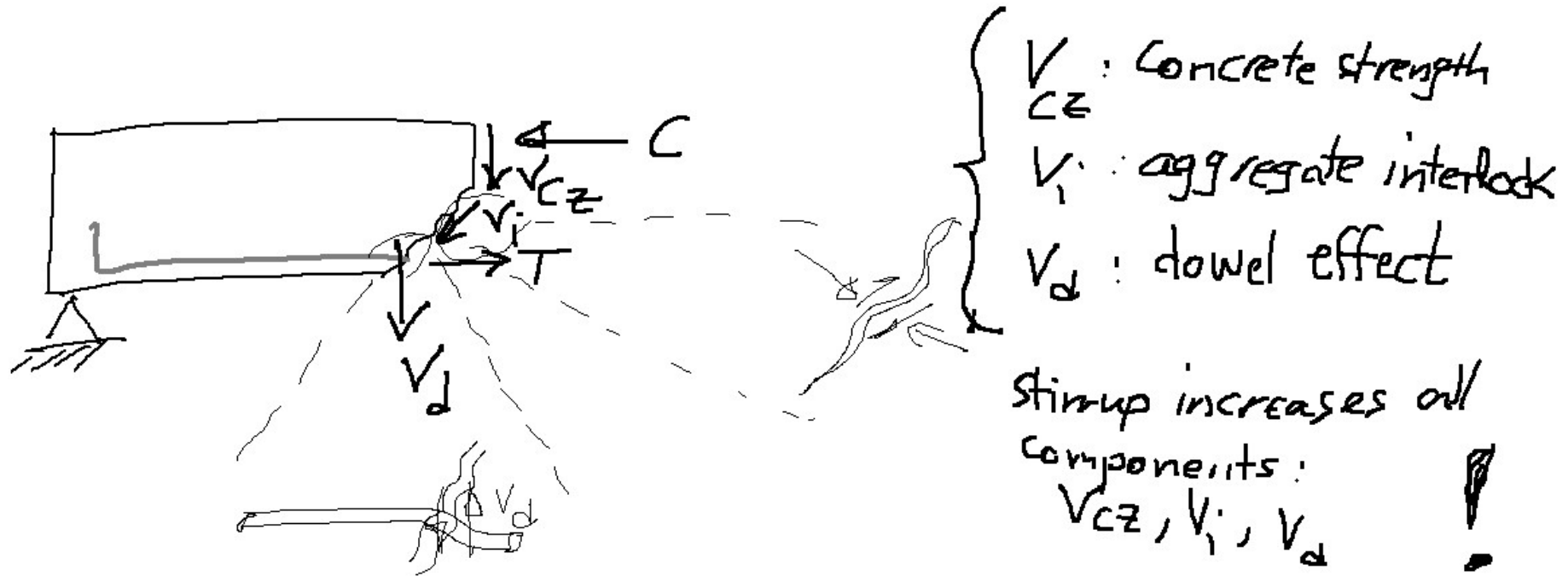


Sudden and brittle

Undesirable failure mode

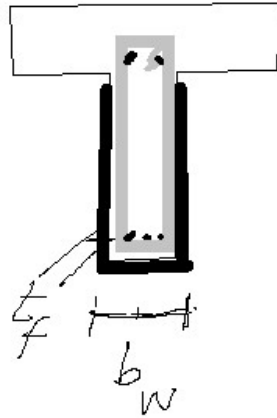
Control shear deformation to avoid sudden failure

Shear capacity of RC

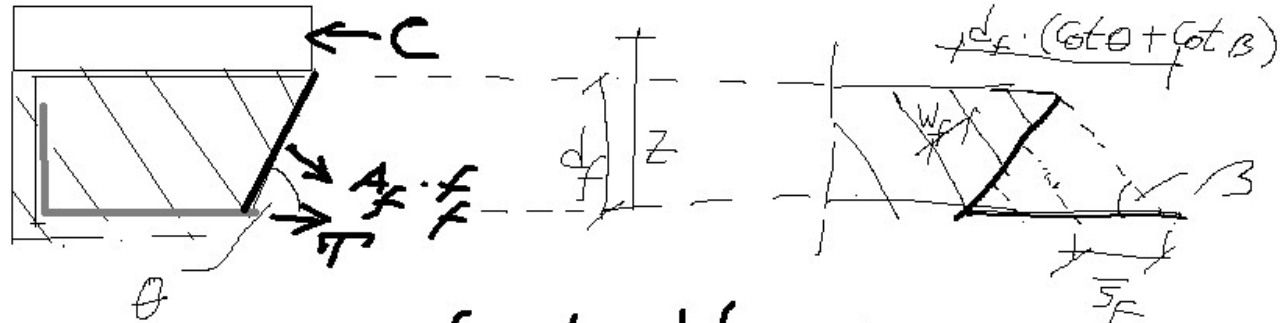


Design model for Ultimate Limit State (ULS)

Assuming that at the ultimate limit state in shear (concrete diagonal tension) the FRP develops an effective strain in the principal material direction, $\epsilon_{f,e}$. The effective strain $\epsilon_{f,e}$ is, in general, less than the tensile failure strain, ϵ_{fu} .



$$V_R = V_C + V_W + V_F$$



$$V_F = 2 f_{fe} \cdot t_f \cdot \frac{W_f}{s_f} \cdot d_f \cdot (\cot \theta + \cot \beta) \cdot \sin \beta$$

$$\rho_f = \frac{2 \cdot t_f \cdot W_f}{b_w \cdot s_f}$$

$$s_f = W_f / \sin \beta \Rightarrow \rho_f = \frac{2 \cdot t_f \cdot \sin \beta}{b_w}$$

$$V_F = 0.9 \cdot E_{fe} \cdot F_f \cdot \rho_f \cdot b_w \cdot d_f \cdot (\cot \theta + \cot \beta) \cdot \sin \beta$$

$$V_f = 0.9 \cdot \varepsilon_{fd,e} \cdot F_f \cdot \rho_f \cdot b_w \cdot d \cdot (\cot \theta + \cot \beta) \cdot \sin \beta$$

Where:

$\varepsilon_{fd,e}$: design value of effective FRP strain.

b_w : minimum width of cross section over the effective depth.

d : effective depth of cross section.

ρ_f : $(2 t_f \sin \beta) / b_w$ for
continuously bonded shear reinforcement of thickness
 t_f (b_w = minimum width of concrete cross section over
the effective depth), or
 $(2t_f/b_w)(b_f/s_f)$ for
FRP reinforcement in the form of strips or sheets of width
 b_f at the spacing s_f .

E_{fu} : elastic modulus of FRP in the principal fiber orientation.

θ : diagonal crack angle, assumed to be 45° .

β : fiber orientation

Circular cross-sections

$$V_{fd} = \frac{\varepsilon_{f_e}}{\gamma_f} E_{fu} \rho_f \frac{1}{2} \frac{\pi D^2}{4} \cot \theta$$

Where:

ε_{f_e} : 0.006 (experimentally verified by Priestley et al, 1995)

D : column diameter

$$V_{cd} = 0.2\Phi_c \cdot b \cdot d \cdot \sqrt{f'_c}$$

$$\Phi_c = 0.6$$

$$V_{wd} = A_s \cdot \varepsilon_s \cdot E_s \cdot \frac{z_s}{s_f} \cdot \text{Cotg} \theta \quad \text{If:} \quad \varepsilon_s \leq \frac{f_y}{E_s}$$

$$V_{wd} = A_s \cdot f_y \cdot \frac{z_s}{s_f} \cdot \text{Cotg} \theta \quad \text{If:} \quad \varepsilon_s > \frac{f_y}{E_s}$$

Where:

z_s : internal lever arm.

s_f : spacing of stirrups.

f_y : yield strain.

θ : crack angle, assumed to be 45° .

- Fully wrapped (or properly anchored) CFRP-FRP fracture controls:

$$\varepsilon_{f,e} = 0.17 \left(\frac{f_{cm}^{2/3}}{E_{fu} \rho_f} \right)^{0.3} \varepsilon_{fu}$$

- Side or U-shaped CFRP jackets:

$$\varepsilon_{f,e} = \min \left[\underbrace{0.65 \left(\frac{f_{cm}^{2/3}}{E_{fu} \rho_f} \right)^{0.56}}_{\text{Peeling off}} \times 10^{-3} \ \& \ \underbrace{0.17 \left(\frac{f_{cm}^{2/3}}{E_{fu} \rho_f} \right)^{0.30}}_{\text{Fracture}} \varepsilon_{fu} \right]$$

Note that in all equations f_{cm} is in *MPa* and E_{fu} is in *GPa*.

Design recommendations

It should point out that the spacing of strips (s_f), if they are used vertically, should not exceed:

$$s_f \leq 0.9d - \frac{b_f}{2} \quad \text{for rectangular cross sections}$$

$$s_f \leq d - h_f - \frac{b_f}{2} \quad \text{for T-beams}$$

In which h_f is slab thickness and b_f is FRP width.

**See IIFC Webinar: Shear Strengthening of RC Beams,
Fib, ACI, Italian and Australian Guidelines,
EBR and NSMR**

Given by: Professor J. Barros, University of Minho, Portugal

<https://www.youtube.com/watch?v=AMWziRZuWHk&list=PLsdGDOBT-H8E9FWUgrURcto9jU9Pr0I45&index=2&t=0s>

Attachments

Beam/One-Way Slab Strengthening

Canadian Guideline

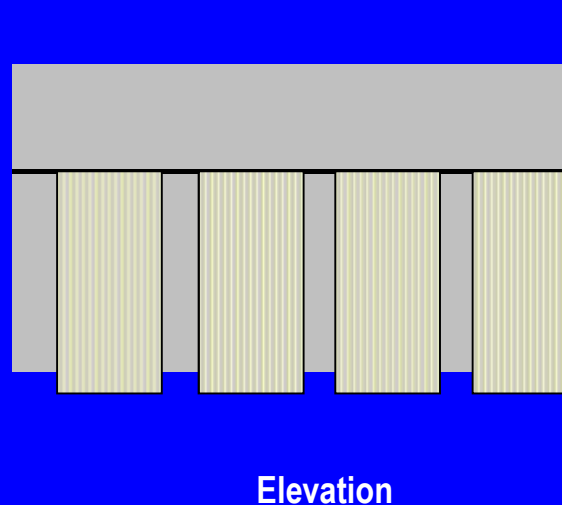
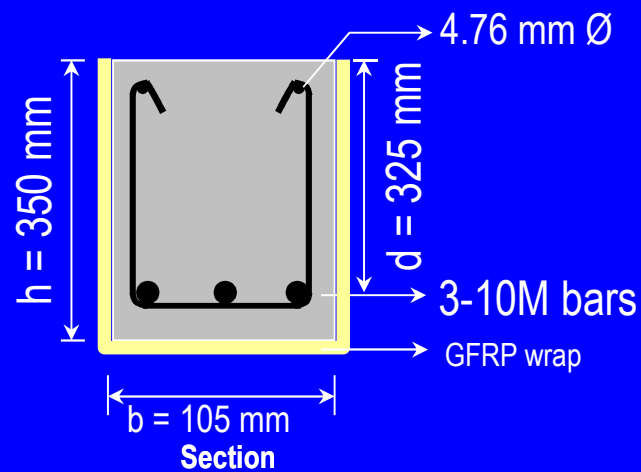
Shear Strengthening

Problem statement

Example

Calculate the shear capacity (V_r) for an FRP-strengthened concrete section

Section information



$$\lambda = 1.0$$

$$f'_c = 45 \text{ MPa}$$

$$\varepsilon_{frpu} = 2.0 \%$$

$$t_{frp} = 1.3 \text{ mm}$$

$$w_{frp} = 100 \text{ mm}$$

$$s_{frp} = 200 \text{ mm}$$

$$E_{frp} = 22.7 \text{ GPa}$$

$$s_s = 225 \text{ mm c/c}$$

$$f_y = 400 \text{ MPa (rebar)}$$

$$f_y = 400 \text{ MPa (stirrup)}$$

Beam/One-Way Slab Strengthening

Shear Strengthening

Solution

Example

Step 1: Calculate concrete and steel contributions

Concrete: $V_c = 0.2 \phi_c \sqrt{f'_c} b_w d$
 $V_c = 0.2 (0.6) \sqrt{45} (105) (325)$
 $V_c = 27470 \text{ N} = 27.47 \text{ kN}$

Steel: $V_s = \frac{\phi_s f_y A_v d}{s} = \frac{0.85 (400) (36) (325)}{225}$
 $V_s = 17680 \text{ N} = 17.68 \text{ kN}$

Beam/One-Way Slab Strengthening

Shear Strengthening

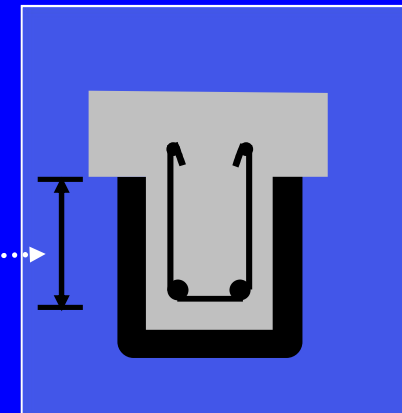
Shear resistance of a beam:

Design Principles

$$V_{frp} = \frac{\phi_{frp} A_{frp} E_{frp} \varepsilon_{frpe} d_{frp} (\sin\beta + \cos\beta)}{S_{frp}}$$

$$\Rightarrow A_{frp} = 2 t_{frp} w_{frp}$$

$\Rightarrow d_{frp}$: distance from free end of FRP to bottom of internal steel stirrups



Beam/One-Way Slab Strengthening

Shear Strengthening

⇒ Effective strain in FRP, ε_{frpe} :

$$\varepsilon_{frpe} = R \varepsilon_{frpu} \leq 0.004$$

Prevents shear cracks from widening
beyond acceptable limits

Ensures aggregate interlock!

Design Principles

⇒ Reduction factor, R:

$$R = \alpha \lambda_1 \left(\frac{f'_c{}^{2/3}}{\rho_{frp} E_{frp}} \right)^{\lambda_2}$$

0.8

Carbon: $\lambda_1 = 1.35$, $\lambda_2 = 0.30$

Glass: $\lambda_1 = 1.23$, $\lambda_2 = 0.47$

Beam/One-Way Slab Strengthening

Shear Strengthening

Design Principles

⇒ FRP shear reinforcement ratio, ρ_{frp} :

$$\rho_{frp} = \left(\frac{2 t_{frp}}{b_w} \right) \left(\frac{W_{frp}}{S_{frp}} \right)$$

Beam/One-Way Slab Strengthening

Shear Strengthening

Design Principles

Another limit on effective strain in FRP, ϵ_{frpe} :

$$\epsilon_{frpe} \leq \frac{\alpha k_1 k_2 L_e}{9525}$$

0.8

⇒ Parameters, k_1 and k_2 :

$$k_1 = \left(\frac{f'_c}{27.65} \right)^{2/3}$$

$$k_2 = \left(\frac{d_{frp} - n_e L_e}{d_{frp}} \right)$$

Beam/One-Way Slab Strengthening

Shear Strengthening

Design Principles

⇒ Effective anchorage length, L_e :

$$L_e = \left[\frac{25350}{\left[t_{frp} E_{frp} \right]^{0.58}} \right]$$

Beam/One-Way Slab Strengthening

Shear Strengthening

Design Principles

Limit on spacing of strips, s_{frp} :

$$s_{frp} \leq w_{frp} + \frac{d}{4}$$

Beam/One-Way Slab Strengthening

Shear Strengthening

Limit on maximum allowable shear strengthening, V_{frp} Design Principles

$$V_r \leq V_c + 0.8\lambda\phi_c\sqrt{f'_c}b_wd$$

Shear contribution due to steel stirrups
and FRP strengthening must be less
than this term

Beam/One-Way Slab Strengthening

Shear Strengthening

Solution

Example

Step 2: Determine A_{frp} , ρ_{frp} , L_e for effective strain calculation

$$A_{frp} = 2 t_{frp} w_{frp} = 2 (1.3) (100)$$
$$A_{frp} = 260 \text{ mm}^2$$

$$\rho_{frp} = \left(\frac{2 t_{frp}}{b_w} \right) \left(\frac{w_{frp}}{s_{frp}} \right) = \left(\frac{2 (1.3)}{105} \right) \left(\frac{100}{200} \right)$$
$$\rho_{frp} = 0.0124$$

Beam/One-Way Slab Strengthening

Shear Strengthening

Solution

Example

Step 2: Determine A_{frp} , ρ_{frp} , L_e for effective strain calculation

$$L_e = \left(\frac{25350}{\left[t_{frp} E_{frp} \right]^{0.58}} \right) = \left(\frac{25350}{\left[1.3 \times 22700 \right]^{0.58}} \right)$$
$$L_e = 64.8 \text{ mm}$$

Beam/One-Way Slab Strengthening

Shear Strengthening

Solution

Example

Step 3: Determine k_1 , k_2 and effective strain, ϵ_{frpe} [Limit 2]

$$k_1: \dots k_1 = \left(\frac{f'_c}{27.65} \right)^{2/3} = \left(\frac{45}{27.65} \right)^{2/3} = 1.38$$

Because of u-wrap.....

$$k_2: \dots k_2 = \left(\frac{d_{frp} - n_e L_e}{d_{frp}} \right) = \left(\frac{325 - 1(64.8)}{325} \right) = 0.80$$

Beam/One-Way Slab Strengthening

Shear Strengthening

Solution

Example

Step 3: Determine k_1 , k_2 and effective strain, ϵ_{frpe} [Limit 2]

Note: This strain is one of three limits placed on the FRP

$$\epsilon_{frpe} \leq \frac{\alpha k_1 k_2 L_e}{9525}$$
$$\epsilon_{frpe} = \frac{0.8 (1.38) (0.80) (64.8)}{9525}$$
$$\epsilon_{frpe} = 0.0060$$

Beam/One-Way Slab Strengthening

Shear Strengthening

Example

Solution

Step 4: Determine R and effective strain, ϵ_{frpe} [Limit 1]

R:

$$R = \alpha \lambda_1$$

$$\left(\frac{f'_c{}^{2/3}}{\rho_{frp} E_{frp}} \right)^{\lambda_2}$$

$$R = 0.8 (1.23)$$

$$\left(\frac{45^{2/3}}{0.0124 (22700)} \right)^{0.47}$$

$$R = 0.229$$

Beam/One-Way Slab Strengthening

Shear Strengthening

Solution

Example

Step 4: Determine R and effective strain, ϵ_{frpe} [Limit 1]

Note: This strain is one of three limits placed on the FRP

$\epsilon_{frpe} :$ $\epsilon_{frpe} = R \epsilon_{frpu} \leq 0.004$

$$\epsilon_{frpe} = 0.229 (0.02)$$

$$\epsilon_{frpe} = 0.0046$$

Beam/One-Way Slab Strengthening

Shear Strengthening

Solution

Example

Step 5: Determine governing effective strain, ϵ_{frpe}

For design purposes, use the smallest limiting value of:

$$\epsilon_{frpe} = 0.0046$$

$$\epsilon_{frpe} = 0.0040$$

$$\epsilon_{frpe} = 0.0060$$

Beam/One-Way Slab Strengthening

Shear Strengthening

Solution

Example

Step 6: Calculate contribution of FRP to shear capacity

V_{frp} :

$$V_{frp} = \frac{\phi_{frp} A_{frp} E_{frp} \epsilon_{frpe} d_{frp} (\sin\beta + \cos\beta)}{S_{frp}}$$

$$V_{frp} = \frac{0.5 (260) (22700) (0.004) (325) (\sin 90 + \cos 90)}{200}$$

$$V_{frp} = 19200 \text{ N} = 19.2 \text{ kN}$$

Beam/One-Way Slab Strengthening

Shear Strengthening

Example

Solution

Step 7: Compute total shear resistance of beam

V_r

$$V_r = V_c + V_s + V_{frp}$$

$$V_r = 27.5 + 17.7 + 19.2$$

$$V_r = 64.4 \text{ kN}$$

Beam/One-Way Slab Strengthening

Shear Strengthening

Solution

Example

Step 8: Check maximum shear strengthening limits

$$V_r \leq V_c + 0.8\lambda\phi_c f'_c b_w d$$

$$64400 \leq 27500 + 0.8 (1) (0.6) (45) (105) (325)$$

$$64400 \leq 137400$$

⇒ OK

Beam/One-Way Slab Strengthening

Shear Strengthening

Example

Solution

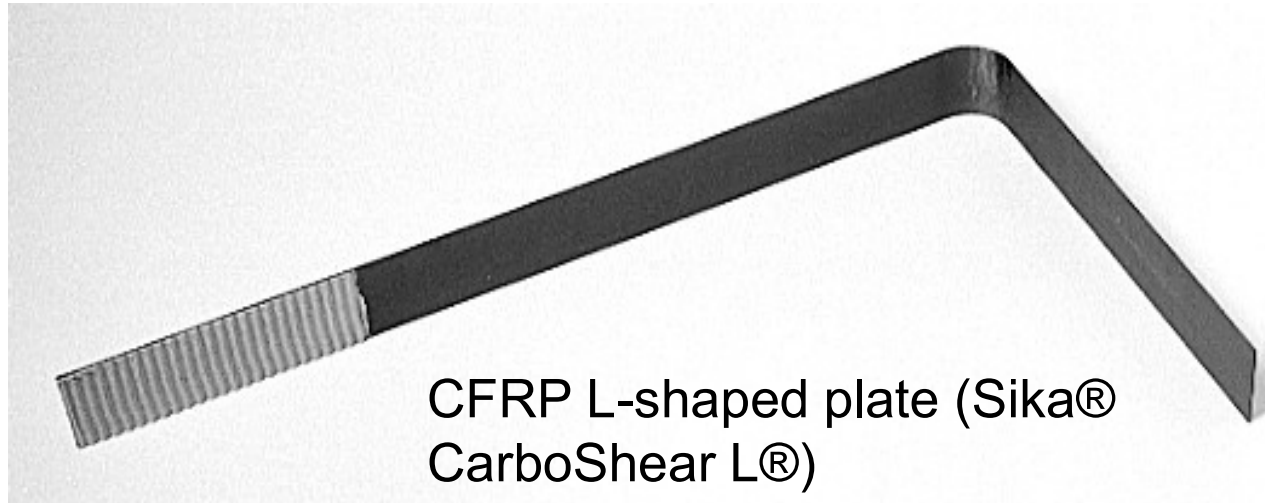
Step 9: Check maximum band spacing

$$s_{frp} \leq w_{frp} + \frac{d}{4}$$
$$200 \leq 100 + \frac{325}{4}$$
$$200 \leq 181$$

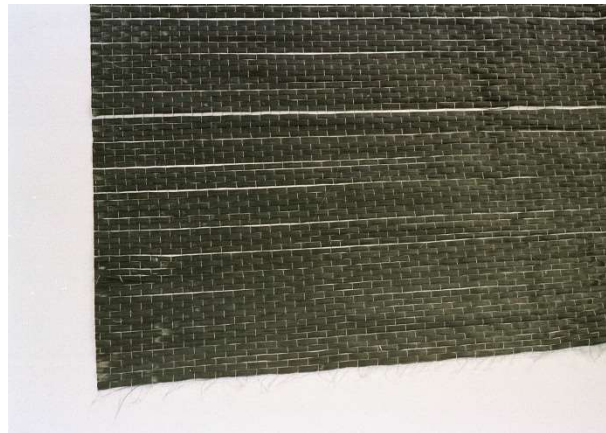
⇒ Not true, therefore use 180 mm spacing

Shear Strengthening of Reinforced Concrete with CFRP: Empa-Project

Material



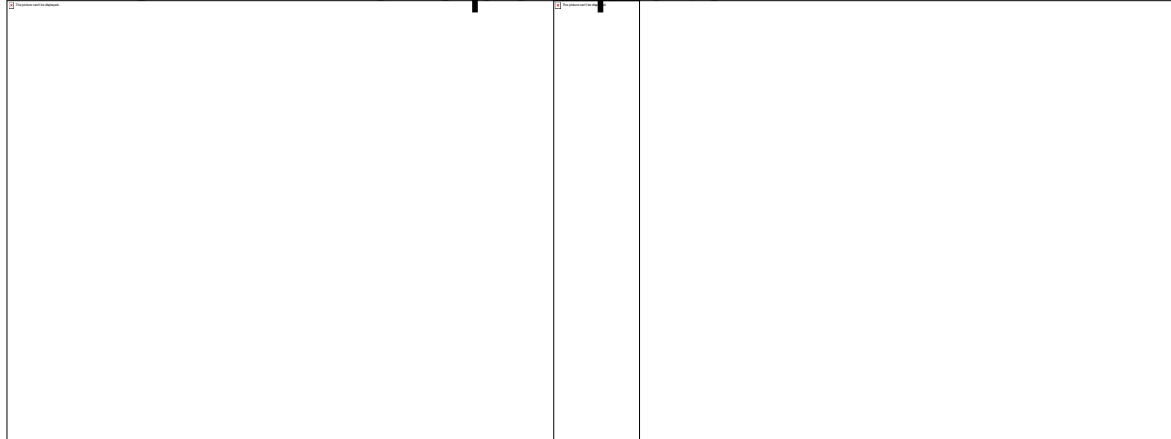
CFRP L-shaped plate (Sika® CarboShear L®)



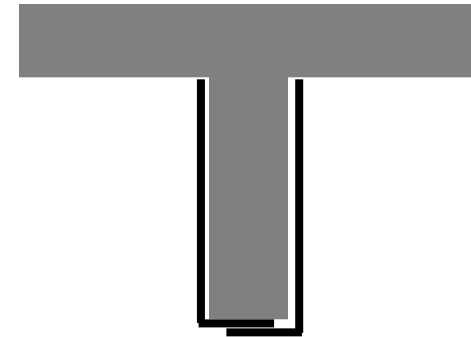
CFRP fabric (SikaWrap® Hex-230C)

Application

CFRP L-shaped plate



CFRP fabric



Referenzproject

Ponte Brogeda (Chiasso, Switzerland)

Photos from 



Externally Bonded FRP: Shear Strengthening

Fibre Composites, FS23

Masoud Motavalli

Referenzproject (3)

Abutment of the Duttweiler-bridge, Zurich

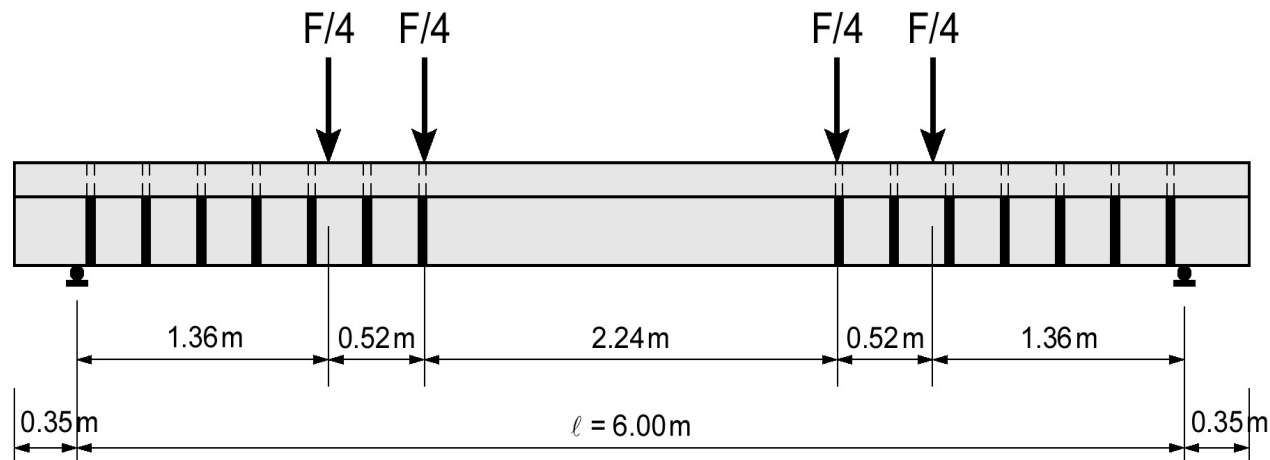


Externally Bonded FRP: Shear Strengthening

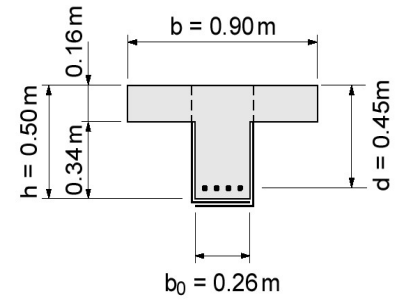
Fibre Composites, FS23

Masoud Motavalli

Beam T1

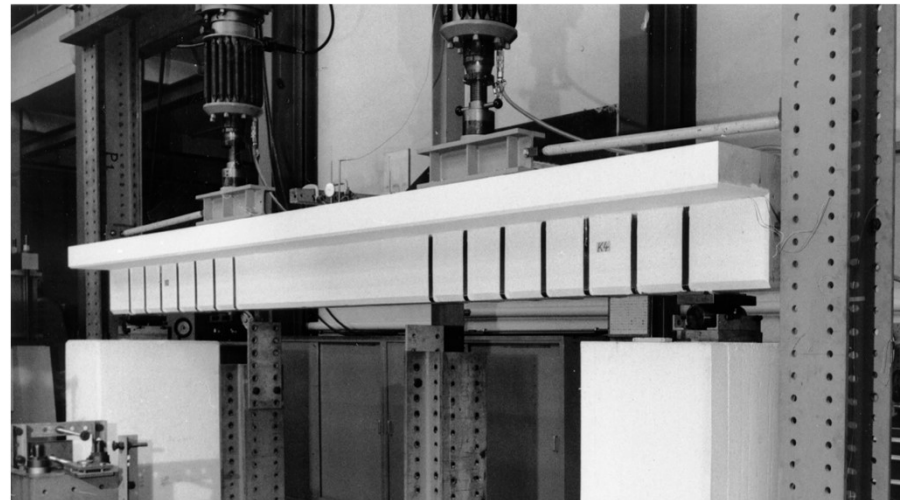


Cross-Section

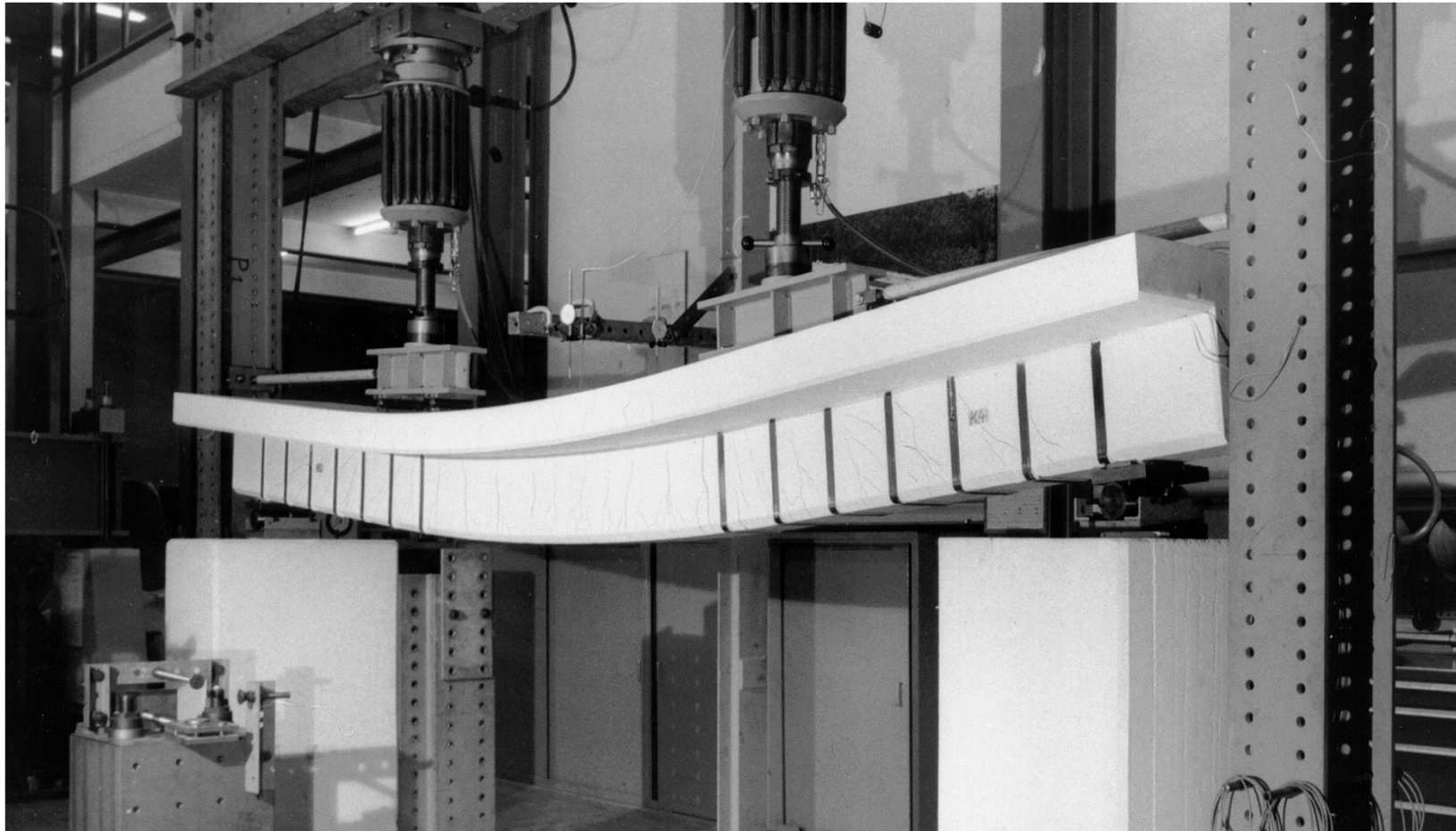


$$l/h = 12:1$$

$$a/d = 3.6$$



Beam T1 after Test

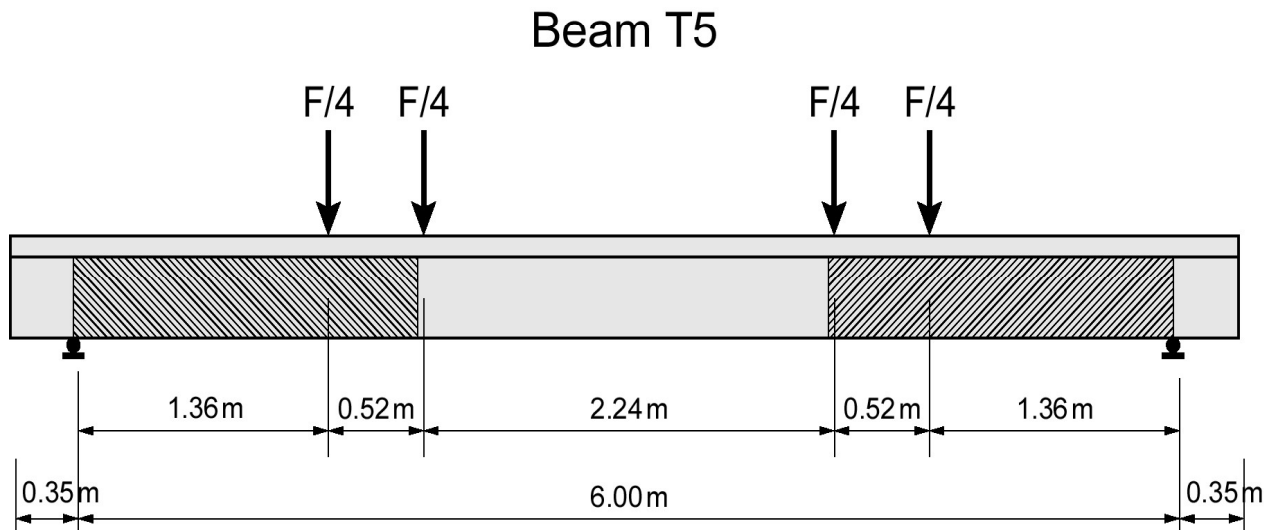
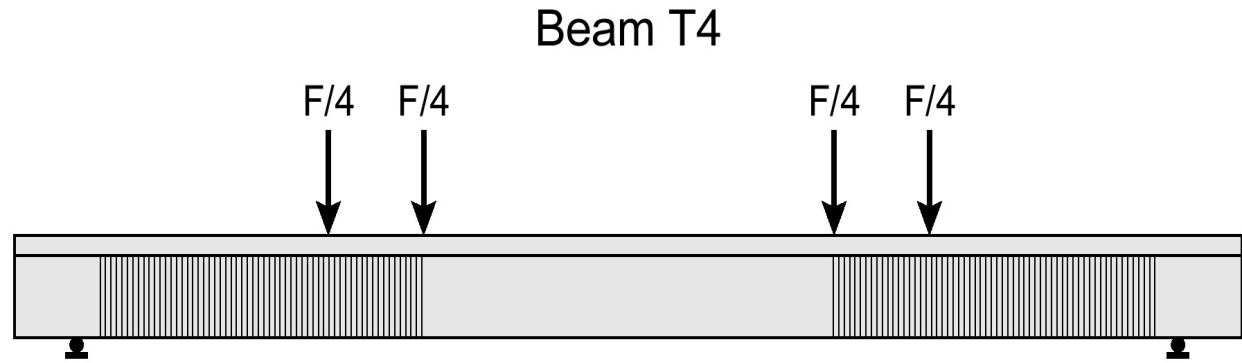


Externally Bonded FRP: Shear Strengthening

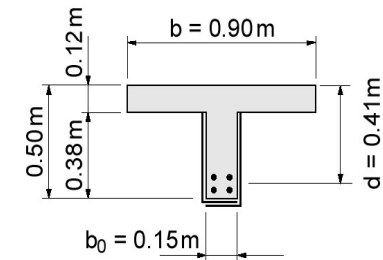
Fibre Composites, FS23

Masoud Motavalli

Strengthening with CFRP Fabric



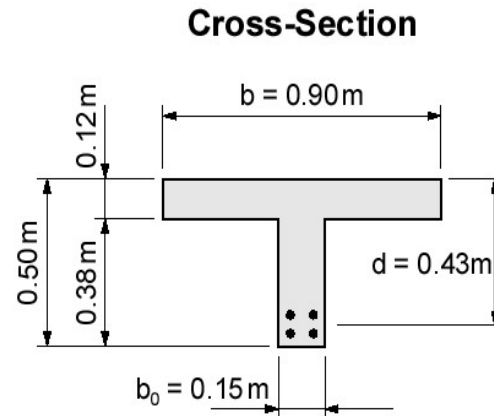
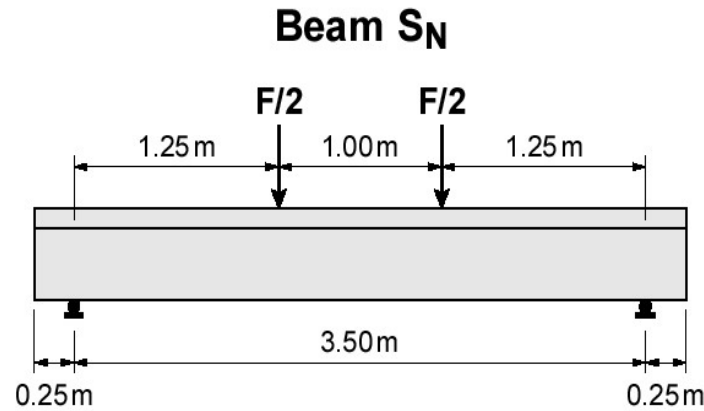
Cross-Section



Debonding of CFRP-Fabric



Systematic Test Program

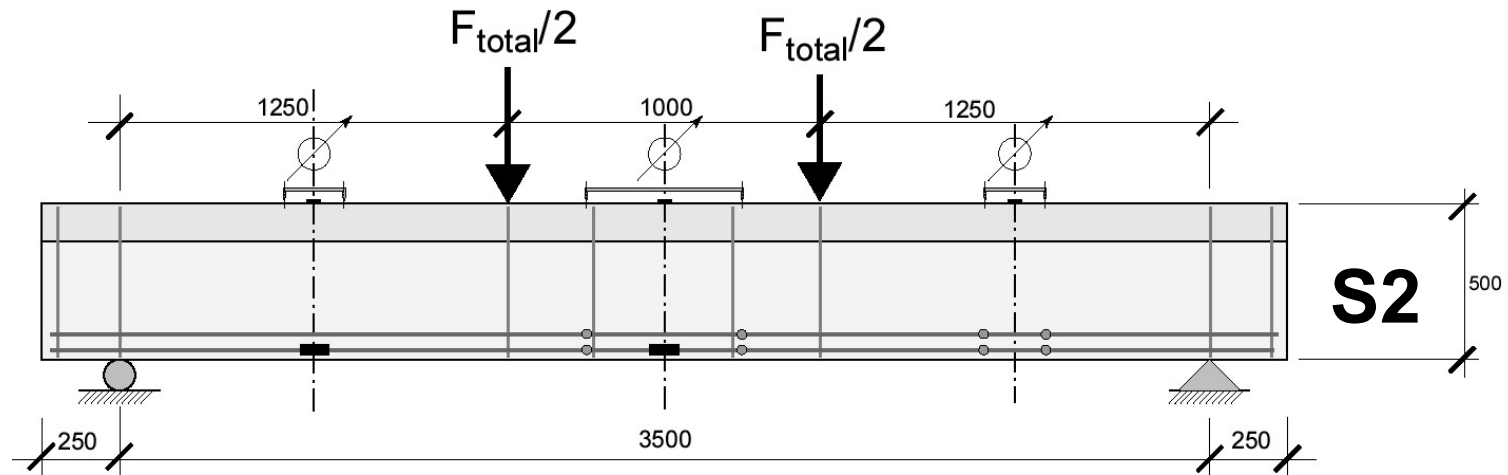
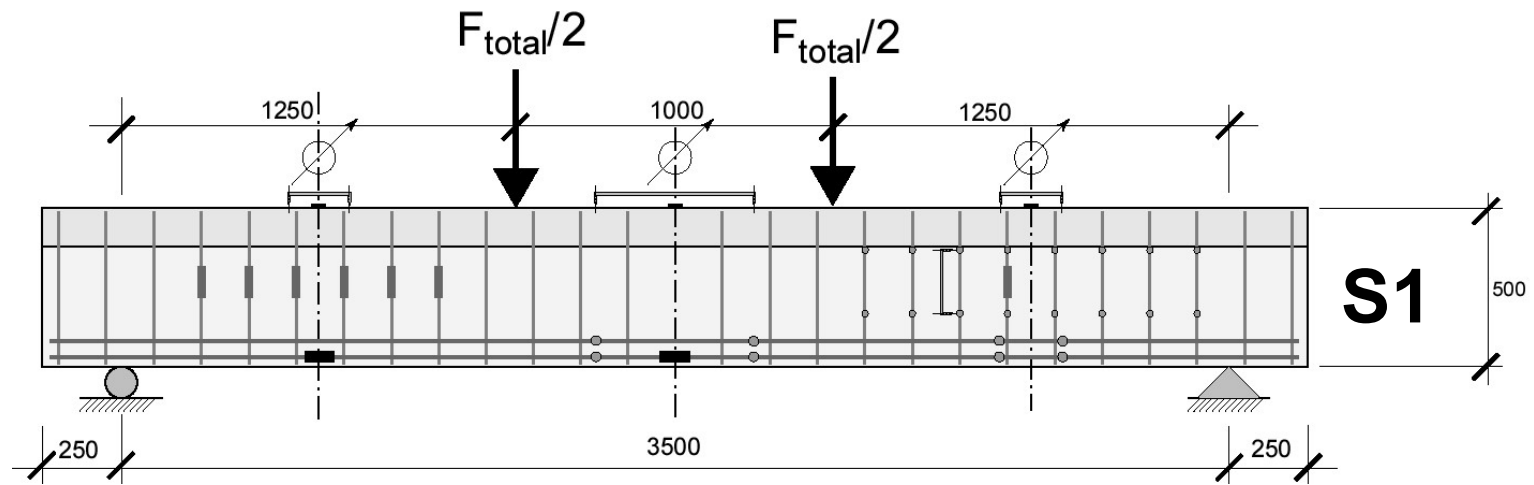


$$a/d = 2.9$$

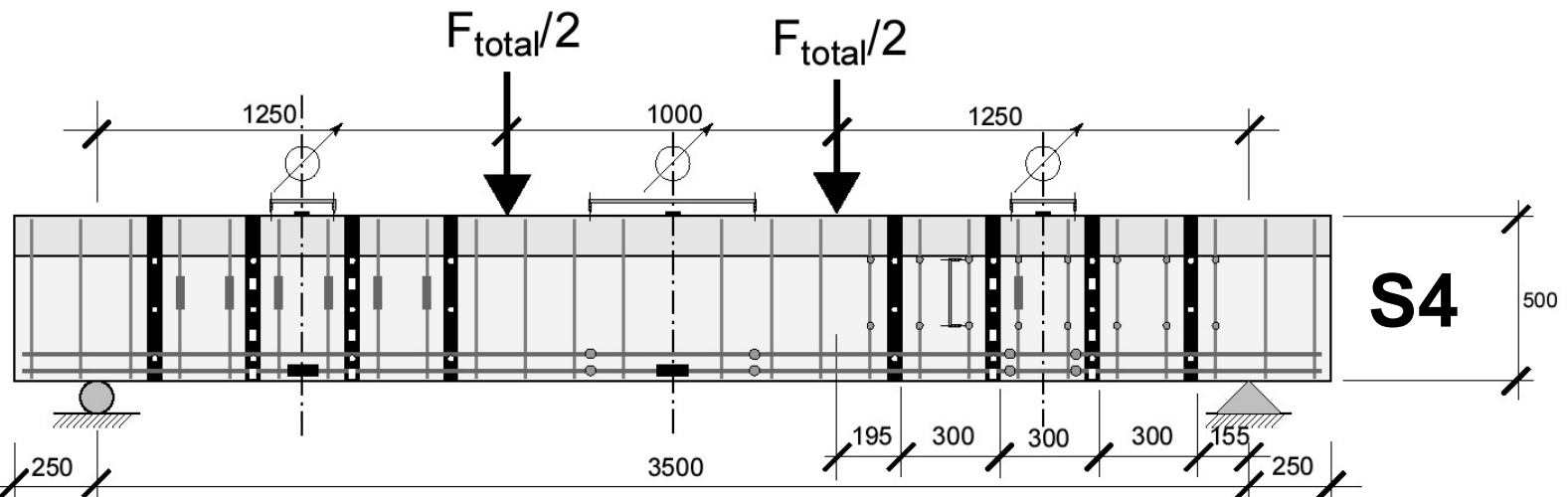
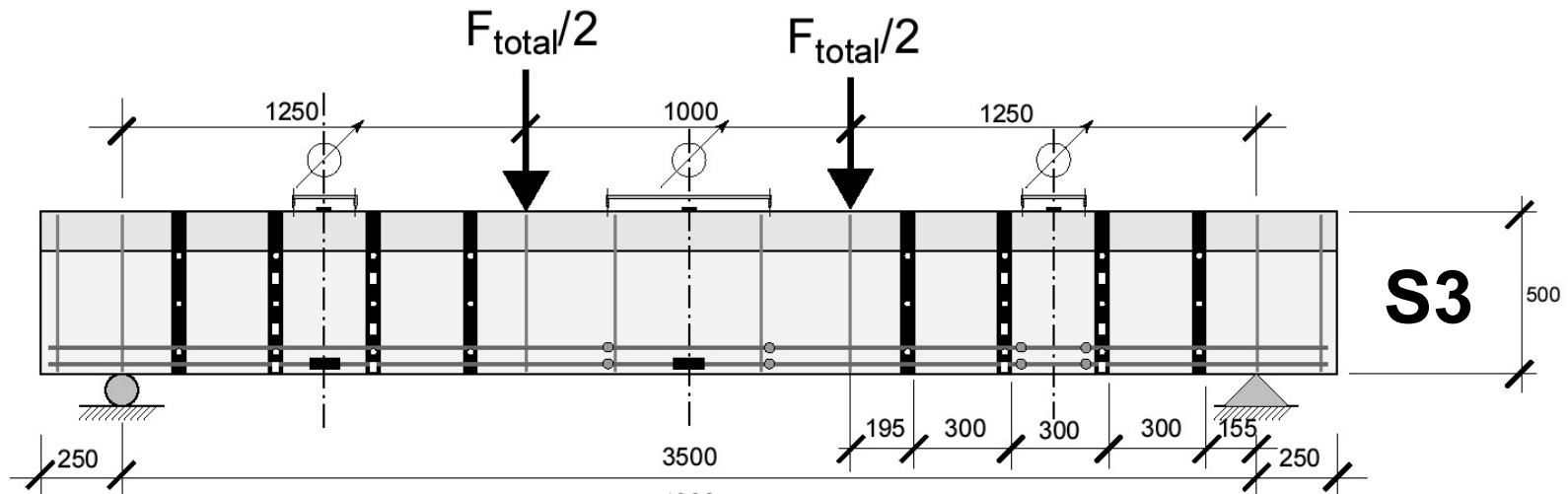
$$b/b_0 = 6.0$$

	Typ of Test	Internal Reinforcement	External Reinforcement
S1	Static Loading	ø8 s = 150 mm	Without
S2	Static Loading	Without	Without
S3	Static Loading	Without	CFRP L- Plates s = 300 mm
S4	Static Loading	ø8 s = 150 mm	CFRP L- Plates s = 300 mm
S5	Pre Loading	ø8 s = 150 mm	CFRP L- Plates s = 300 mm
S6	Fatigue	ø8 s = 150 mm	CFRP L- Plates s = 300 mm

Test Beams



Test Beams



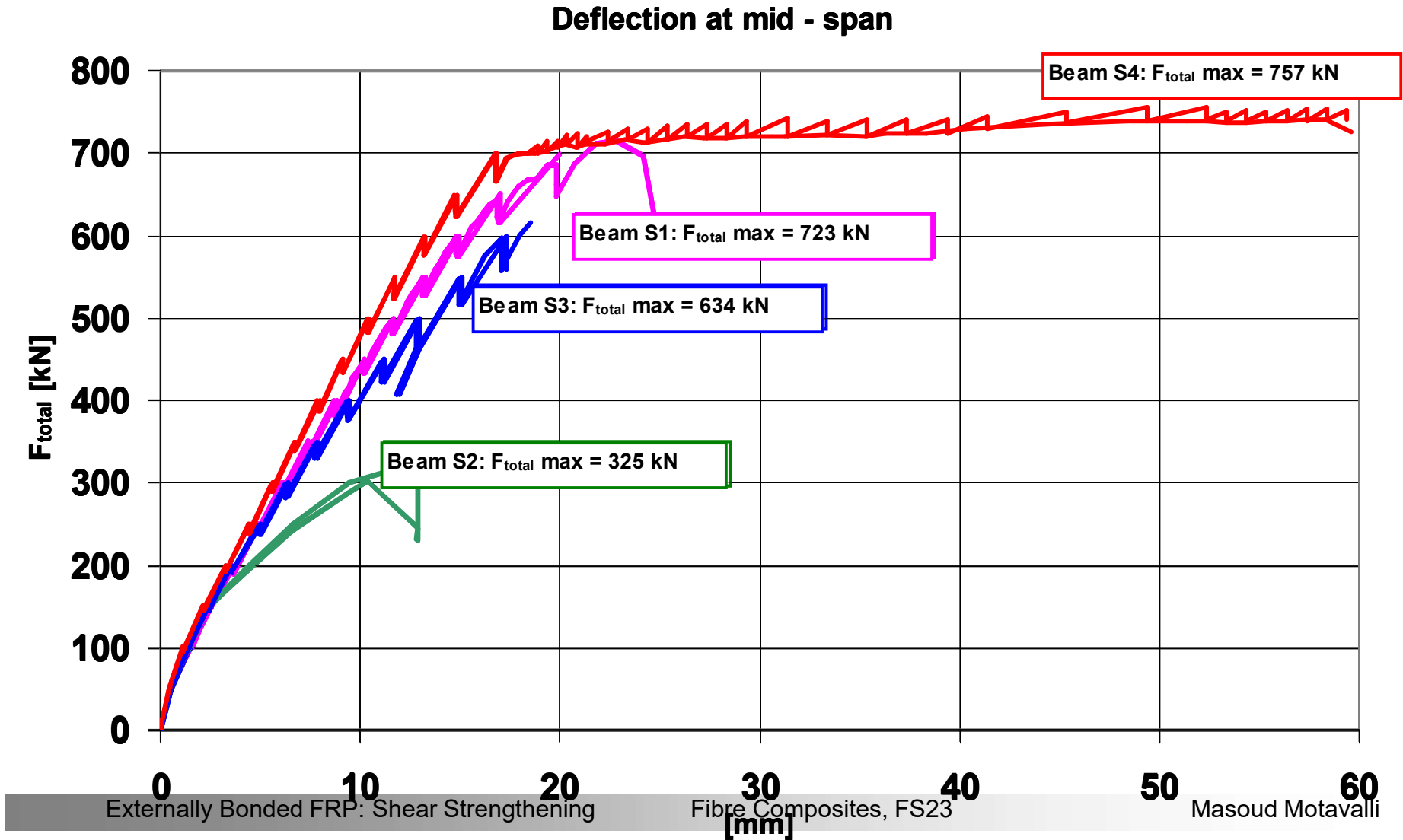
Externally Bonded FRP: Shear Strengthening

Fibre Composites, FS23

Masoud Motavalli

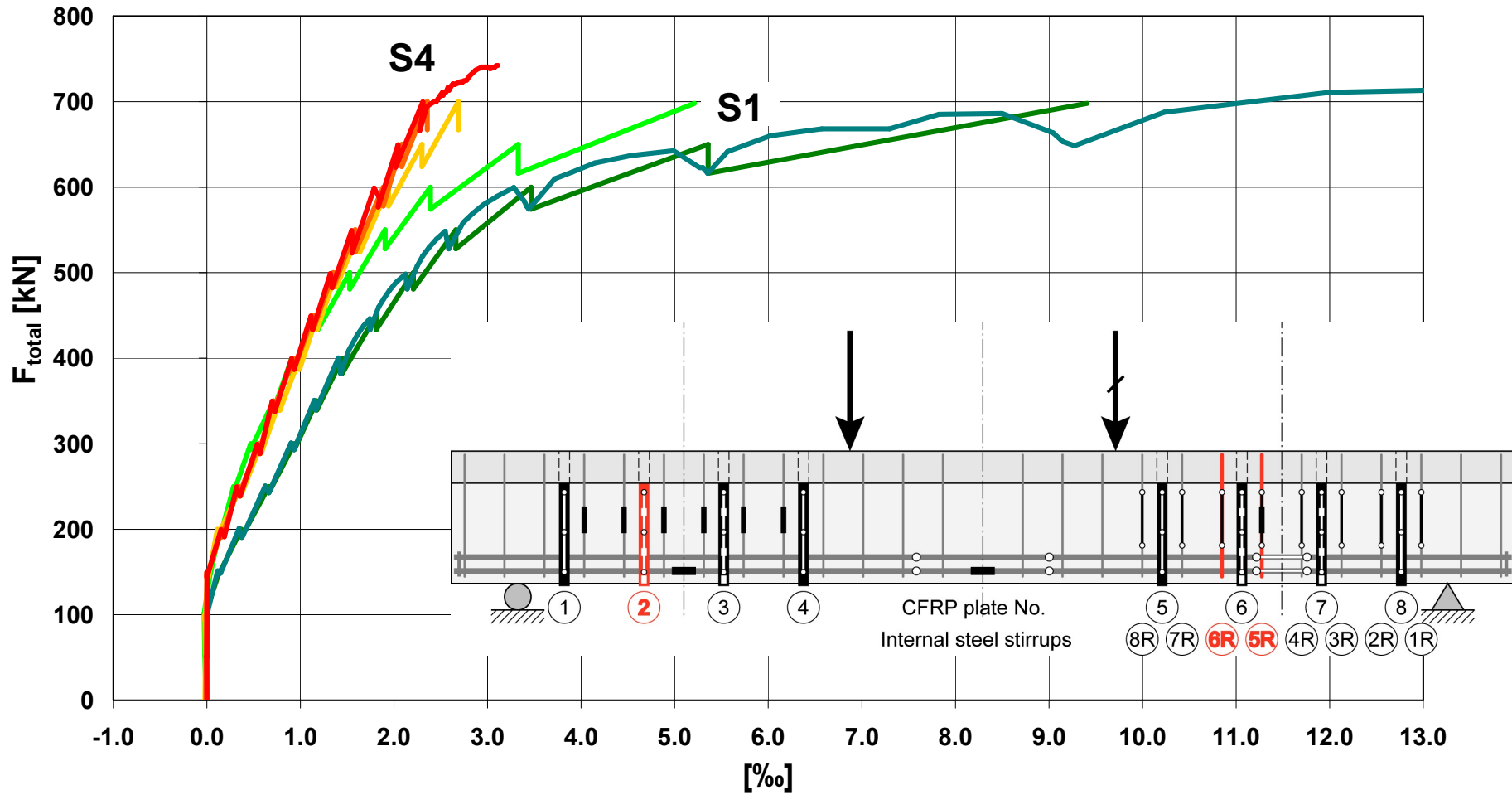


Deflection at mid-span

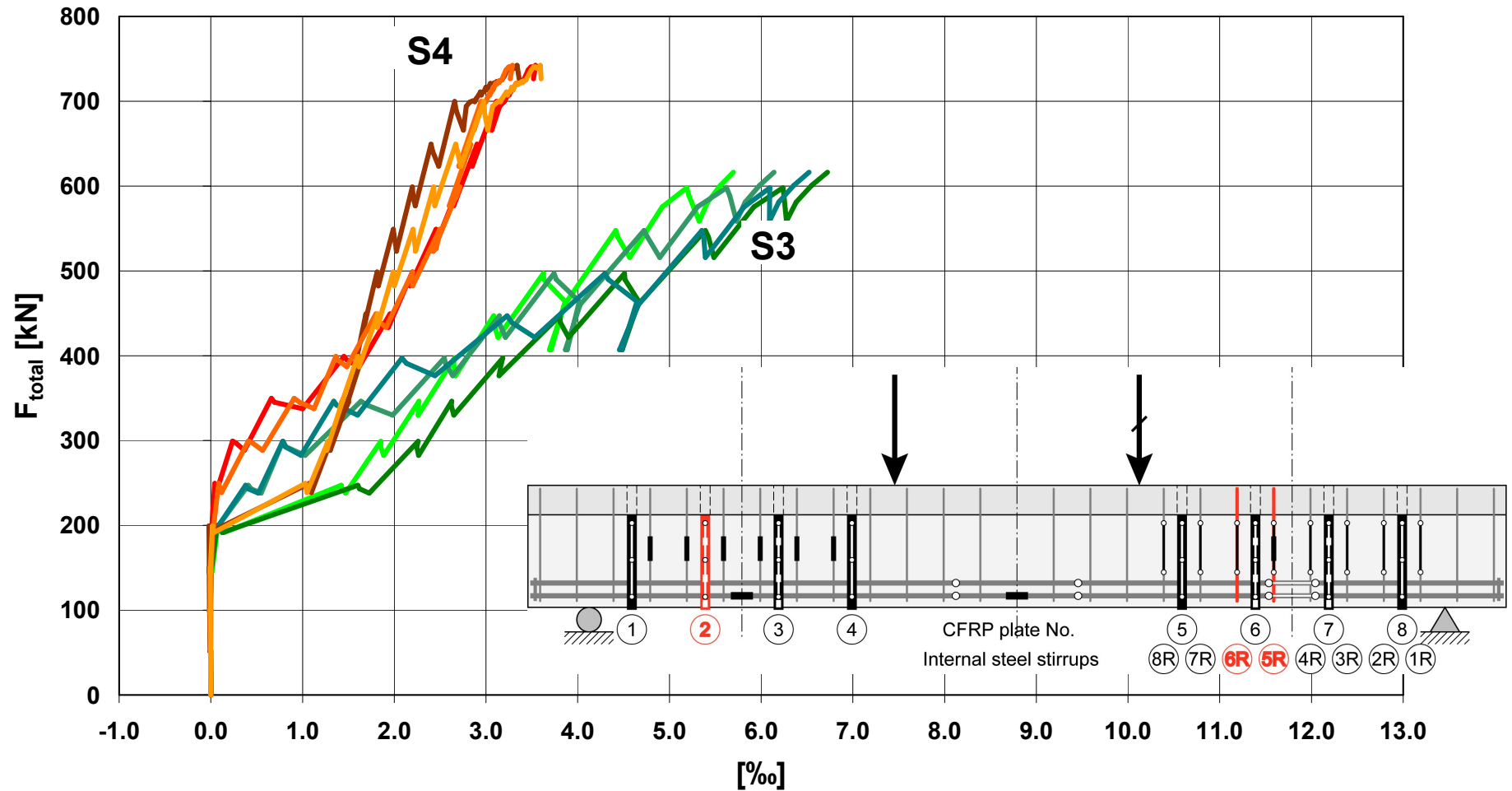


Strain in steel stirrup

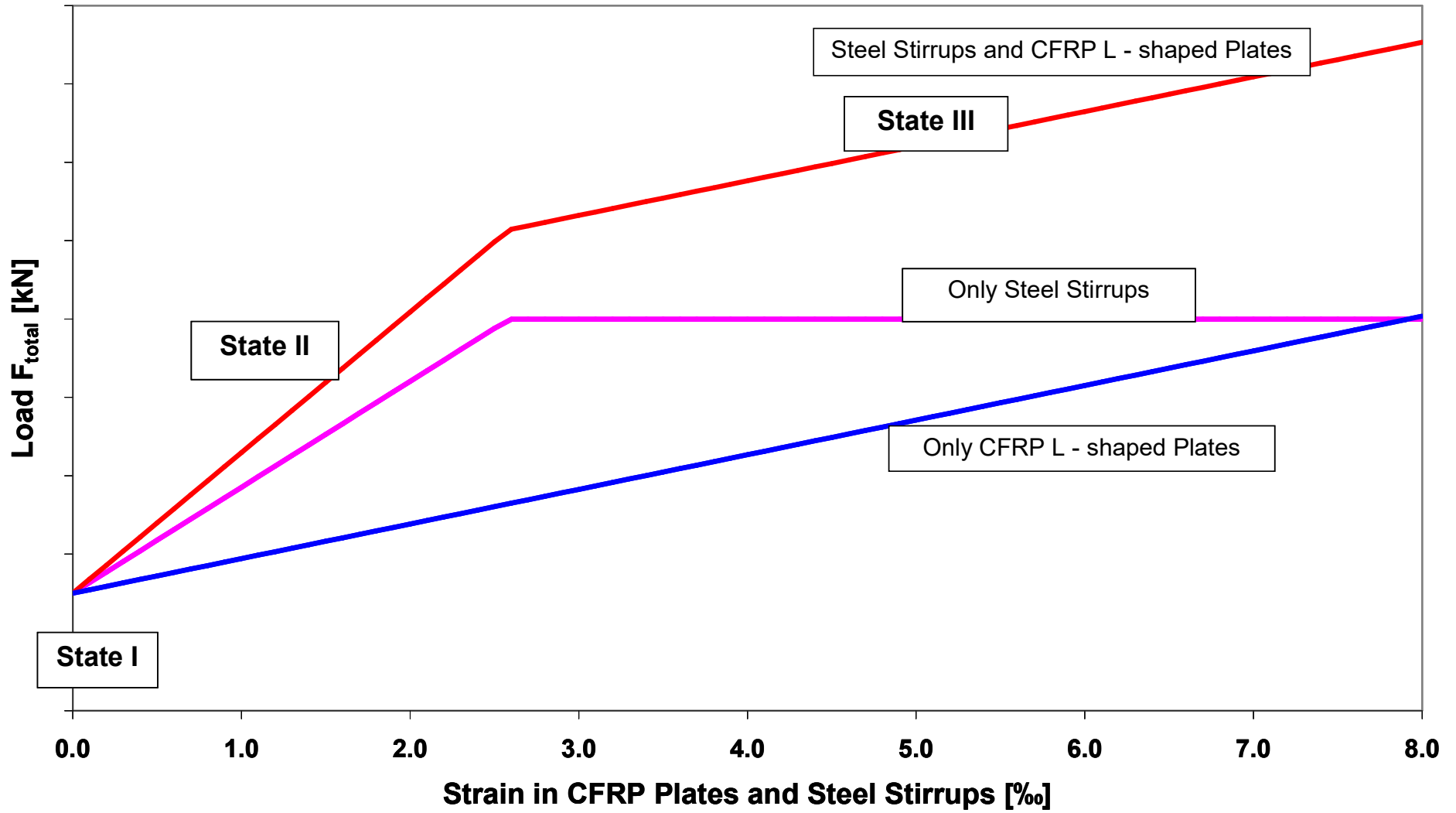
Tensile strain steel stirrups 6R and 5R



Strain in CFRP L-shaped plate



Design



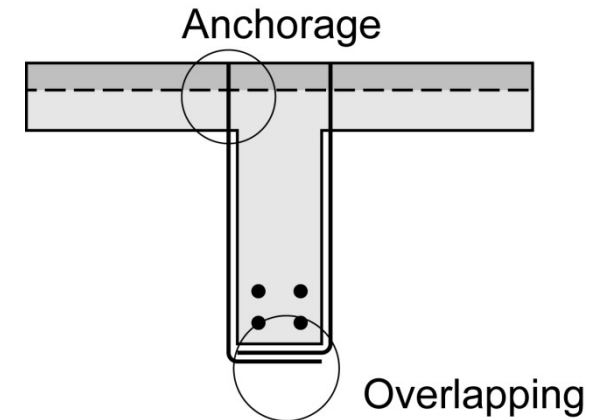
Design

$$V_R = V_C + V_S + V_F$$

- V_R = Shear resistance of RC member
- V_C = Concrete contribution (first shear crack)
- V_S = Contribution of internal steel stirrups
- V_F = Contribution of external CFRP L-shaped plates

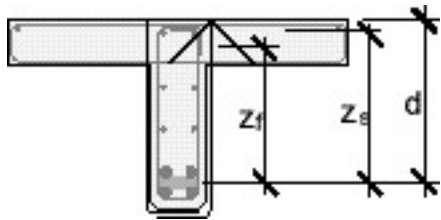
Contribution of CFRP L-shaped plates

- Ultimate limit state (ULS)
 - Failure mode “opening of the overlapping”
 - Failure of the anchorage
- Serviceability limit state (SLS)
- Analysis of unstrengthened section
 - After plate failure remaining safety factor > 1



Failure of anchorage

- Should be prevented if:
 - Anchor length \rightarrow whole height of flange (if possible)



- Carefully filling of the anchorage holes
- Anchor length > 100 mm (see reference [2])

Design equations for Sika[®] CarboShear L[®]

Equations without any safety factors!

Verification ULS:

$$V_R = A_s \cdot f_y \cdot \frac{Z_s}{S_s} \cdot \cot \alpha + F_f \cdot \frac{Z_f}{S_f} \cdot \cot \alpha \quad \text{with } \alpha \geq 45^\circ$$

Verification SLS:

$$V_{ser} = V_C + A_s \cdot 0.8 \cdot f_y \cdot \frac{Z_s}{S_s} \cdot \cot \alpha + A_f \cdot \left(0.8 \cdot \frac{f_y}{E_s}\right) \cdot E_f \cdot \frac{Z_f}{S_f} \cdot \cot \alpha \quad \text{with } \alpha \geq 45^\circ$$

Verification accidental situation:

$$V_{acc} = V_C + A_s \cdot f_y \cdot \frac{Z_s}{S_s} \cdot \cot \alpha$$

Shear design

All the usual design verifications for RC (failure of the concrete struts, shift of moment line, etc.) have to be considered. For ductility reasons, the member should have a minimum internal shear reinforcement ratio, otherwise a strengthening is not recommended.

Fatigue

- See design concept in reference [3]

References

- [1] Czaderski, C., *Nachträgliche Schubverstärkung mit CFK-Winkeln*. Schweizer Ingenieur und Architekt SI+A, 1998(43): p. Seite 822-826.
- [2] Meier, H., *CFK-Schubverstärkungselemente*. Schweizer Ingenieur und Architekt SI+A, 1998(43): p. Seite 819-821.
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"List of Symbols"

< shear strengthening >

S_f : FRP spacing, or (S_{FRP})

b_f : FRP width, or (W_{FRP})

θ : diagonal crack angle, assumed to be 45°

α : fibre direction, or (β)

$\epsilon_{fd,e}$: design value of effective FRP strain

V_{Rd} : design value of shear force capacity of the cross-section

V_{cd} : design value of shear force capacity concrete contribution

V_{wd} or (V_s) : " " " " steel "

V_{fd} or (V_{FRP}) : " " " " FRP "

T_{Fd} : Contribution of FRP to torsional capacity